Massless scalar scattering by Ayón-Beato-García regular black holes

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Outline

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Scattering properties of charged black holes in nonlinear and Maxwell's electrodynamics

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Abstract We investigate the scattering properties of a massless scalar field in the background of a charged Ayón-Beato-García regular black hole solution. Using a numerical approach, we compute the differential scattering cross section for arbitrary values of the scattering angle and of the incident wave frequency. We compare our results with those obtained via the classical geodesic scattering of massless particles, as well as with the semiclassical glory approximation, and show that they present an excellent agreement in the corresponding limits. We also show that Ayón-Beato-García and Reissner-Nordström black hole solutions present similar scattering properties, for low-to-moderate values of the black hole electric charge, for any value of the scattering angle.

VOLUME 80, NUMBER 23 PHYSICAL REVIEW LETTERS 8 JUNE 1998

Regular Black Hole in General Relativity Coupled to Nonlinear Electrodynamics

Eloy Ayón-Beato* and Alberto García

Departamento de Física, Centro de Investigación y Estudios Avanzados del IPN, Apdo. Postal 14-740, 07000 México DF, Mexico

(Received 14 November 1997)

The first regular *exact* black hole solution in general relativity is presented. The source is a nonlinear electrodynamic field satisfying the weak energy condition, which in the limit of weak field becomes the Maxwell field. The solution corresponds to a charged black hole with $|q| \leq 2 s_c m \approx 0.6 m$, having the metric, the curvature invariants, and the electric field regular everywhere. [S0031-9007(98)06332-7]

Nonlinear electrodynamics

[redacted]

General relativity

Monthly Notices

Volume 163B, number 1,2,3,4 PHYSICS LETTERS 21 November 1985

NON-LINEAR ELECTRODYNAMICS FROM QUANTIZED STRINGS

E.S. FRADKIN and A.A. TSEYTLIN
P.N. Lebedev Physical Institute, Leninsky pr. 53, Moscow 117924, USSR

Received 6 June 1985

We compute the effective action for an abelian vector field coupled to the virtual open Bose string. The problem is exactly solved (in the "tree" and "one-loop" approximation for the string theory) for the case of a constant field strength and the number of space–time dimensions $D = 26$. The resulting tree-level effective lagrangian is shown to coincide with the Born–Infeld lagrangian, $[\det(\delta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})]^{1/2}$.

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with the standard model predictions.

