Synchronized bosonic hair: equilibrium solutions

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JHEP **07** (2020) 010, *Phys. Lett. B* **824** (2022) 136835, arXiv:2207.10089 **[gr-qc]**

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SFRH/BD/143407/2019

Outline

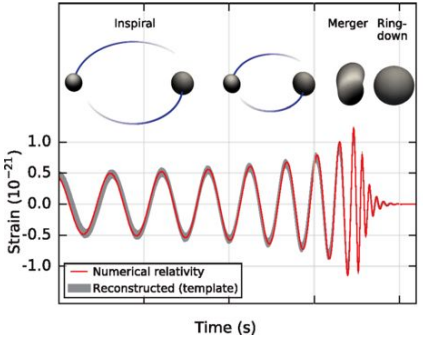
1. Motivation
2. Black holes with synchronized hair
   - Kerr-like and non-Kerr-like solutions
   - dynamical formation and a bound on the hairiness
   - thermodynamic stability
3. Summary and future work

Motivation

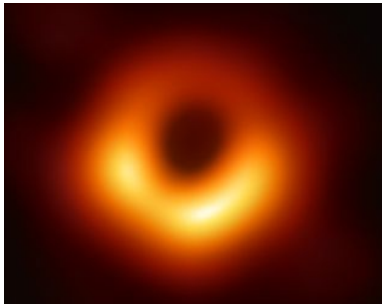
Kerr hypothesis

All **astrophysical black hole** candidates are described by the **Kerr metric**.

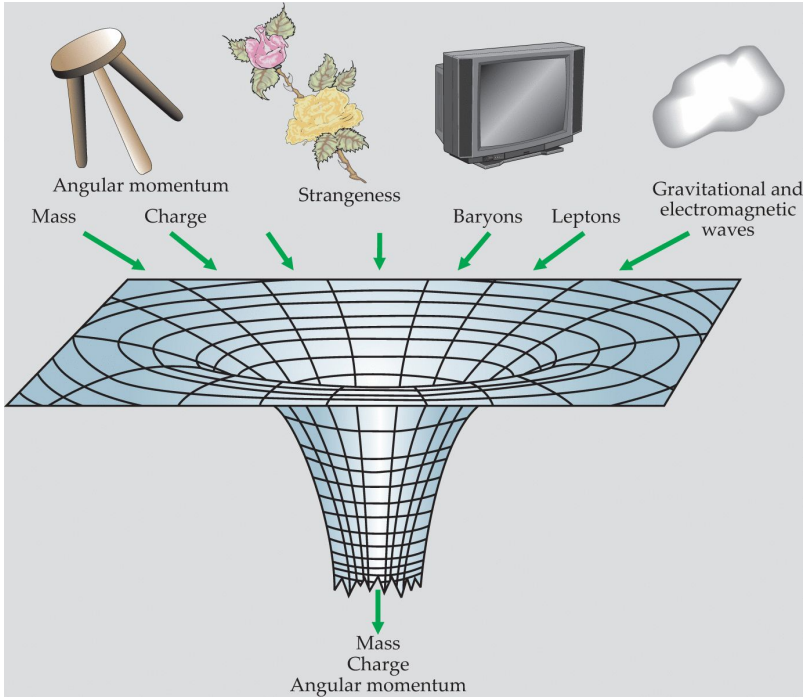
Are they?



LIGO/Virgo, *PRL* **116** (2016) 061102



EHT, *APL* **875** (2019) L1



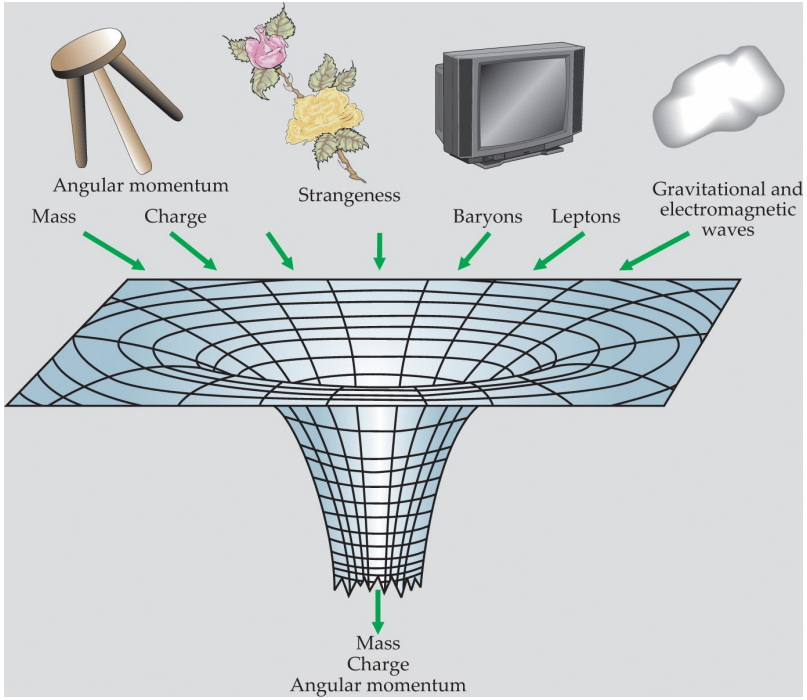
Ruffini & Wheeler, *Phys. Today* **24** (1971) 1, 30

Kerr hypothesis

Non-Kerr black holes

General relativity → Modify gravity

Particle Physics → Add new particle(s)



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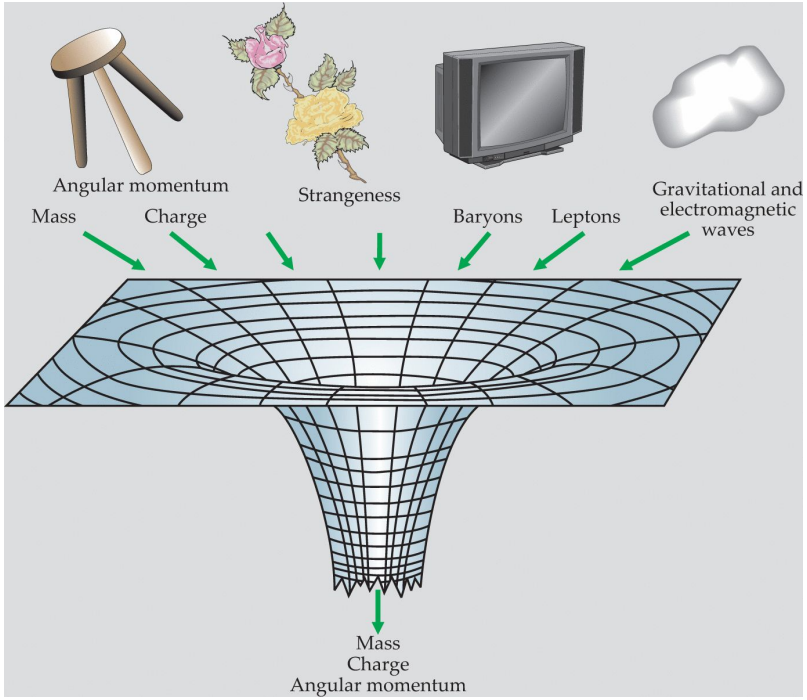


Ruffini & Wheeler, *Phys. Today* **24** (1971) 1, 30

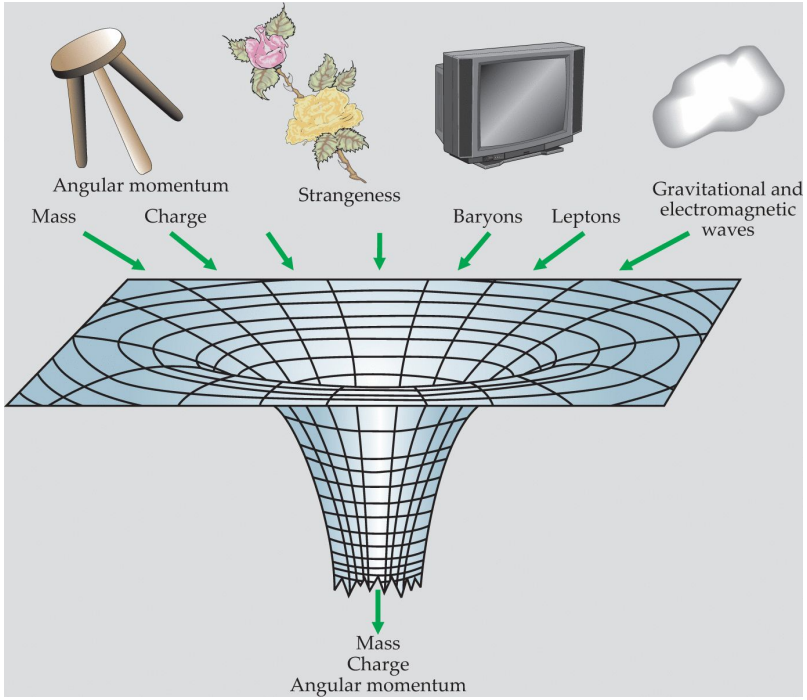
Kerr hypothesis

Non-Kerr black holes

General relativity → Modify gravity

Particle Physics → Add new particle(s)

ultralight bosonic fields



Ruffini & Wheeler, *Phys. Today* **24** (1971) 1, 30

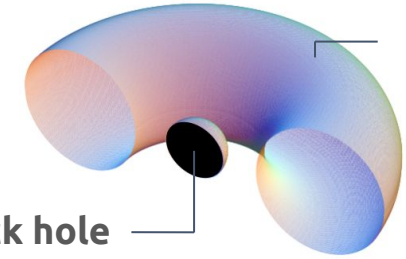
Black holes with synchronized hair

N. M. Santos, C. L. Benone, L. C. B. Crispino, C. A. R. Herdeiro and E. Radu, *JHEP* **07** (2020) 010
C. A. R. Herdeiro, E. Radu and **N. M. Santos**, *Phys. Lett. B* **824** (2022) 136835

