Light rings and their topological charges

Pedro V.P. Cunha

University of Aveiro
Department of Mathematics - CIDMA

Several work in collaboration with: C. Herdeiro, E. Radu, E. Berti, N. Sanchis-Gual

What is the nature of *astrophysical* Black Holes?

What is the nature of *astrophysical* Black Holes?

Are they described by the paradigmatic Kerr solution?

What is the nature of *astrophysical* Black Holes?

Are they described by the paradigmatic Kerr solution?

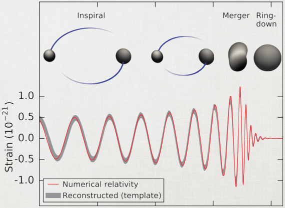
$\cdots$ or could models beyond Kerr *mimic* its phenomenology?

Strong gravity has entered the precision era:

- breakthroughs in gravitational wave astrophysics. LIGO/Virgo, PRL 116, 061102 (2016)
- unveiling of the first black hole (BH) shadow image. EHT, AJ 875 L1 (2019)

observed image M87*

EHT, AJ 875 L1 (2019)



GW ringdown after BH merger

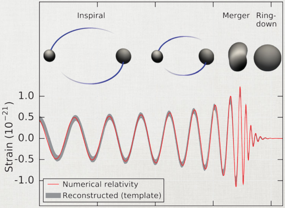
LIGO/Virgo, PRL 116, 061102 (2016)

Strong gravity has entered the precision era:

- breakthroughs in gravitational wave astrophysics. LIGO/Virgo, PRL 116, 061102 (2016)
- unveiling of the first black hole (BH) shadow image. EHT, AJ 875 L1 (2019)

observed image M87*

EHT, AJ 875 L1 (2019)



GW ringdown after BH merger

LIGO/Virgo, PRL 116, 061102 (2016)

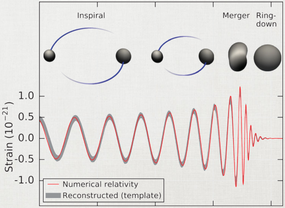
These observations can be used to test the true nature of black Holes (**BHs**).

Strong gravity has entered the precision era:

- breakthroughs in gravitational wave astrophysics. LIGO/Virgo, PRL 116, 061102 (2016)
- unveiling of the first black hole (BH) shadow image. EHT, AJ 875 L1 (2019)

observed image M87*

EHT, AJ 875 L1 (2019)



GW ringdown after BH merger

LIGO/Virgo, PRL 116, 061102 (2016)

These observations can be used to test the true nature of black Holes (**BHs**).

Some key signatures are connected to a special set of bound null orbits: *Light Rings* (**LRs**).

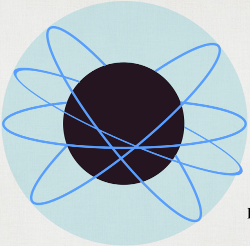
Plan of the seminar

1. Why Light Rings (**LRs**) are relevant for observations.
2. Topological charge of critical points
3. LRs around horizonless compact objects
4. LRs around Black Holes (**BHs**)

Plan of the seminar

1. Why Light Rings (**LRs**) are relevant for observations.
2. Topological charge of critical points
3. LRs around horizonless compact objects
4. LRs around Black Holes (**BHs**)

A Light Ring (**LR**) is a (spatially closed) circular null geodesic orbit.

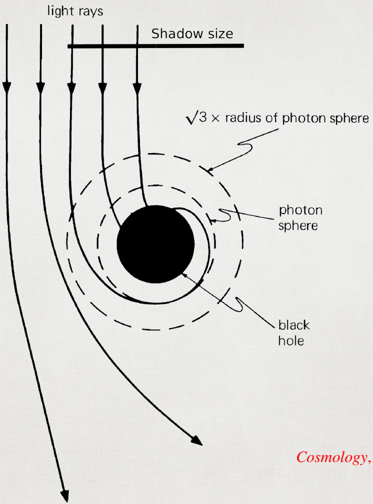


Photon Sphere as a collection of LRs

In spherically symmetry, the clustering of LRs forms a *Photon Sphere*.

LRs exist around Schwarzschild and Kerr BHs and very compact *horizonless* stars.