New Journal of Physics

The open-acce ... for physics

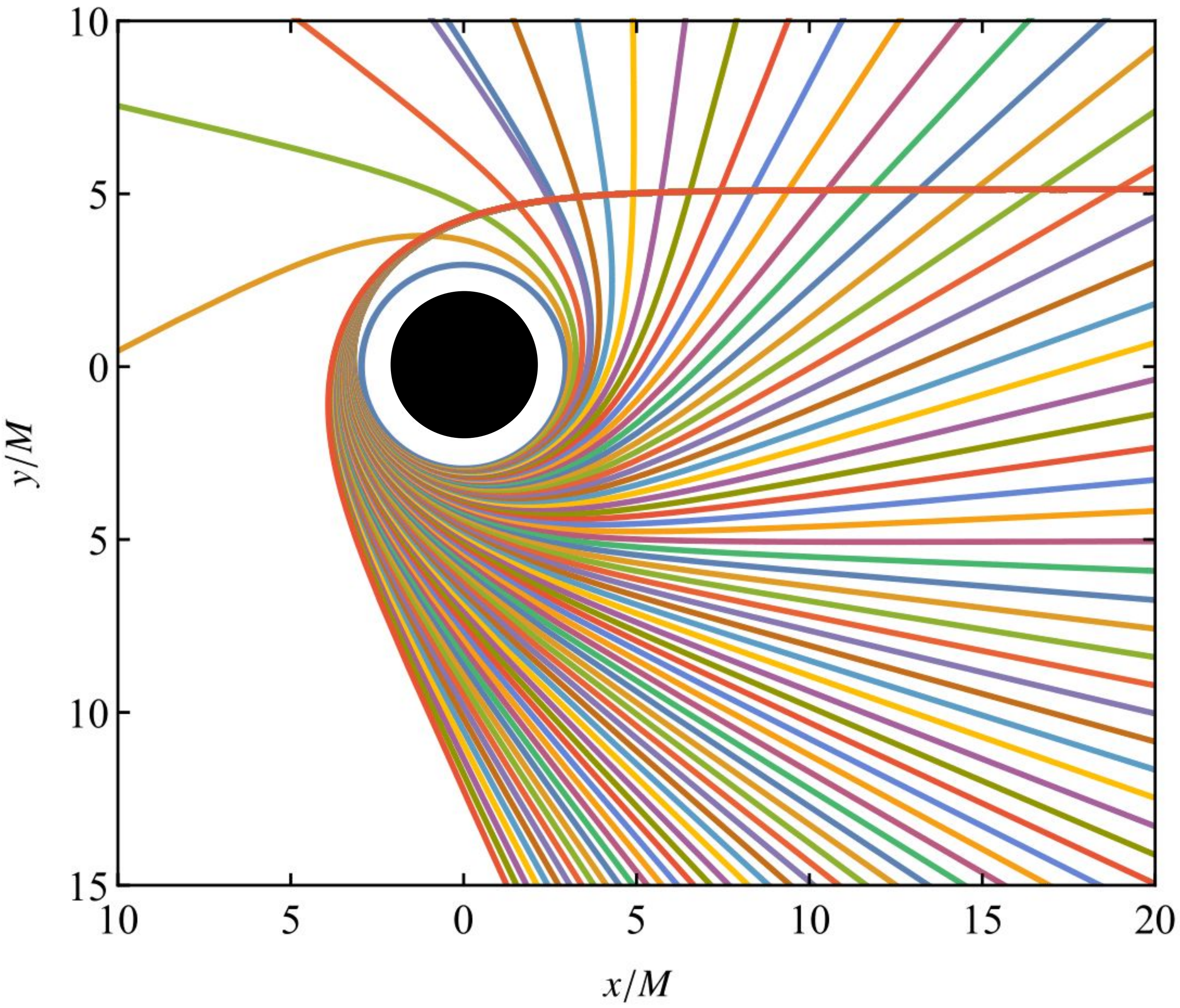
Black holes in as...

Ramesh Na...

Harvard Colleg... ...n Center for Astrophysics,
Cambridge, MA...
E-mail: rnarayan...

New Journal of Physic...
Received 7 September 2004
Published 29 September 2005
Online at http://www.njp.org/
doi:10.1088/1367-2630/7/1/199

M, Q
Ayón-Beato-García
Phys. Rev. Lett. 80, 5056 (1998)


10
5
0
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y/M
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15
20
x/M

The line element of the Ayón-Beato-García (ABG) solution is given by (Ayón-Beato and García, 1998)

[redacted] (1)

with

[redacted] (2)

At infinity, the ABG metric function (2) behaves as

$$f^{\mathrm{ABG}}(r) = \underbrace{1 - \frac{2M}{r} + \frac{Q^2}{r^2}}_{\text{Reissner–Nordström (RN) metric function}} + \frac{3MQ^2}{r^3} + \mathcal{O}\left[\frac{1}{r^4}\right], \tag{3}$$

whereas as we approach the center, we have

$$f^{\mathrm{ABG}}(r) = \underbrace{1 - \frac{1}{3}\Lambda(M,Q)r^2}_{\text{de Sitter metric function}} - \left(\frac{2}{Q^4} - \frac{3MQ}{Q^6}\right) r^4 + \mathcal{O}\left[r^5\right]. \tag{4}$$