Black holes with synchronized hair

Scalar field

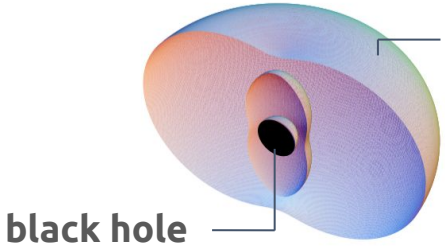
$$\mathcal{S} = \int \mathrm{d}^4x \left[\frac{R}{16\pi} - (\nabla_a \bar{\Psi})(\nabla^a \Psi) - \mu^2 |\Psi|^2 \right]$$



Herdeiro & Radu, *PRL* **112** (2014) 221101

Vector field

$$\mathcal{S} = \int \mathrm{d}^4x \left[\frac{R}{16\pi} - \frac{1}{4} F_{ab} \bar{F}^{ab} - \frac{1}{2} \mu^2 A_a \bar{A}^a \right]$$



Herdeiro & Radu, *CQG* **33** (2016) 154001

bosonic field — $\psi \sim e^{i(m\varphi - \omega t)}$

azimuthal index — m

oscillation frequency — ω

Black holes with synchronized hair

Scalar field

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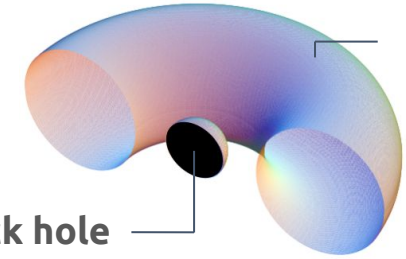

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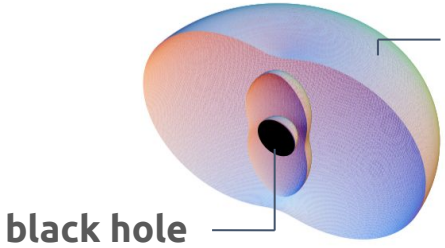

Herdeiro & Radu, *CQG* **33** (2016) 154001

synchronization

horizon angular velocity — $\Omega_H = \dfrac{\omega}{m}$ — phase angular velocity

Black holes with synchronized hair

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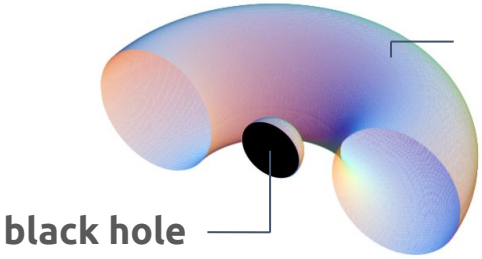

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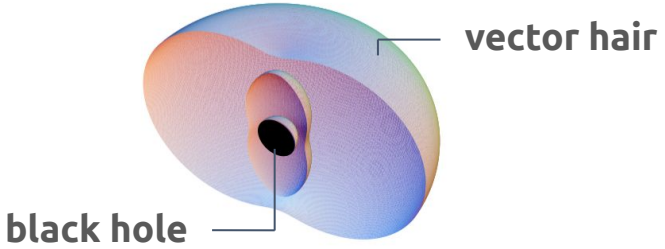
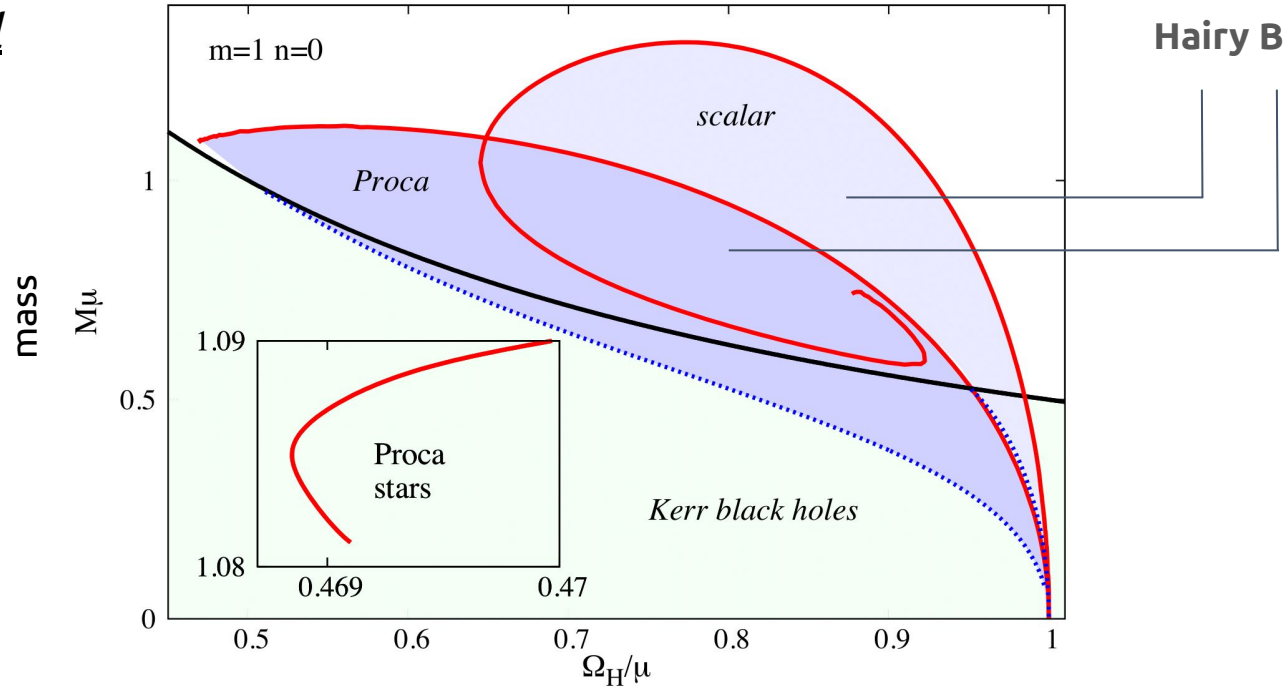


Herdeiro & Radu, *CQG* **33** (2016) 154001

M	J	Q	n	m
mass	angular momentum	Noether charge	node number	azimuthal index

Domain of existence

Fundamental solutions



Santos *et al.*, *JHEP* **07** (2020) 010

Domain of existence

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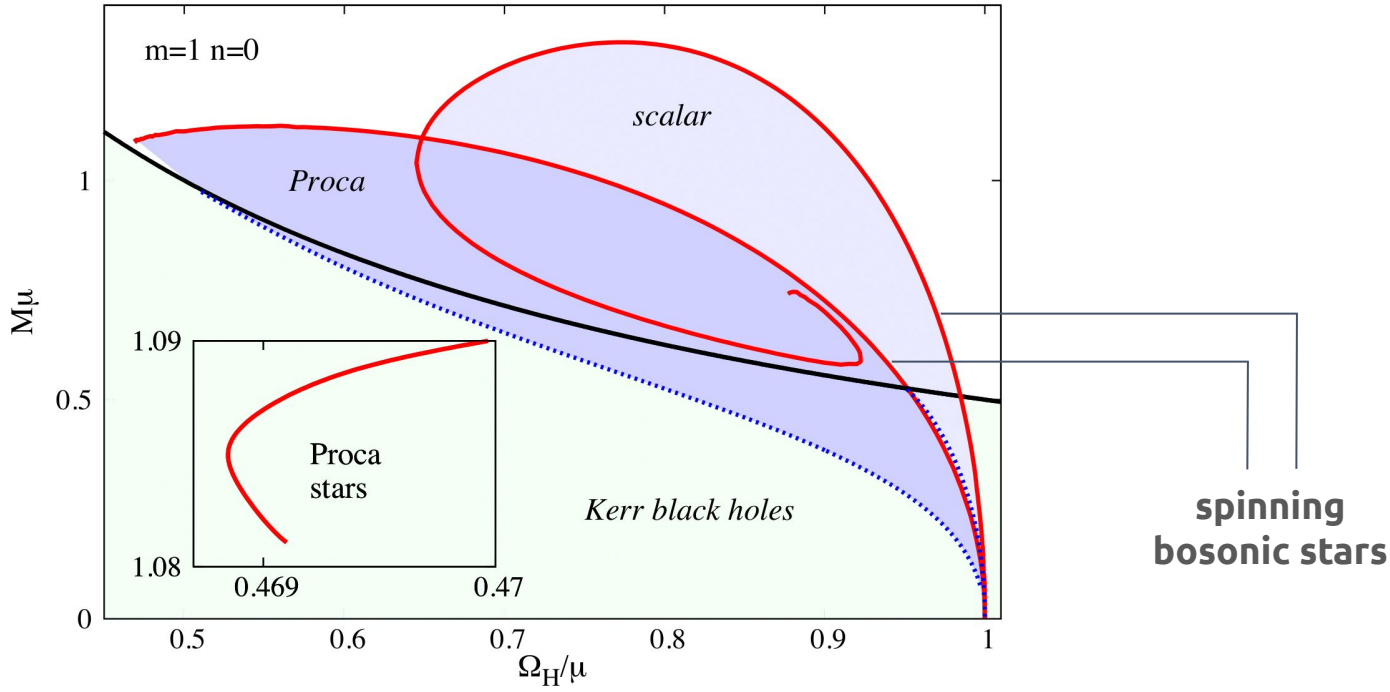