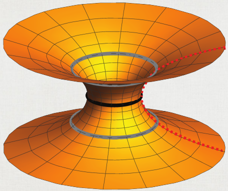
Example 1: the scattering of light rays around a Schwarzschild BH:



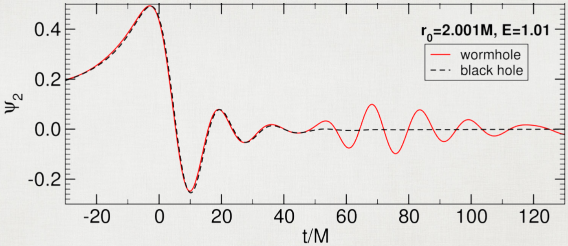
Cosmology, E. Harrisson

- Light rays need an impact parameter large enough to *escape* the BH.
- The shadow edge corresponds to rays that approach a *circular photon orbit*: a LR.

Example 2: GW ringdown as an horizon probe? Cardoso+ 2016 PRL**117**, 089902



(wormhole)

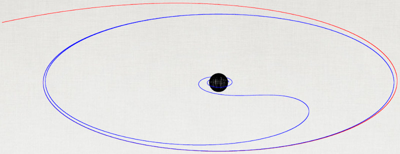


PRL**117**, 089902

- The GW ringdown typically contains a *signature* of the Light Ring (LR).

Goebel, Astro. Jour. **172** (1972)

- As a case study, a perturbed wormhole with a LR vibrates like a BH (initially).
- In principle, it could mimic a BH ringdown...



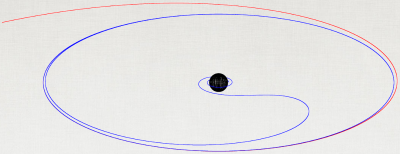
Definition:

Ultra-Compact Objects (UCO) $\iff$ any object with a LR (with or without an horizon).

Motivation:

LRs are closely connected to important astrophysical observables:

- Electromagnetic channel $\rightarrow$ BH shadow.
- GW channel $\rightarrow$ BH ringdown and Quasi-Normal modes.



Definition:

Ultra-Compact Objects (UCO) $\iff$ any object with a LR (with or without an horizon).

Motivation:

LRs are closely connected to important astrophysical observables:

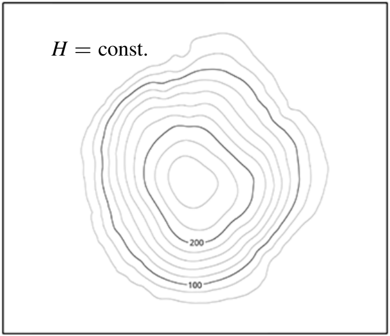
- Electromagnetic channel $\rightarrow$ BH shadow.
- GW channel $\rightarrow$ BH ringdown and Quasi-Normal modes.

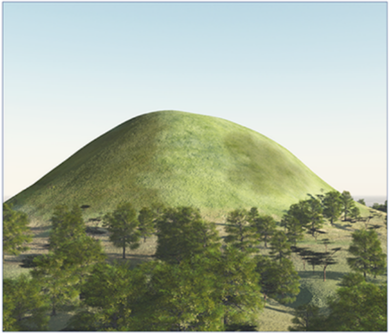
Interest of UCOs:

Hypothetical exotic UCOs might *mimic* Kerr phenomenology because of LRs.

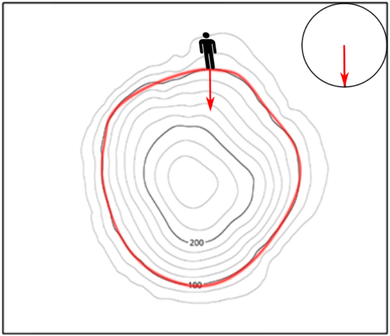
Plan of the seminar

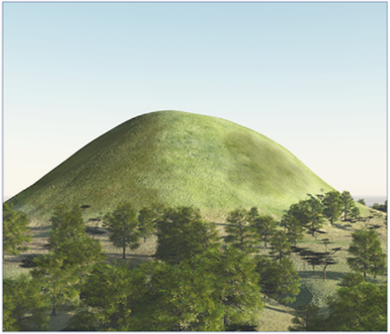
1. Why Light Rings (**LRs**) are relevant for observations.
2. Topological charge of critical points
3. LRs around horizonless compact objects
4. LRs around Black Holes (**BHs**)