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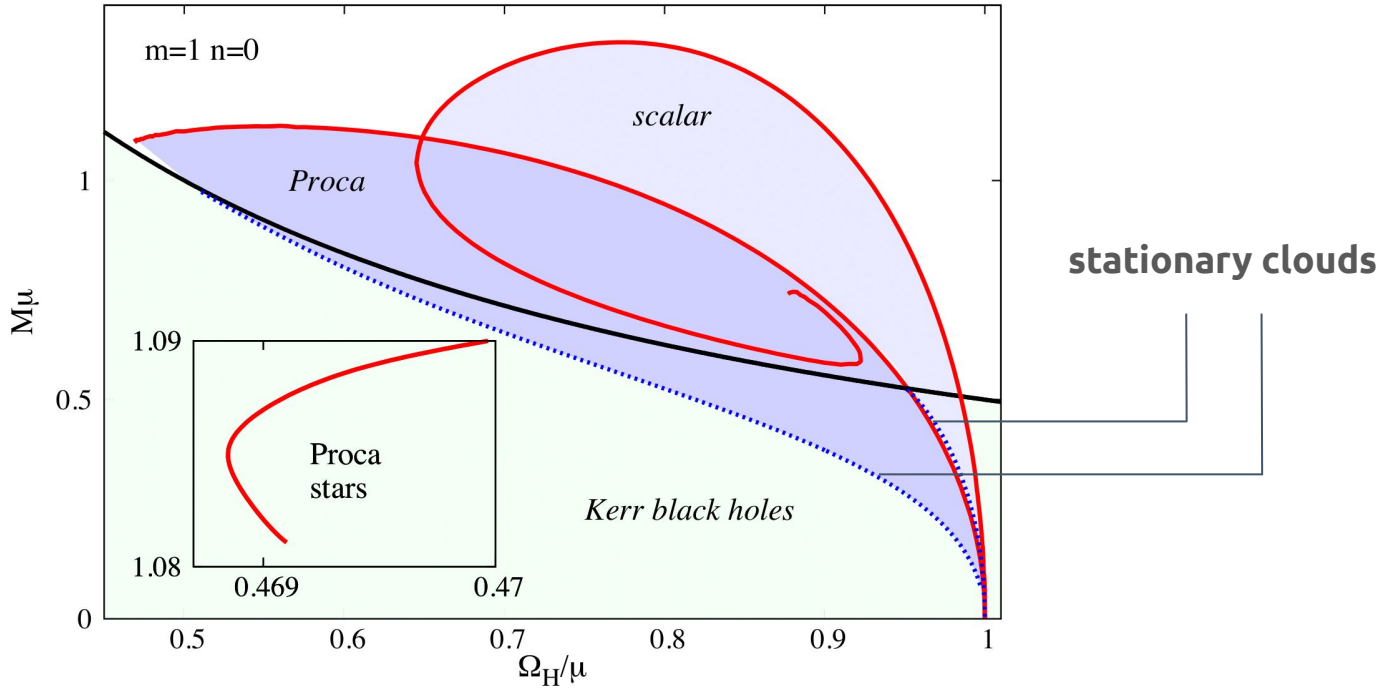
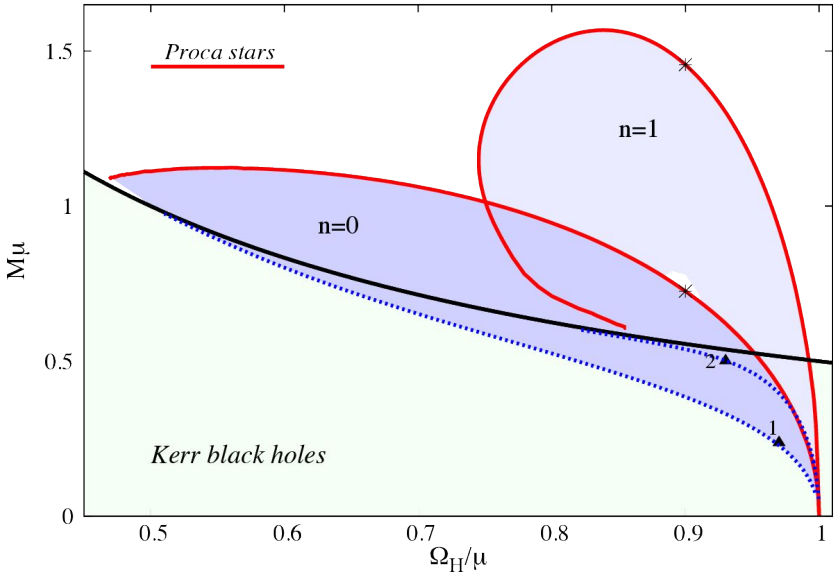


Santos *et al.*, *JHEP* **07** (2020) 010

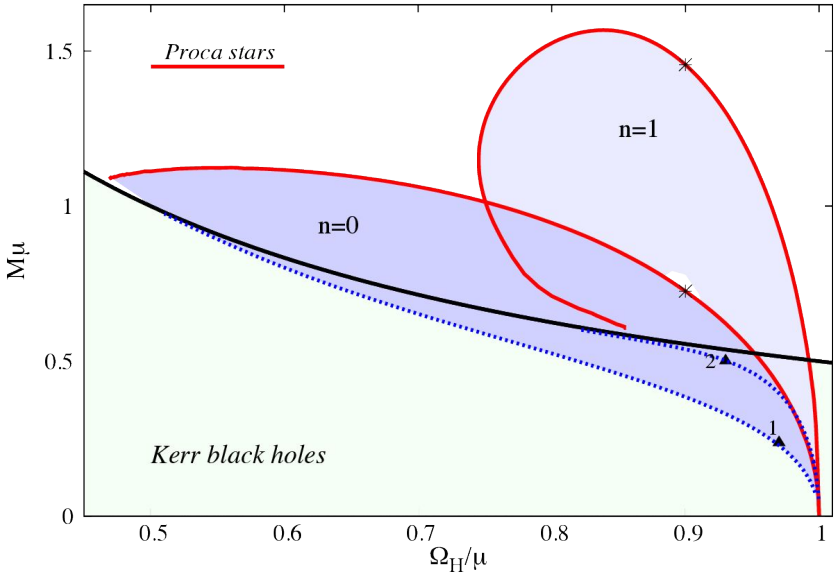
Hairstyle

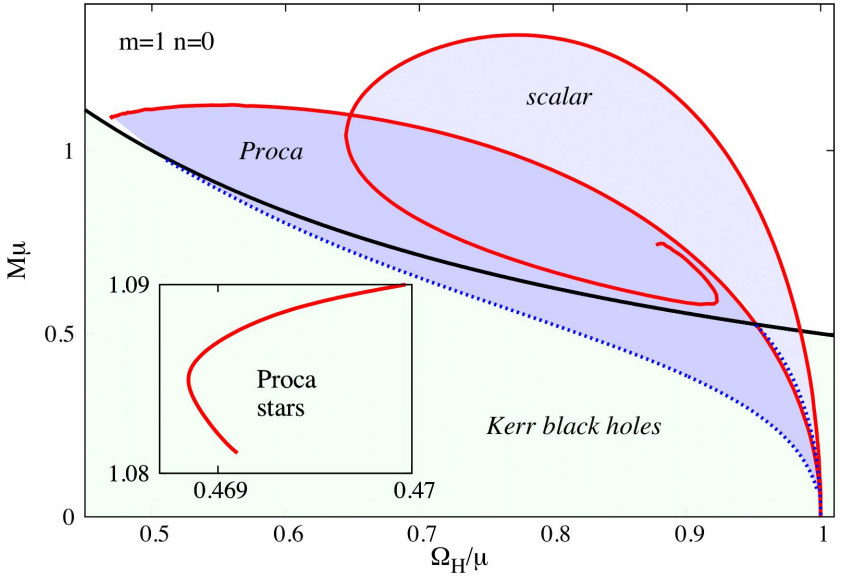


$n = 0$ fundamental state ⟶ larger ω-range

$n = 1$ excited state ⟶ larger M-range

Hairstyle





$n = 0$ fundamental state ⟶ larger ω-range

$n = 1$ excited state ⟶ larger M-range

scalar ⟶ larger ω-range

vector ⟶ larger M-range

Black holes with synchronized hair

Kerr-like and Non-Kerr-like solutions

N. M. Santos, C. L. Benone, L. C. B. Crispino, C. A. R. Herdeiro and E. Radu, *JHEP* **07** (2020) 010

How Kerr are you?

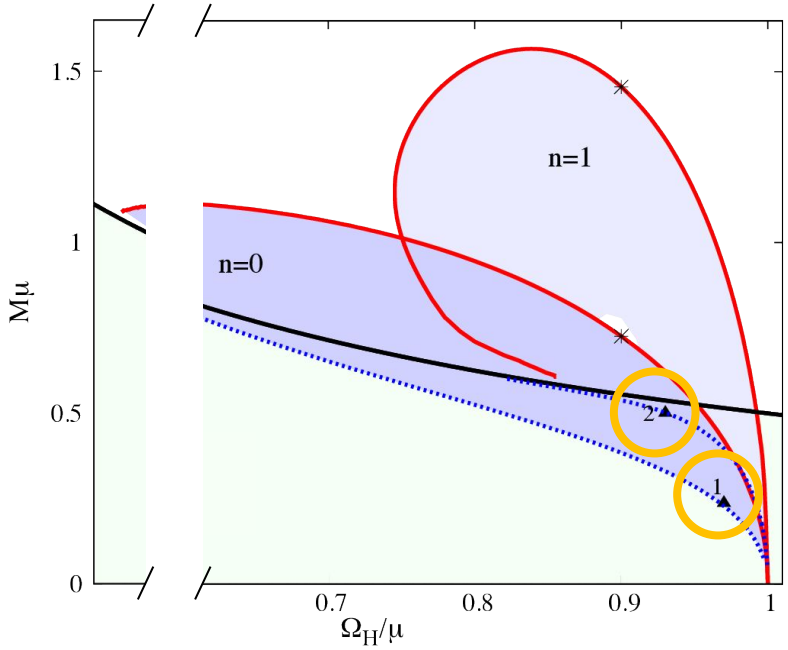
Fundamental solutions

1. Kerr-like (obeys the Kerr bound)

$$\frac{M_P}{M} \approx 9.5\% \ , \quad \frac{J_P}{J} \approx 39.3\%$$

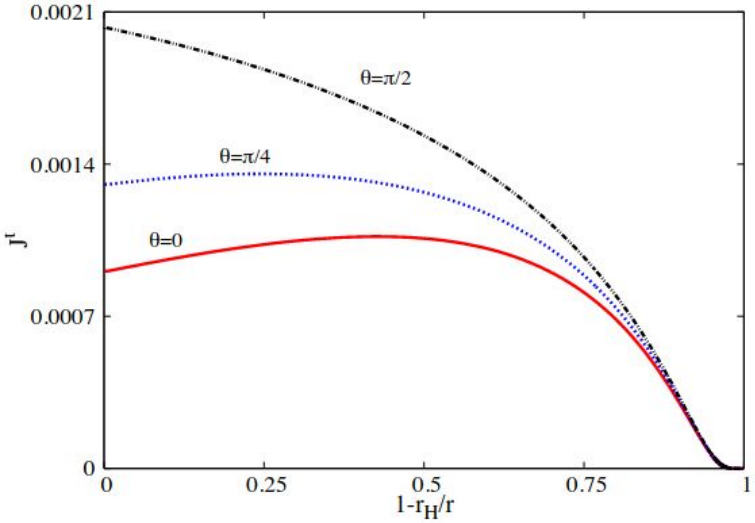
2. Non-Kerr-like (violates the Kerr bound)

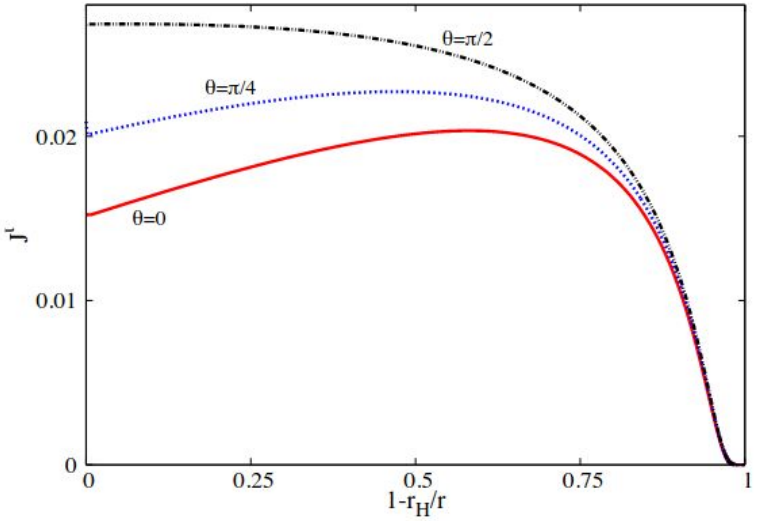
$$\frac{M_P}{M} \approx 76.9\% \ , \quad \frac{J_P}{J} \approx 97.8\%$$



How Kerr are you?

Noether charge density



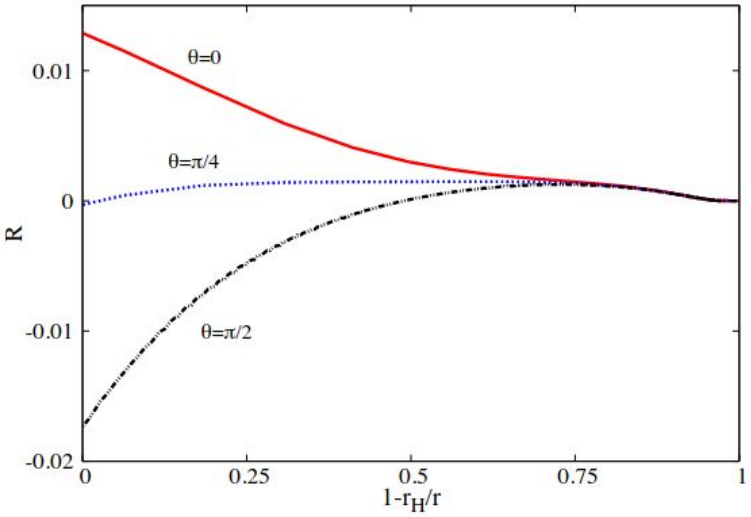


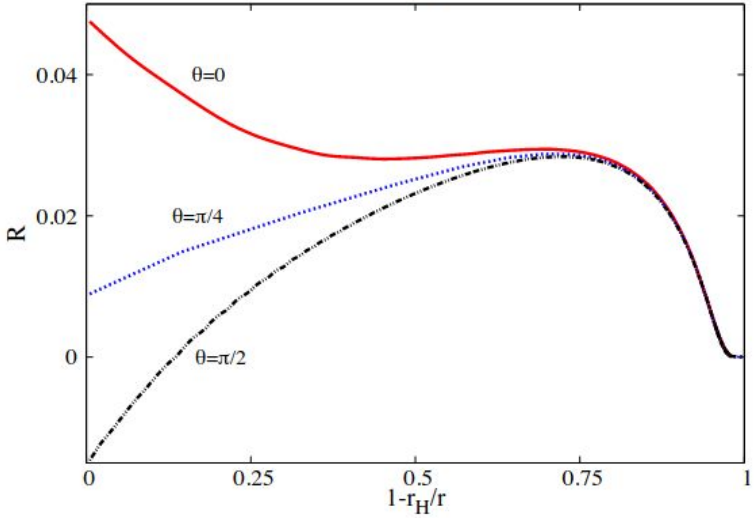
1 Kerr-like

2 Non-Kerr-like

How Kerr are you?

Ricci scalar curvature



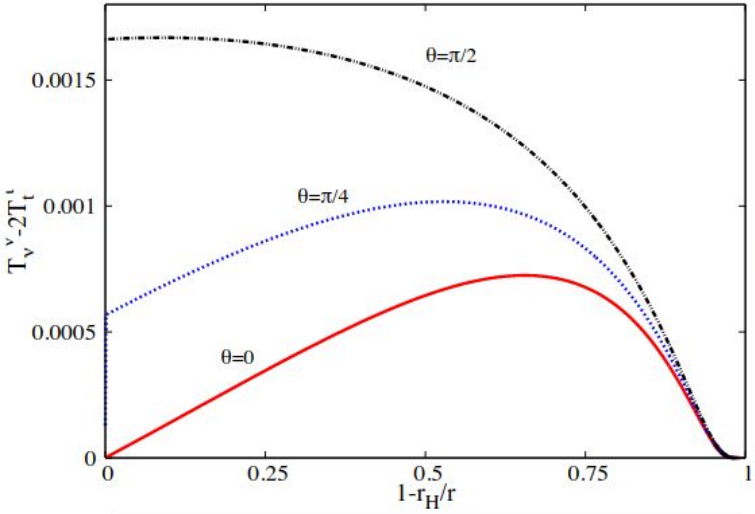


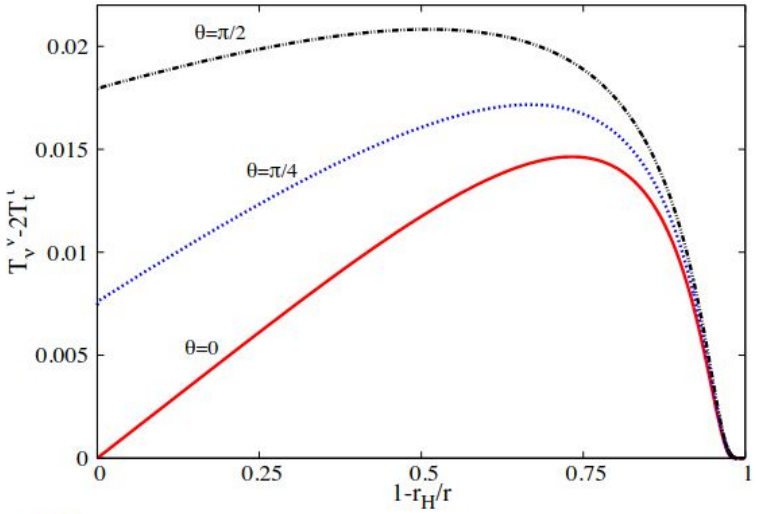
1 Kerr-like

2 Non-Kerr-like

How Kerr are you?