$H = \text{const.}$
200
100

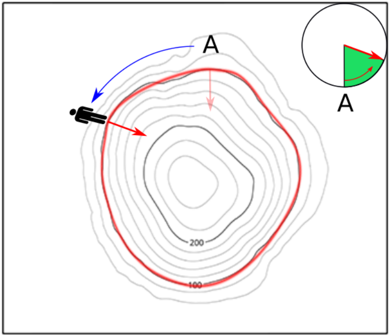


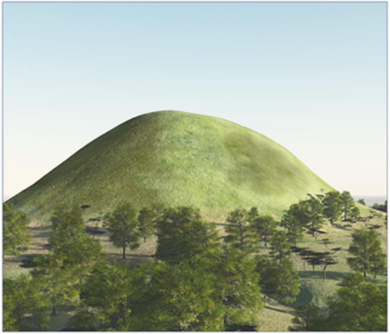
$\nearrow = \nabla H$



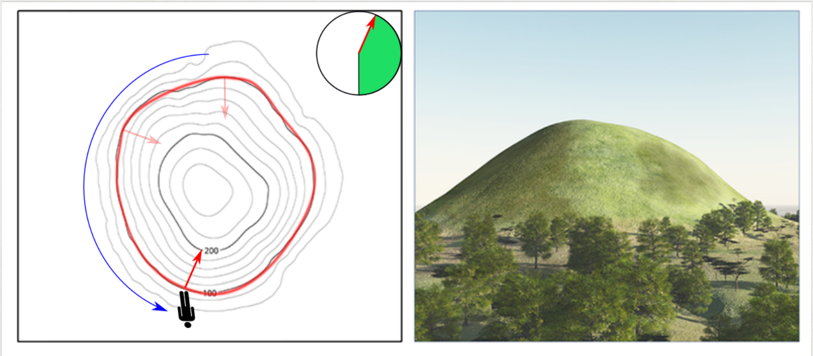


$$\color{red}{\nearrow} = \nabla H$$

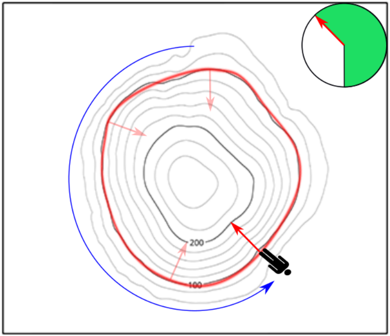


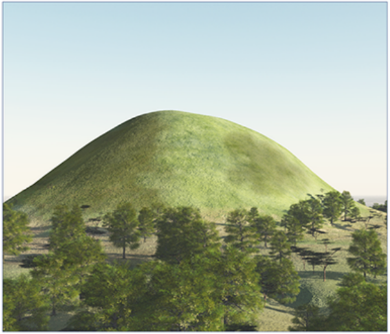


$\nearrow = \nabla H$

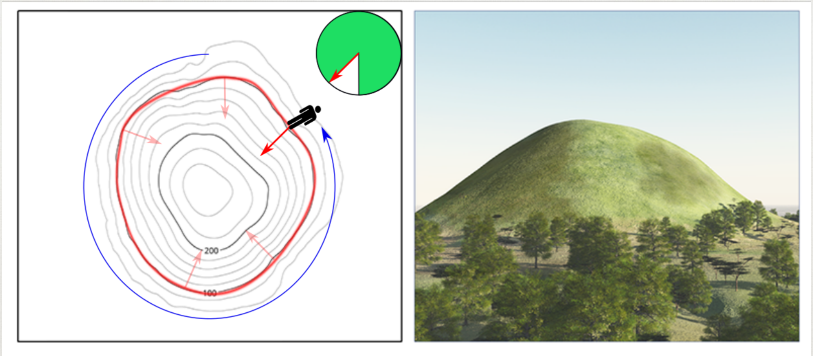


$\uparrow = \nabla H$

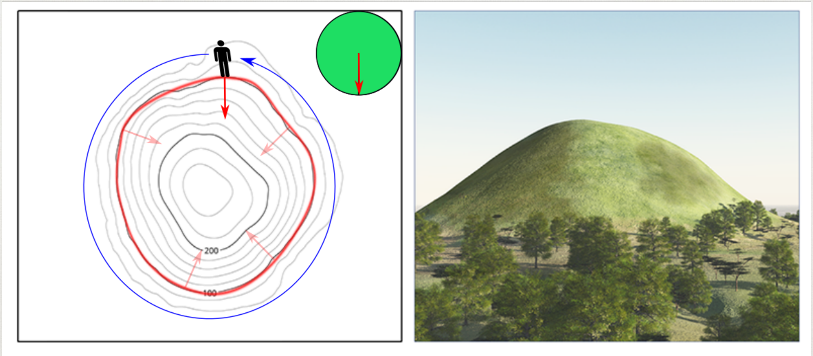




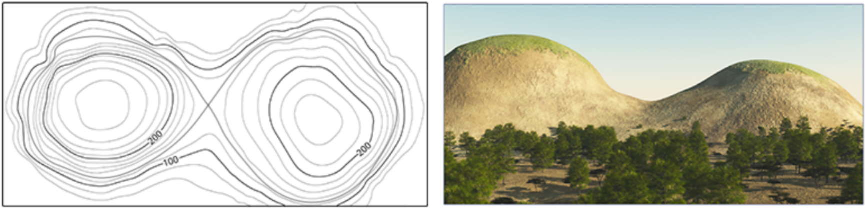
$\nearrow = \nabla H$



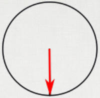
$\nearrow = \nabla H$

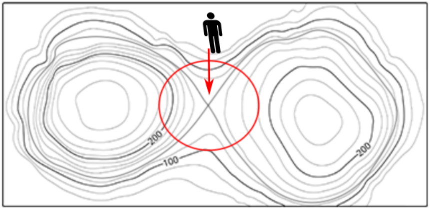


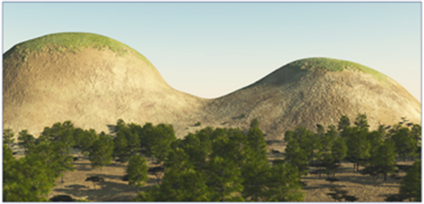
Rule 1 $\Longrightarrow$ a Maximum (or Min.) leads to $(+\mathbf{1})$ full turns of vector field.