Komar mass density



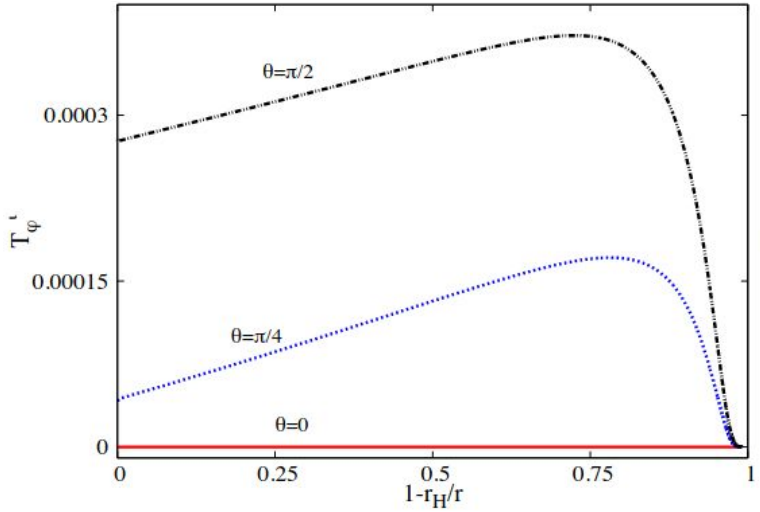


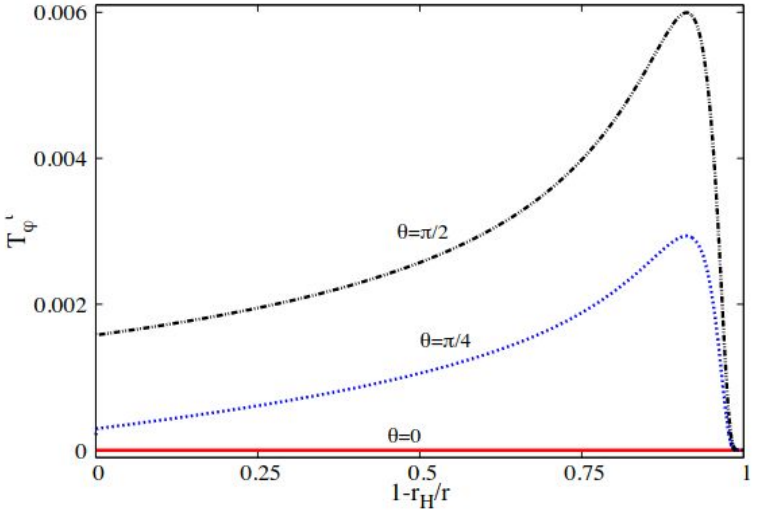
1 Kerr-like

2 Non-Kerr-like

How Kerr are you?

Komar angular momentum density





1 Kerr-like

2 Non-Kerr-like

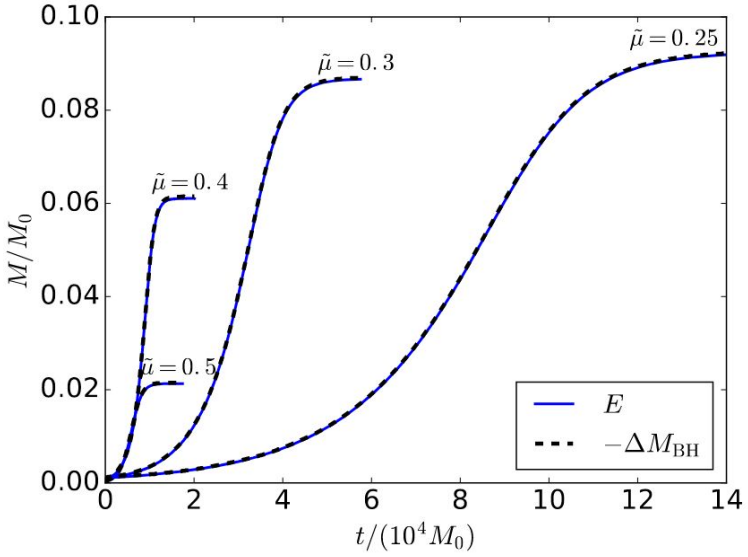
Black holes with synchronized hair

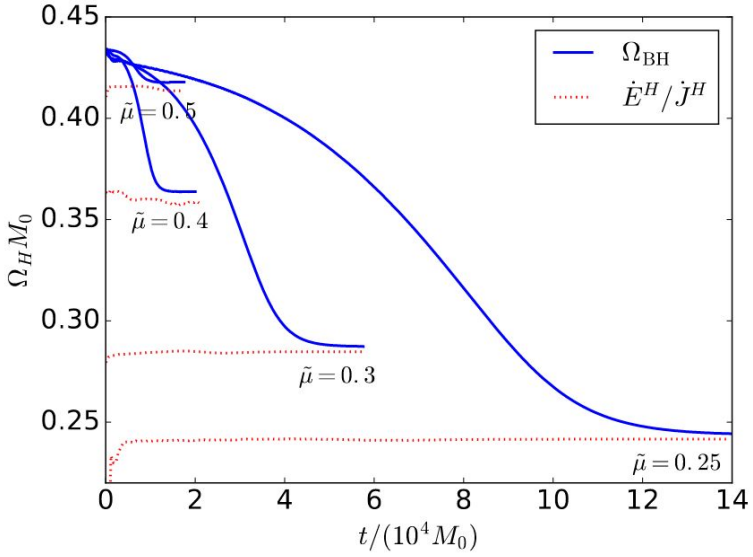
Dynamical formation and a bound on the hairiness

C. A. R. Herdeiro, E. Radu and **N. M. Santos**, *Phys. Lett. B* **824** (2022) 136835

Dynamical formation

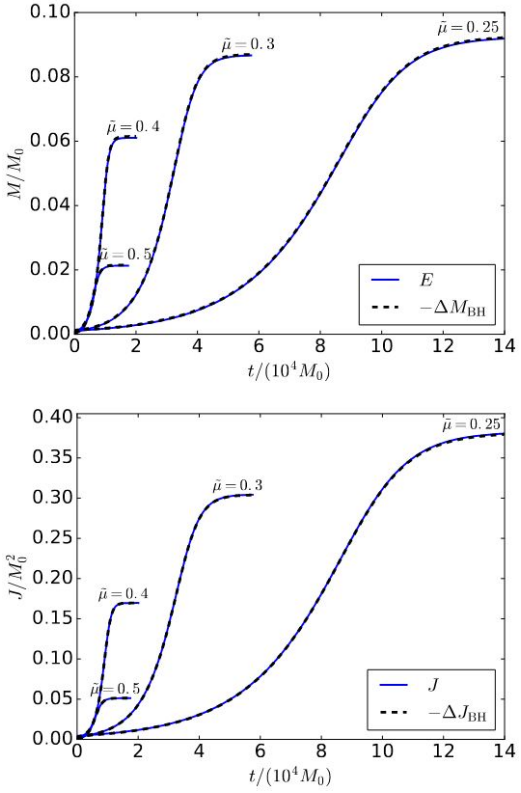
Black holes may grow hair via **superradiance**





East & Pretorius, *PRL* **119** (2017) 041101

Efficiency of energy extraction



East & Pretorius, *PRL* **119** (2017) 041101

$$M_H \Downarrow M_\psi$$

$$J_H \Downarrow J_\psi$$

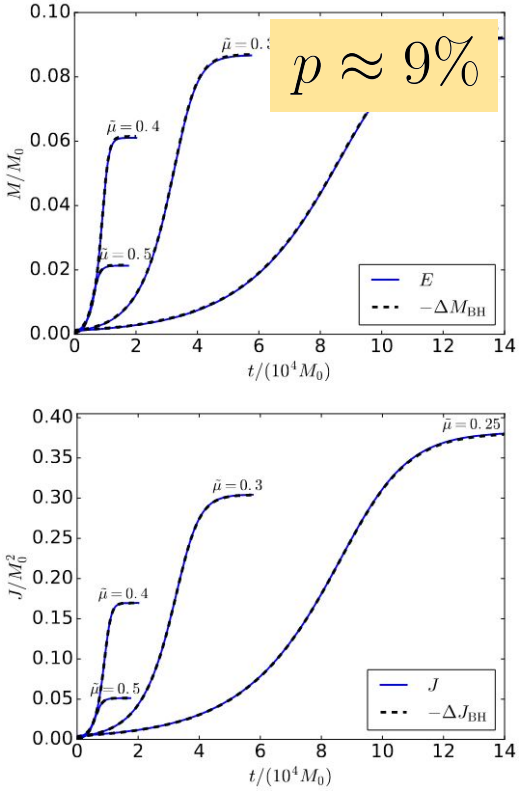
The «**hairiness**» is approximately equal to the **efficiency of energy extraction**

$$p = \epsilon$$

$$\epsilon \lesssim 29\% \longrightarrow p \lesssim 29\%$$

Hawking's area theorem

Efficiency of energy extraction



East & Pretorius, *PRL* **119** (2017) 041101

M_H

$\Downarrow$

M_ψ

J_H

$\Downarrow$

J_ψ

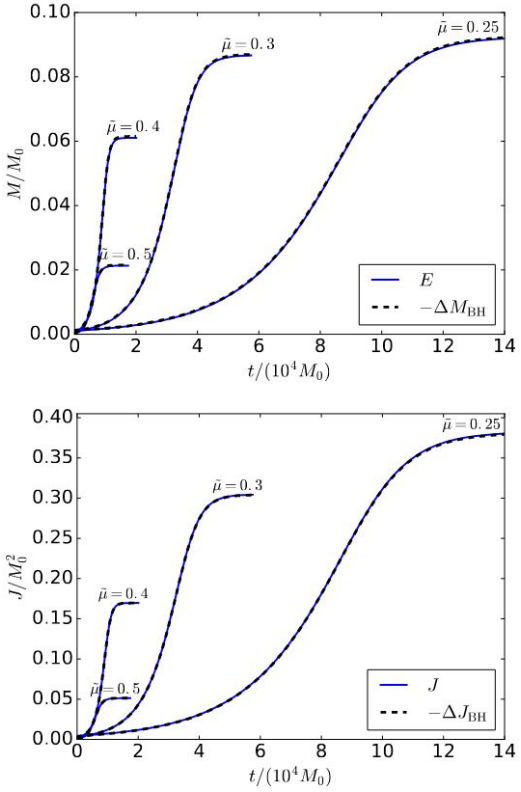
The «**hairiness**» is approximately equal to the **efficiency of energy extraction**

$$p = \epsilon$$

$\epsilon \lesssim 29\%$ $\longrightarrow$ $p \lesssim 29\%$

Hawking's area theorem

Conservativeness



East & Pretorius, *PRL* **119** (2017) 041101

$$M_H \Rightarrow M_\psi$$

$$J_H \Rightarrow J_\psi$$

The evolution of superradiant instabilities is nearly **conservative**

M — CONSERVED — $j = \dfrac{J}{M^2}$ — **CONSERVED**

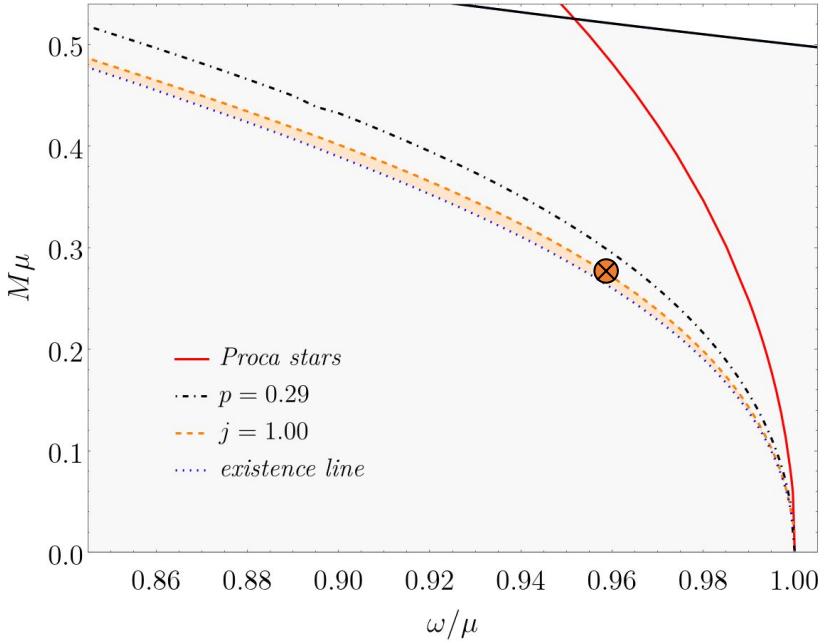
J

Kerr black hole → **Hairy black hole**

$|j| \leq 1$ → $|j| \leq 1$

Region of interest

Vector field $(n, m) = (0, 1)$



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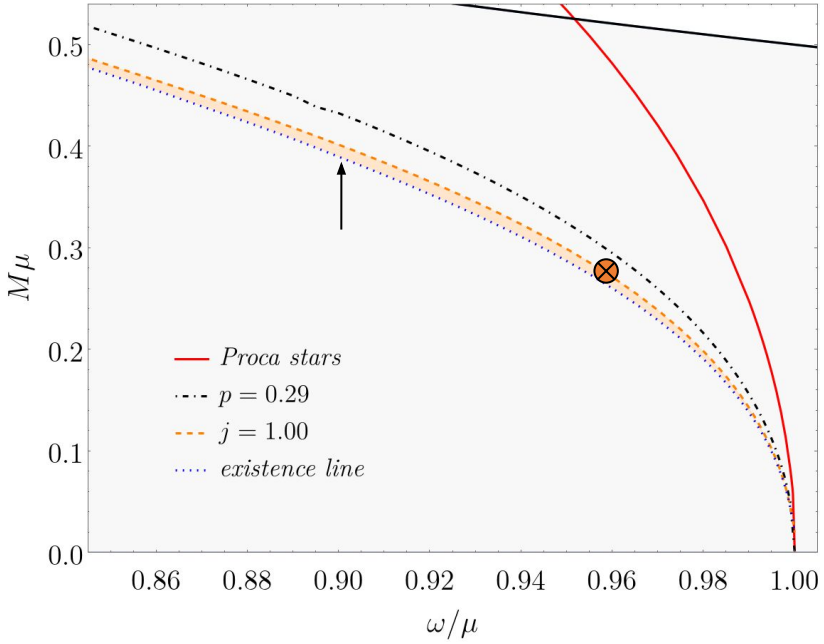

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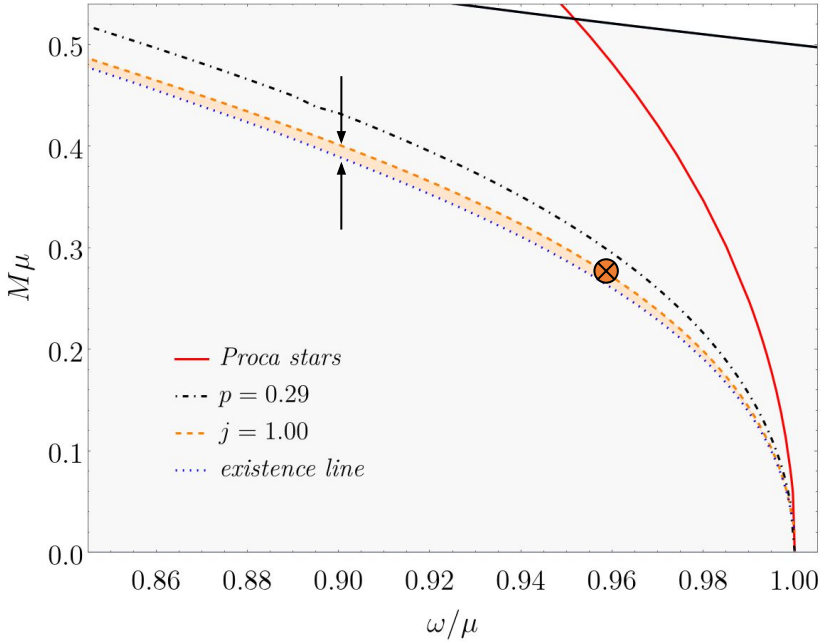

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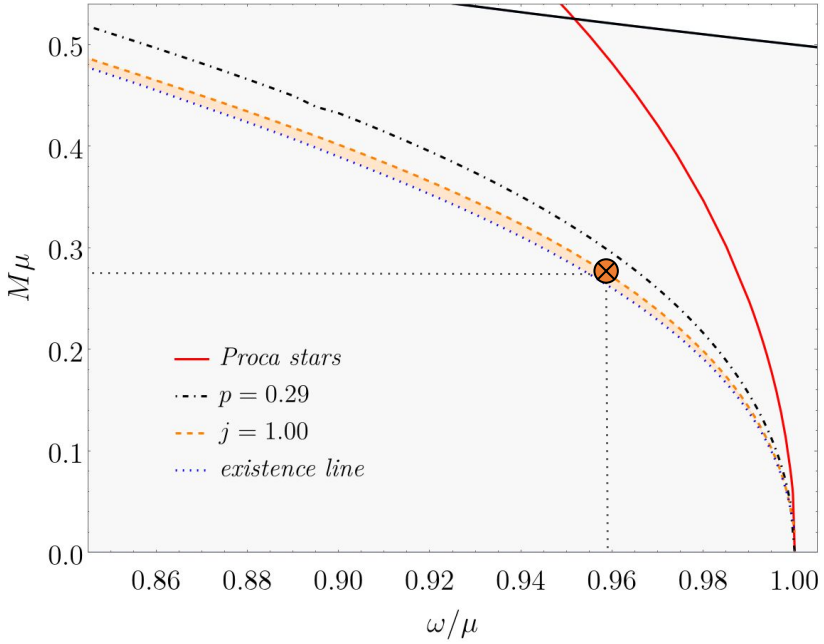

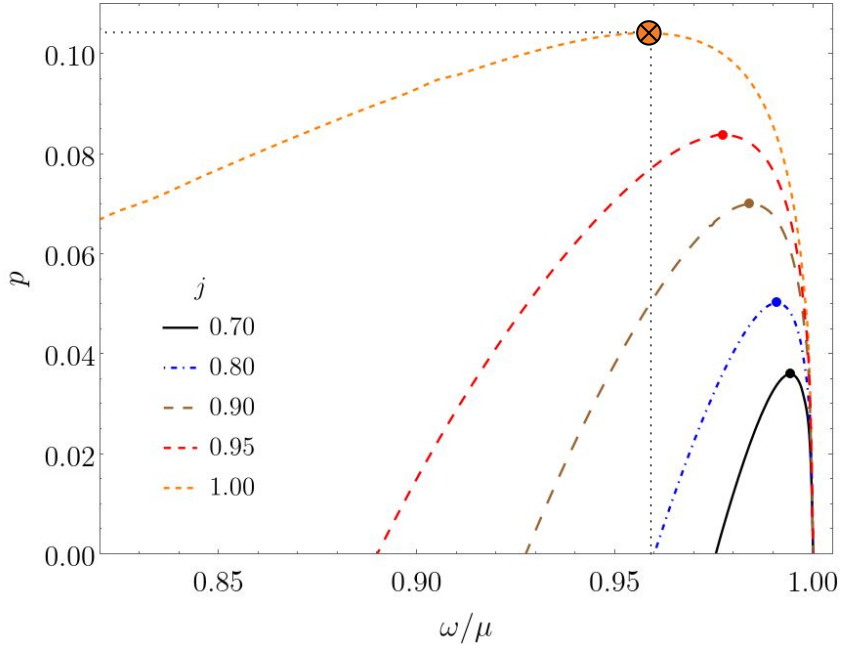

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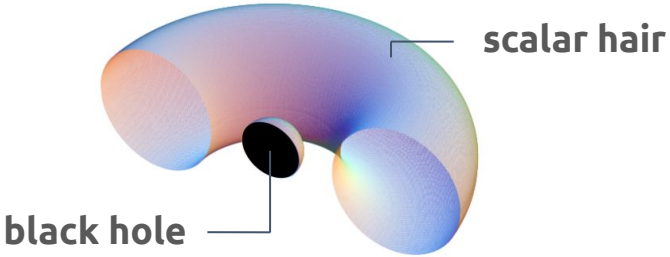
A bound on «hairiness»