200
100
200

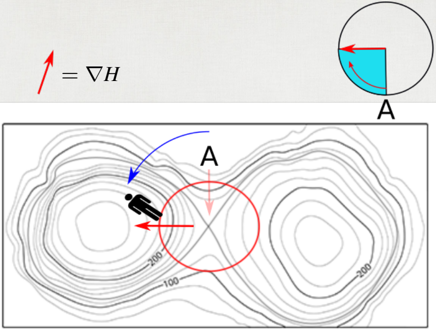
$\nearrow = \nabla H$

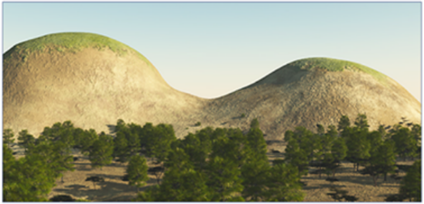




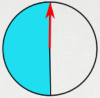


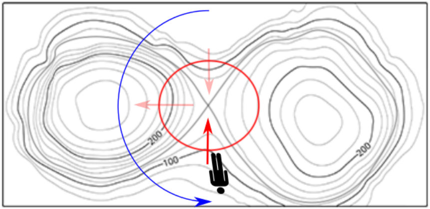
$= \nabla H$

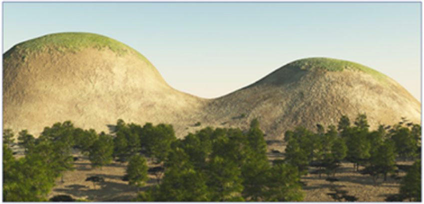




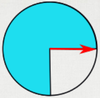
 $= \nabla H$

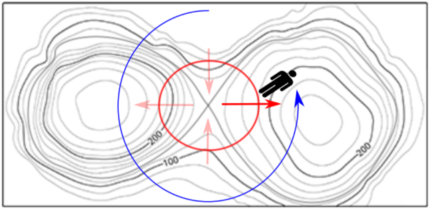


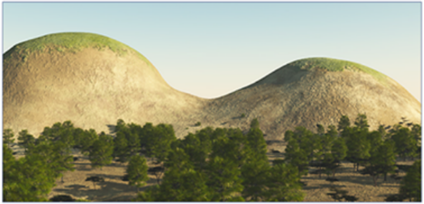




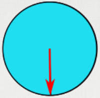
 $= \nabla H$

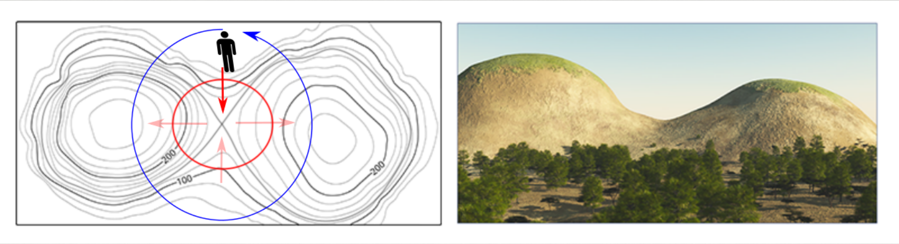






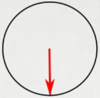
$= \nabla H$

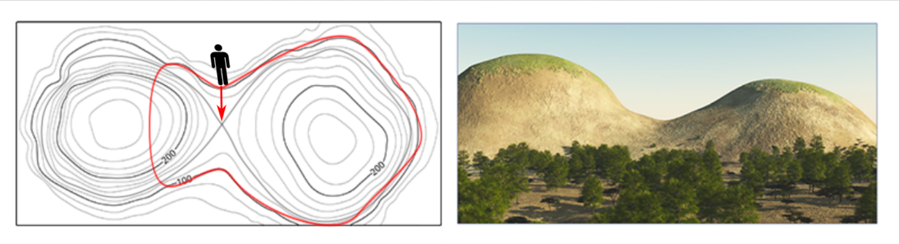




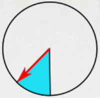
Rule 2 $\Longrightarrow$ Saddle point leads to $(-\mathbf{1})$ full turns of vector field ($i.e.$ inverse sense).

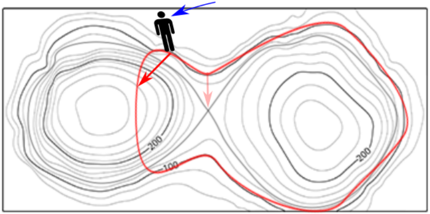
 $= \nabla H$

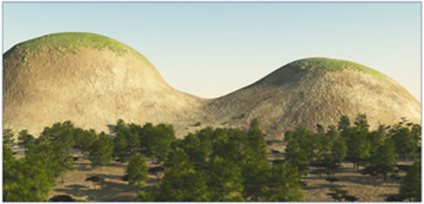




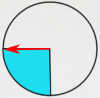
$\nearrow = \nabla H$

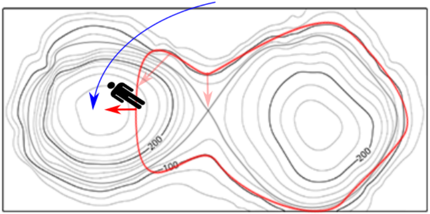


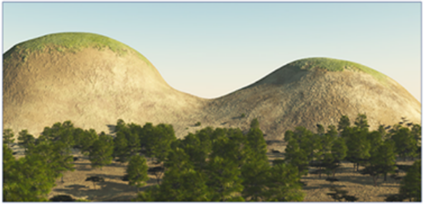




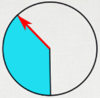
$= \nabla H$

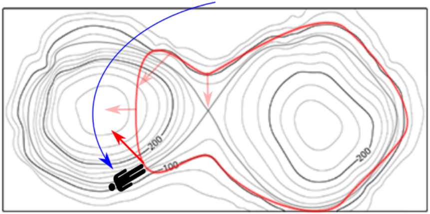


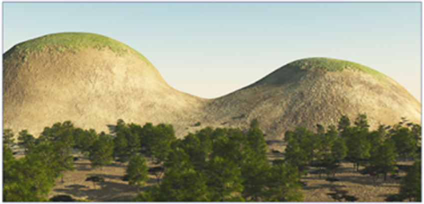




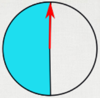
 $= \nabla H$

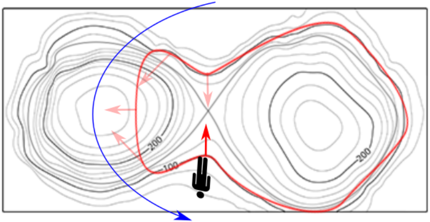


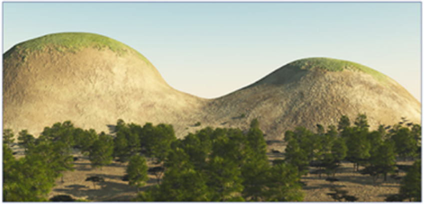




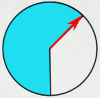
 $= \nabla H$

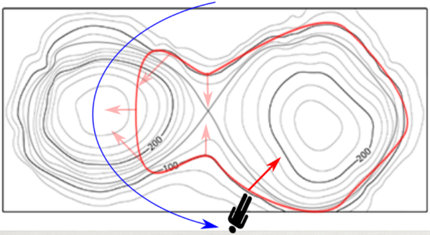


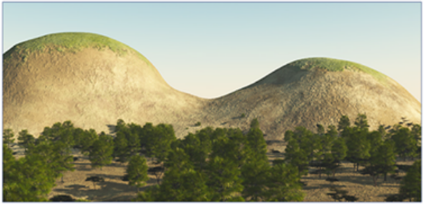


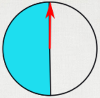


 $= \nabla H$