Scalar field



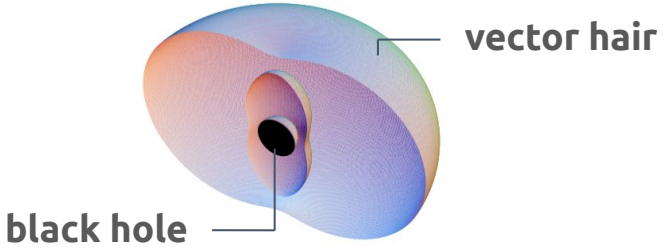
$$p_{\max} = \epsilon_{\max} \approx 9.9\%$$

$\omega/\mu \approx 0.993$

$M\mu \approx 0.24$

PREDICTION

Vector field



$$p_{\max} = \epsilon_{\max} \approx 10.4\%$$

$\omega/\mu \approx 0.958$

$M\mu \approx 0.27$

AGREEMENT

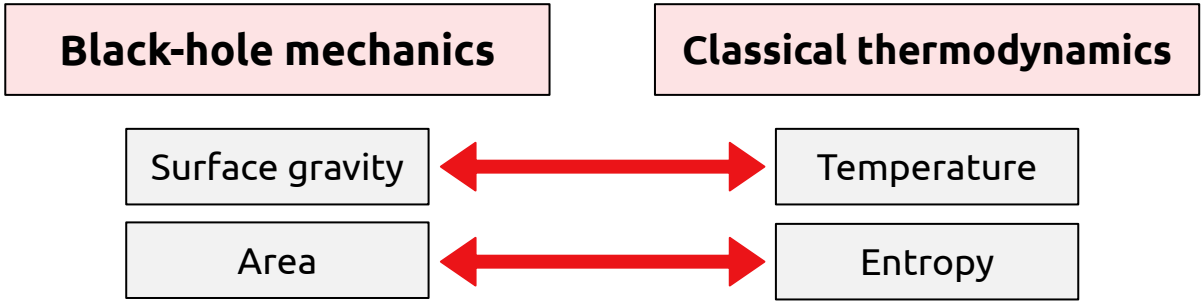
Herdeiro, Radu & **Santos**, *PLB* **824** (2022) 136835

Black holes with synchronized hair

Thermodynamic stability

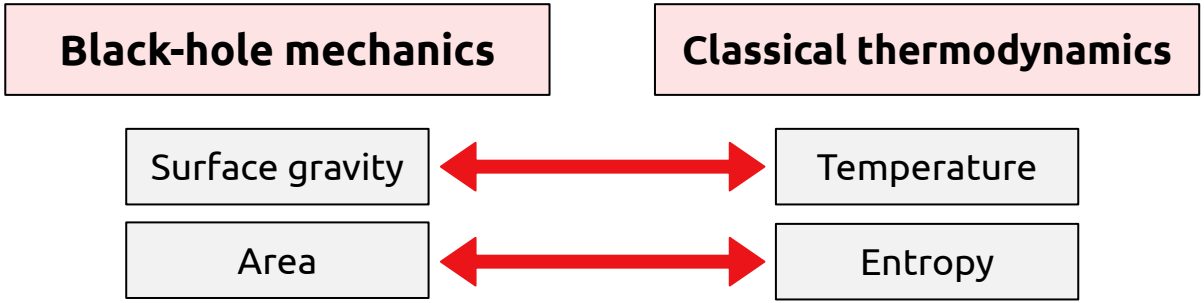
N. M. Santos, C. A. R. Herdeiro and E. Radu, arXiv:2207.10089 **[gr-qc]**

Black holes as thermodynamic systems



Bardeen, Carter & Hawking, *CMP* **31** (1973) 161

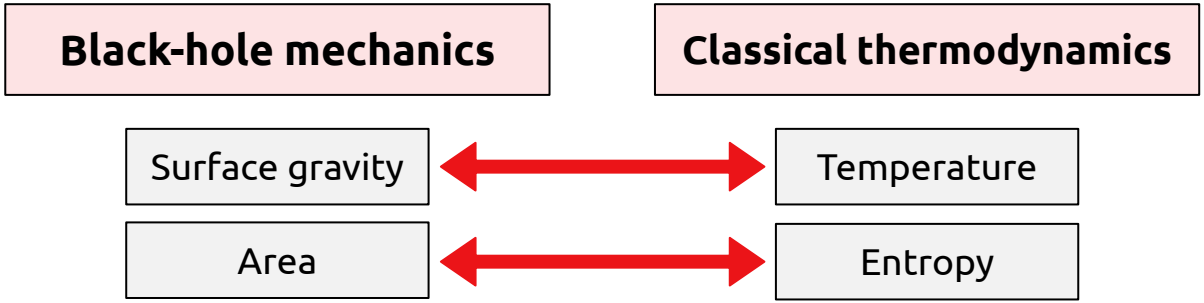
Black holes as thermodynamic systems



Bardeen, Carter & Hawking, *CMP* **31** (1973) 161

Are black holes thermodynamically stable?

Black holes as thermodynamic systems

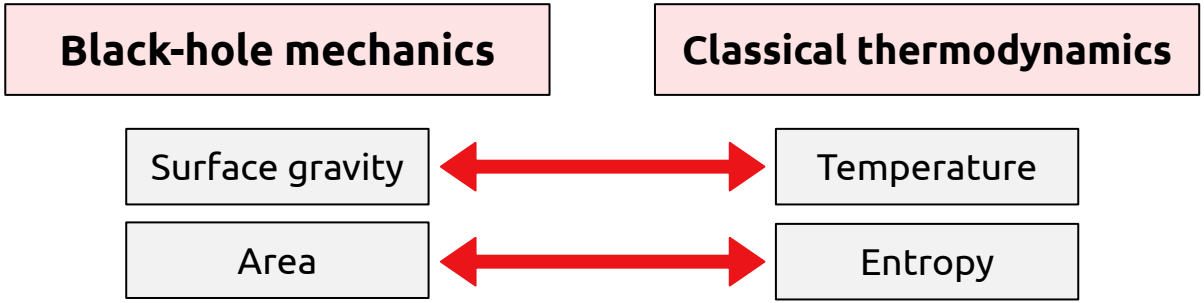


Bardeen, Carter & Hawking, *CMP* **31** (1973) 161

Are black holes thermodynamically stable?

Local stability

Black holes as thermodynamic systems



Bardeen, Carter & Hawking, *CMP* **31** (1973) 161

Are black holes thermodynamically stable?

Local stability

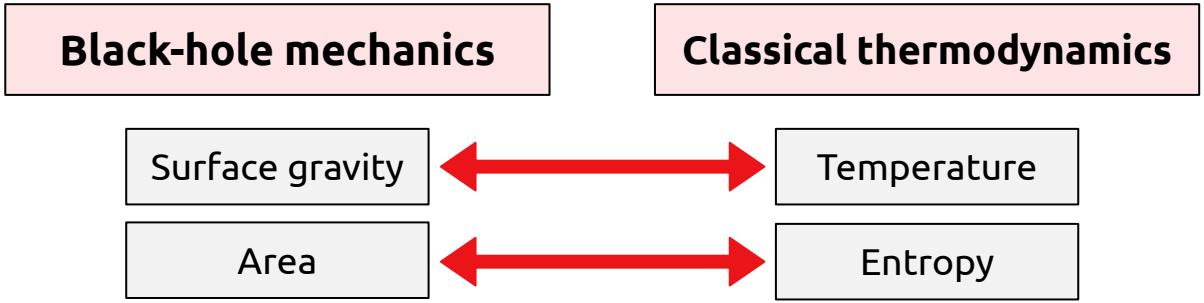
$$C = T\left(\frac{\partial S}{\partial T}\right)$$

entropy

specific heat

temperature

Black holes as thermodynamic systems



Bardeen, Carter & Hawking, *CMP* **31** (1973) 161

Are black holes thermodynamically stable?