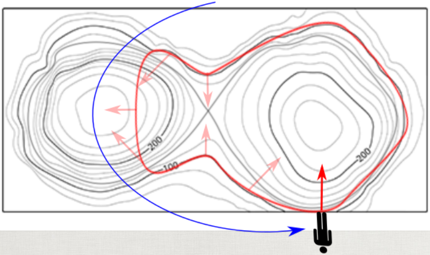


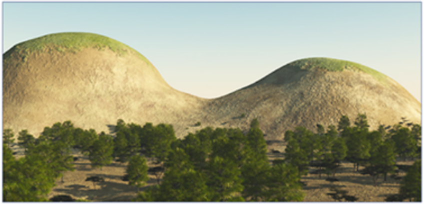




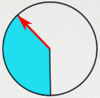


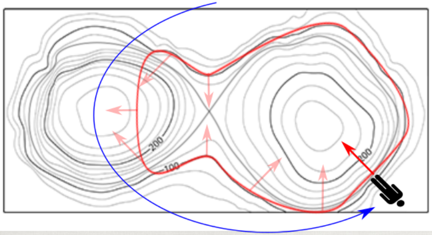
 $= \nabla H$

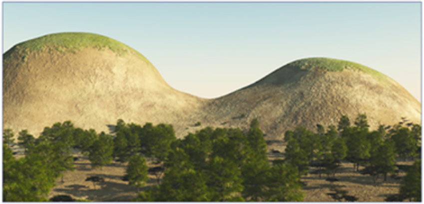




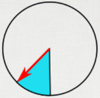
 $= \nabla H$

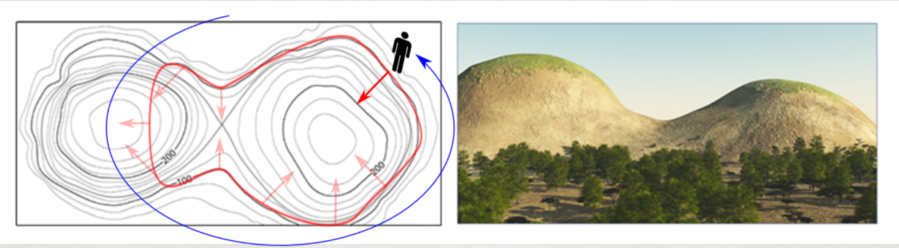




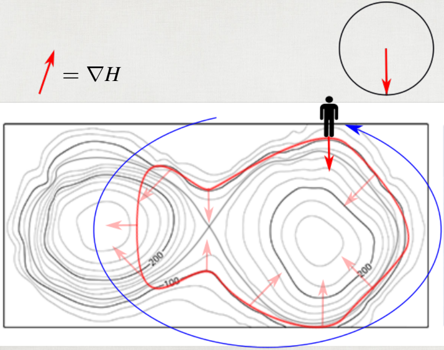


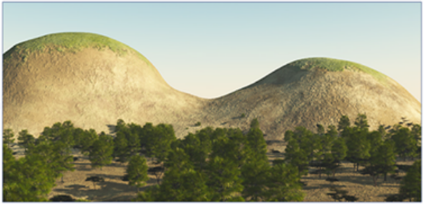
 $= \nabla H$



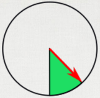


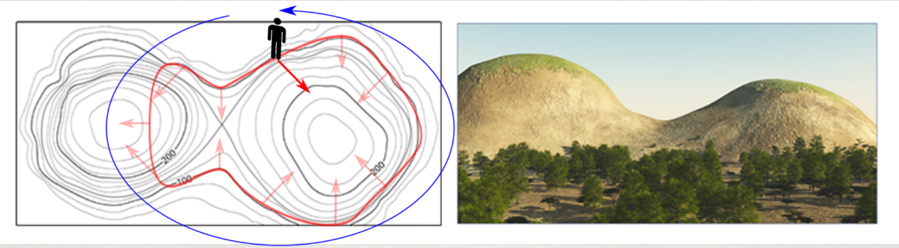
$= \nabla H$



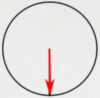


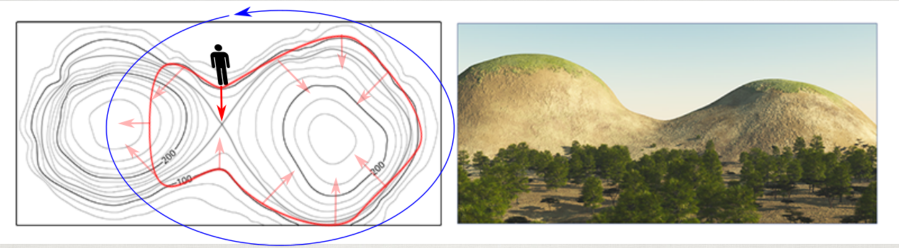
$= \nabla H$





$= \nabla H$





Rule 3 $\Longrightarrow$ number of full turns is additive, *e.g.* Saddle point (-1) + Max (+1) = 0.

Some more details on why we can associate a *topological charge* to LRs:

Some more details on why we can associate a *topological charge* to LRs:

- 1º) Introduce a field $\mathbf{v} = (v_r, v_\theta)$, as a normalized gradient of $H_\pm$:

$$v_r \equiv \frac{\partial_r H_\pm}{\sqrt{g_{rr}}} \qquad v_\theta \equiv \frac{\partial_\theta H_\pm}{\sqrt{g_{\theta\theta}}}$$

$v^2 \equiv v_r^2 + v_\theta^2 = (\nabla H_\pm)^2$. In terms of $\mathbf{v}$, a LR occurs if and only if $\mathbf{v} = 0 \iff v = 0$.

Some more details on why we can associate a *topological charge* to LRs:

- 1º) Introduce a field $\mathbf{v} = (v_r, v_\theta)$, as a normalized gradient of $H_\pm$:

$$v_r \equiv \frac{\partial_r H_\pm}{\sqrt{g_{rr}}} \qquad v_\theta \equiv \frac{\partial_\theta H_\pm}{\sqrt{g_{\theta\theta}}}$$

$v^2 \equiv v_r^2 + v_\theta^2 = (\nabla H_\pm)^2$. In terms of $\mathbf{v}$, a LR occurs if and only if $\mathbf{v} = 0 \iff v = 0$.

- 2º) We define an angle Ω such that:

$$v_r = v\cos\Omega, \qquad v_\theta = v\sin\Omega.$$

Both $\{\Omega, v\}$ together parameterize an auxiliary 2-space $\mathcal{V}$, spanned by $\mathbf{v}$.

Some more details on why we can associate a *topological charge* to LRs:

- 1º) Introduce a field $\mathbf{v} = (v_r, v_\theta)$, as a normalized gradient of $H_\pm$:

$$v_r \equiv \frac{\partial_r H_\pm}{\sqrt{g_{rr}}} \qquad v_\theta \equiv \frac{\partial_\theta H_\pm}{\sqrt{g_{\theta\theta}}}$$

$v^2 \equiv v_r^2 + v_\theta^2 = (\nabla H_\pm)^2$. In terms of $\mathbf{v}$, a LR occurs if and only if $\mathbf{v} = 0 \iff v = 0$.

- 2º) We define an angle Ω such that:

$$v_r = v\cos\Omega, \qquad v_\theta = v\sin\Omega.$$

Both $\{\Omega, v\}$ together parameterize an auxiliary 2-space $\mathcal{V}$, spanned by $\mathbf{v}$.

- 3º) In the space (r, θ) consider a simple closed curve C (piecewise smooth and positive oriented).

  After a full revolution the angle Ω must be the same, modulo 2π:

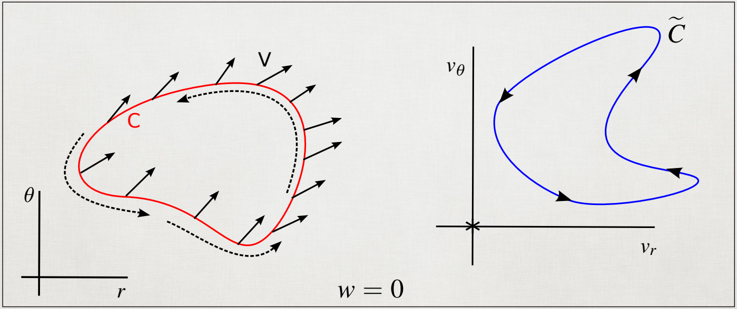
$$\oint_C d\Omega = 2\pi w, \qquad w \in \mathbb{Z}$$

.

This quantity w is a well defined topological quantity: *winding number*.

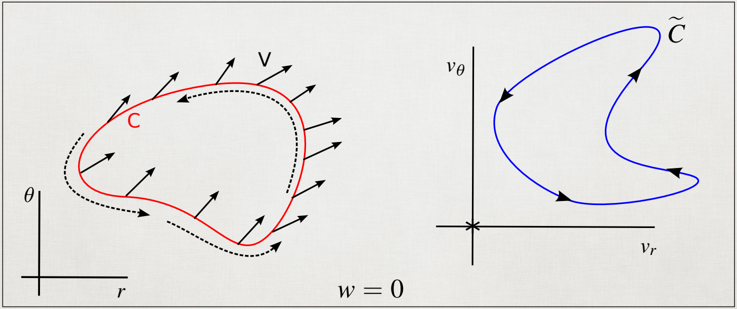
This quantity w is a well defined topological quantity: *winding number*.