Schwarzschild BHs

unstable

$$C = T\left(\frac{\partial S}{\partial T}\right) < 0$$

Black holes in the canonical ensemble

Local thermodynamic stability

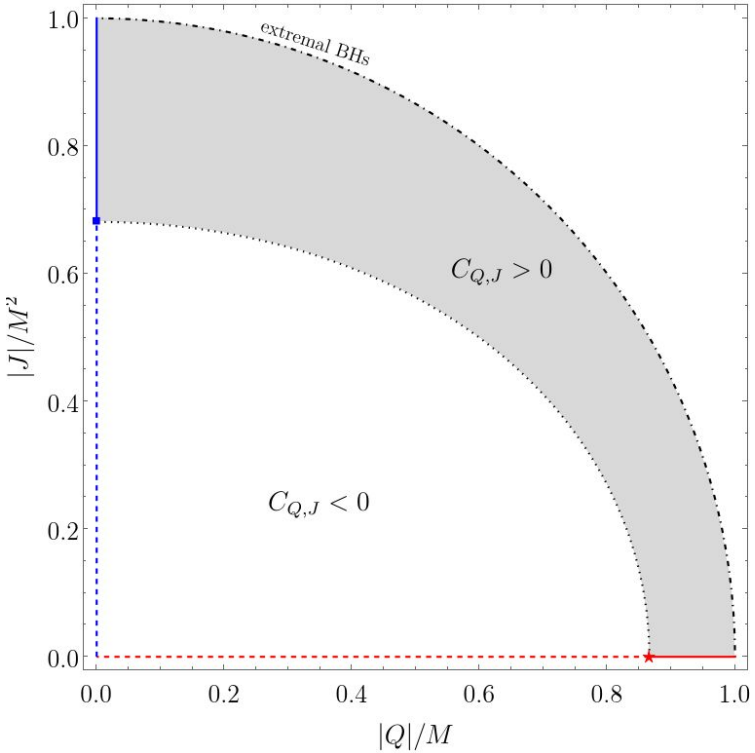
Kerr-Newman BHs

$$C_{Q,J} = T\left(\frac{\partial S}{\partial T}\right)_{Q,J} > 0$$

electric charge

angular momentum

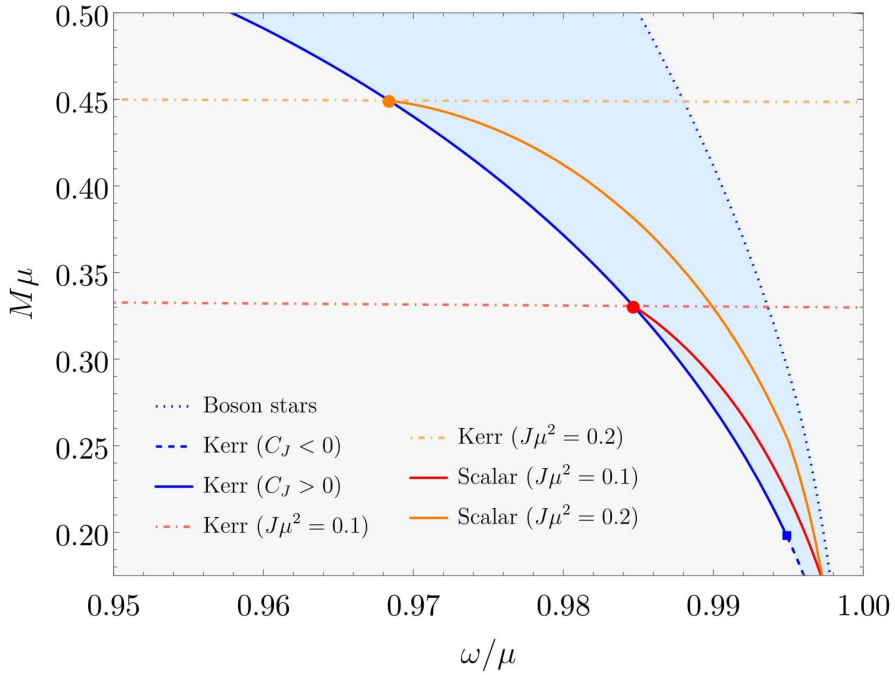
$$J^4 + 6J^2M^4 + 4Q^2M^6 - 3M^8 > 0$$



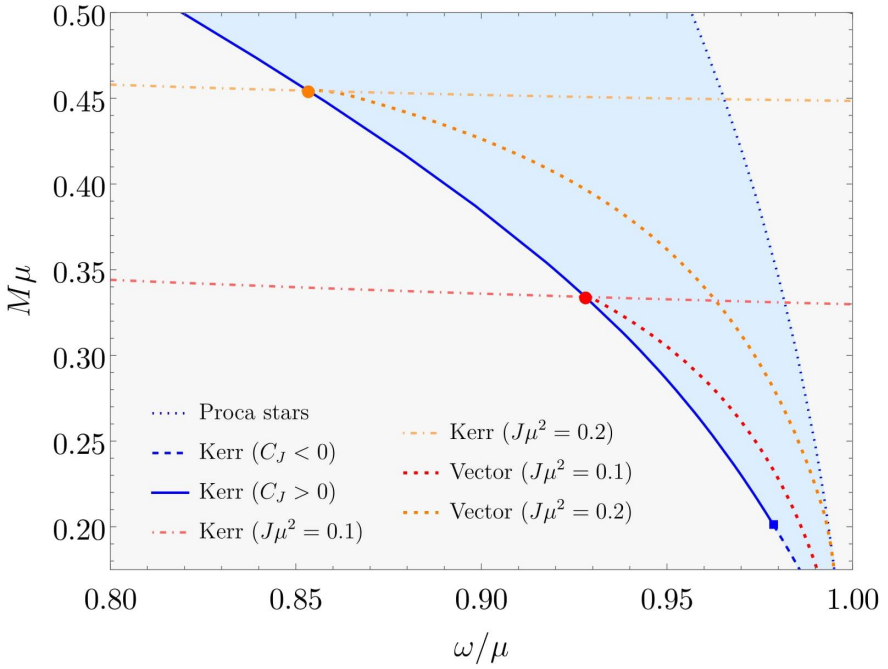
Davies, *RSL A* **353** (1977) 499

Local thermodynamic stability

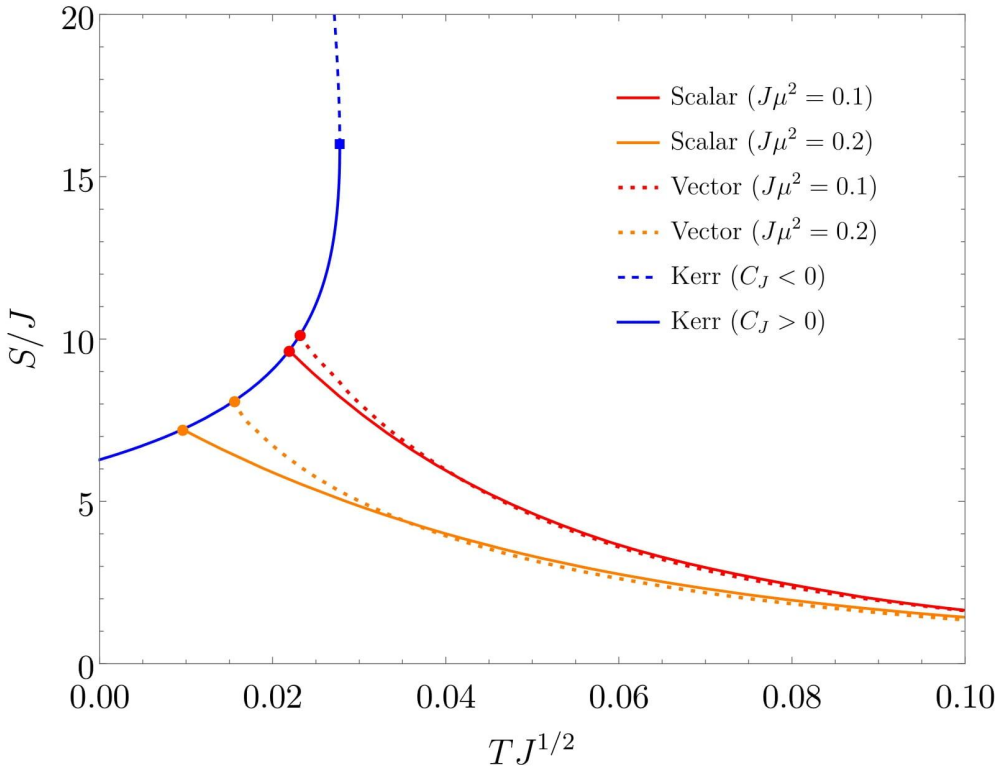
Scalar field



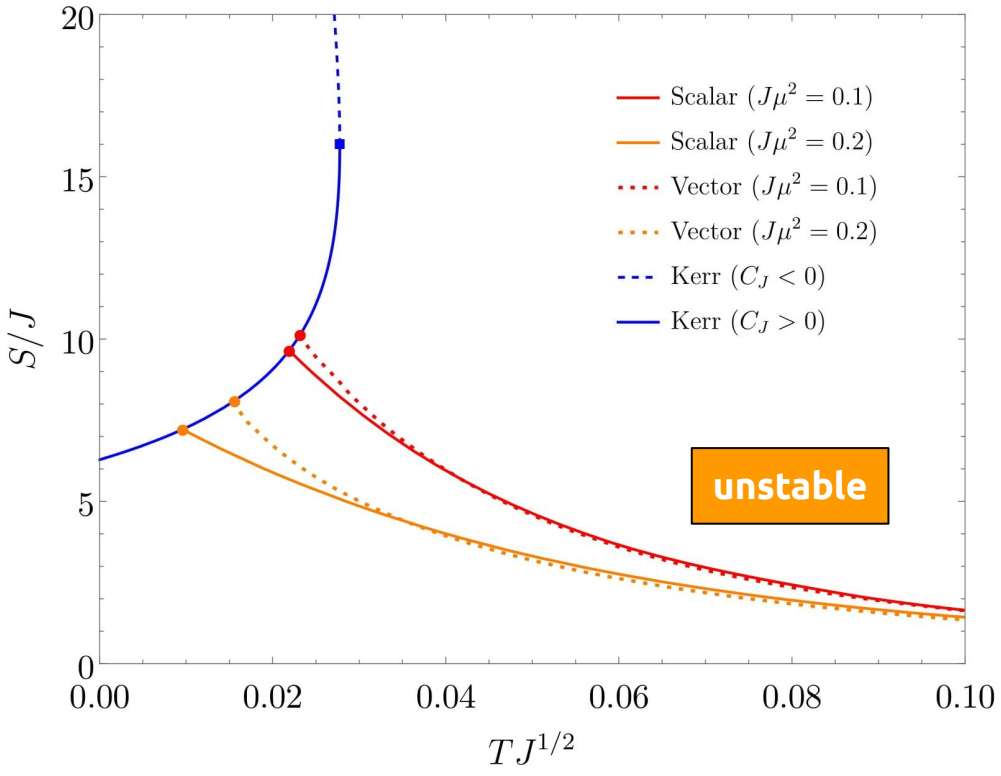
Vector field



Local thermodynamic stability



Local thermodynamic stability



Summary and future work

Summary and future work

Black holes with synchronized hair are a theoretical possibility of dynamically robust non-Kerr black holes that could manifest themselves only at some scales of mass and angular momentum.

- They can be sufficiently akin to Kerr black holes.
- Kerr-like solutions form from superradiance, with a hairiness up to 10%.
  - ➢ bound from the merger of bosonic stars?
- They are locally unstable in the canonical ensemble.
  - ➢ grand-canonical ensemble?

Synchronized bosonic hair: equilibrium solutions

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