Take a closed 2D contour $C(r, \theta)$ with the 2D field $\mathbf{v} = \{v_r, v_\theta\}$:



This quantity w is a well defined topological quantity: *winding number*.

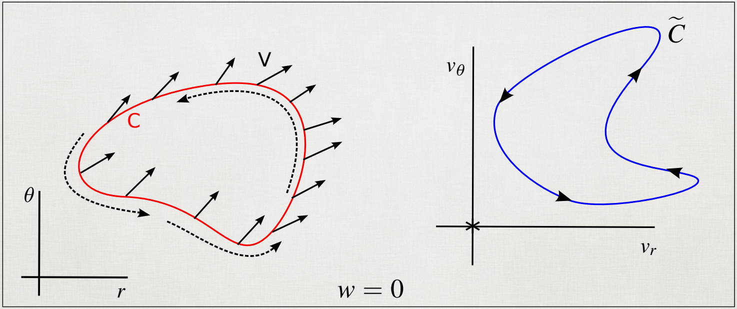
Take a closed 2D contour $C(r, \theta)$ with the 2D field $\mathbf{v} = \{v_r, v_\theta\}$:



The circulation of $\mathbf{v}$ around C is mapped to a curve $\widetilde{C}(v_r, v_\theta)$ in 2-space $\mathcal{V}$.

This quantity w is a well defined topological quantity: *winding number*.

Take a closed 2D contour $C(r, \theta)$ with the 2D field $\mathbf{v} = \{v_r, v_\theta\}$:

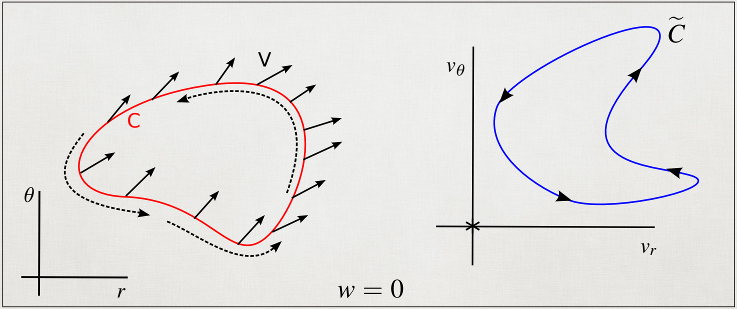


The circulation of $\mathbf{v}$ around C is mapped to a curve $\widetilde{C}(v_r, v_\theta)$ in 2-space $\mathcal{V}$.

The number of times $\widetilde{C}$ travels anti-clockwise around origin $(\mathbf{v} = 0)$ is the **winding number** w.

This quantity w is a well defined topological quantity: *winding number*.

Take a closed 2D contour $C(r, \theta)$ with the 2D field $\mathbf{v} = \{v_r, v_\theta\}$:

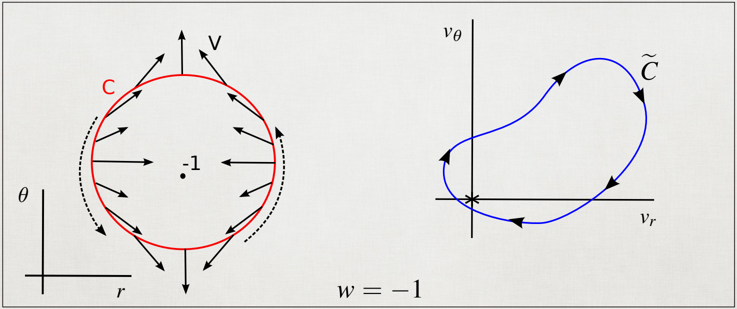


The circulation of $\mathbf{v}$ around C is mapped to a curve $\widetilde{C}(v_r, v_\theta)$ in 2-space $\mathcal{V}$.

The number of times $\widetilde{C}$ travels anti-clockwise around origin ($\mathbf{v} = 0$) is the **winding number** w.

Example shown: there are no LRs within $C \implies$ curve $\widetilde{C}$ does not enclose the origin ($\mathbf{v} = 0$)

Example 2: one “standard” LR (saddle-point type) within $C \implies w = -1$.



In this case $\widetilde{C}$ encircles the origin ($\mathbf{v} = 0$) in the **negative** sense.