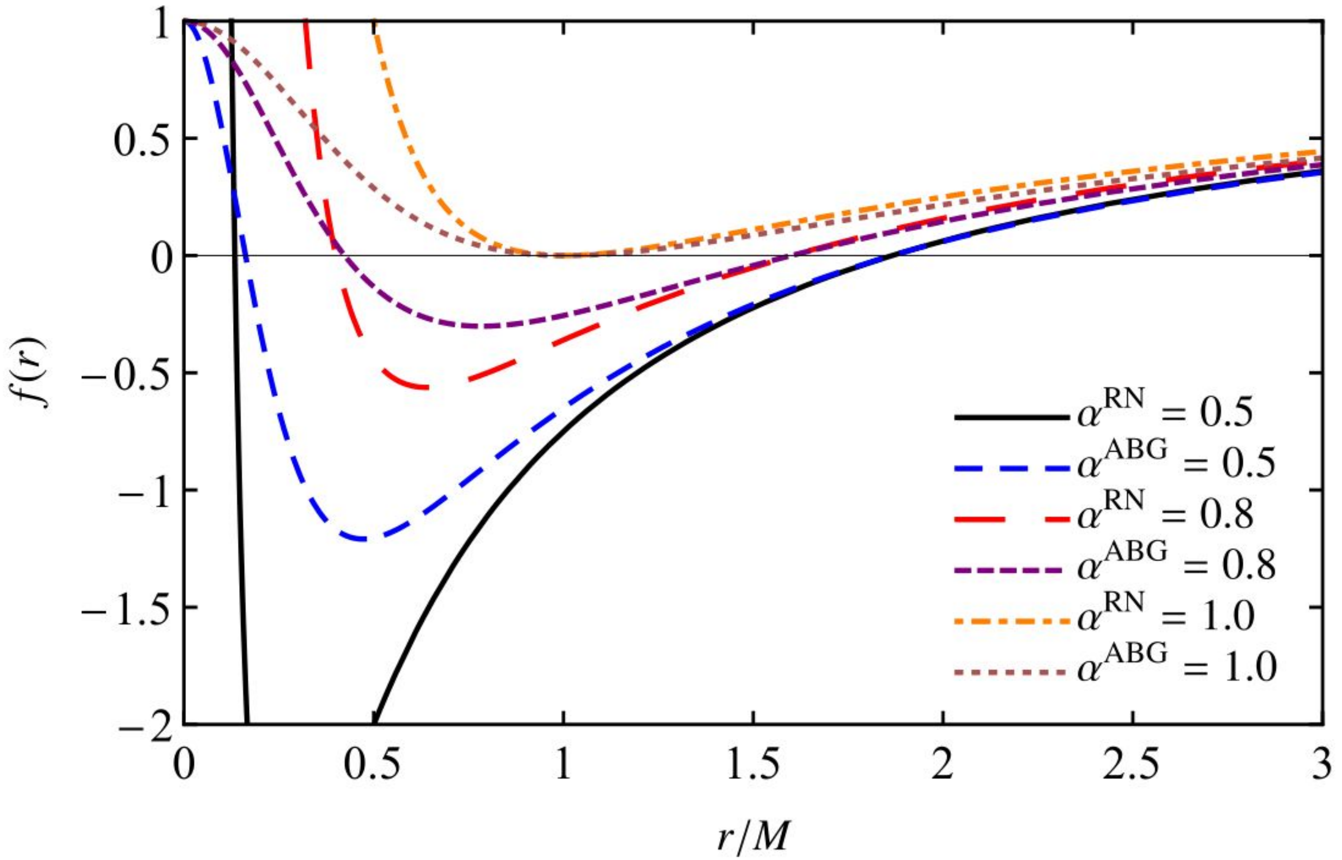


Figure 1: Comparison between the metric functions of ABG and RN BHs, for different choices of the normalized electric charge, defined as $\alpha = Q/Q_{\text{ext}}$. For ABG, we have RBHs for $Q \leq Q_{\text{ext}} \equiv 0.6341M$, whereas for RN, we have BHs for $Q \leq Q_{\text{ext}} \equiv M$.

The deflection angle of scattered massless particles is given by

$$\Theta(b) = 2\gamma(b) - \pi, \tag{5}$$

where

$$\gamma(b) = \int_{r_0}^{\infty} \frac{1}{\sqrt{\mathcal{T}(r)}} dr, \tag{6}$$

with

[redacted] (7)

The classical differential scattering cross section (SCS) is given by (Newton, 2013)

$$\frac{d\sigma_{\mathrm{cl}}}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\Theta}\right|, \tag{8}$$

where $\Theta = \theta + 2n\pi$, with $n \in \mathbb{Z}^{+}$. By using the geodesic method and considering the weak-field expansion of the deflection angle, we can write the deflection angle for ABG RBH as

RN's weak deflection angle

$$\Theta(b) = \frac{4M}{b} + \frac{3\pi\left(5M^2 - Q^2\right)}{4b^2} + \frac{8M\left(16M^2 - 9Q^2\right)}{3b^3} + \mathcal{O}\left[\frac{1}{b^4}\right]. \tag{9}$$

Einstein's deflection angle

The classical differential SCS for small scattering angles reads

[redacted] (10)

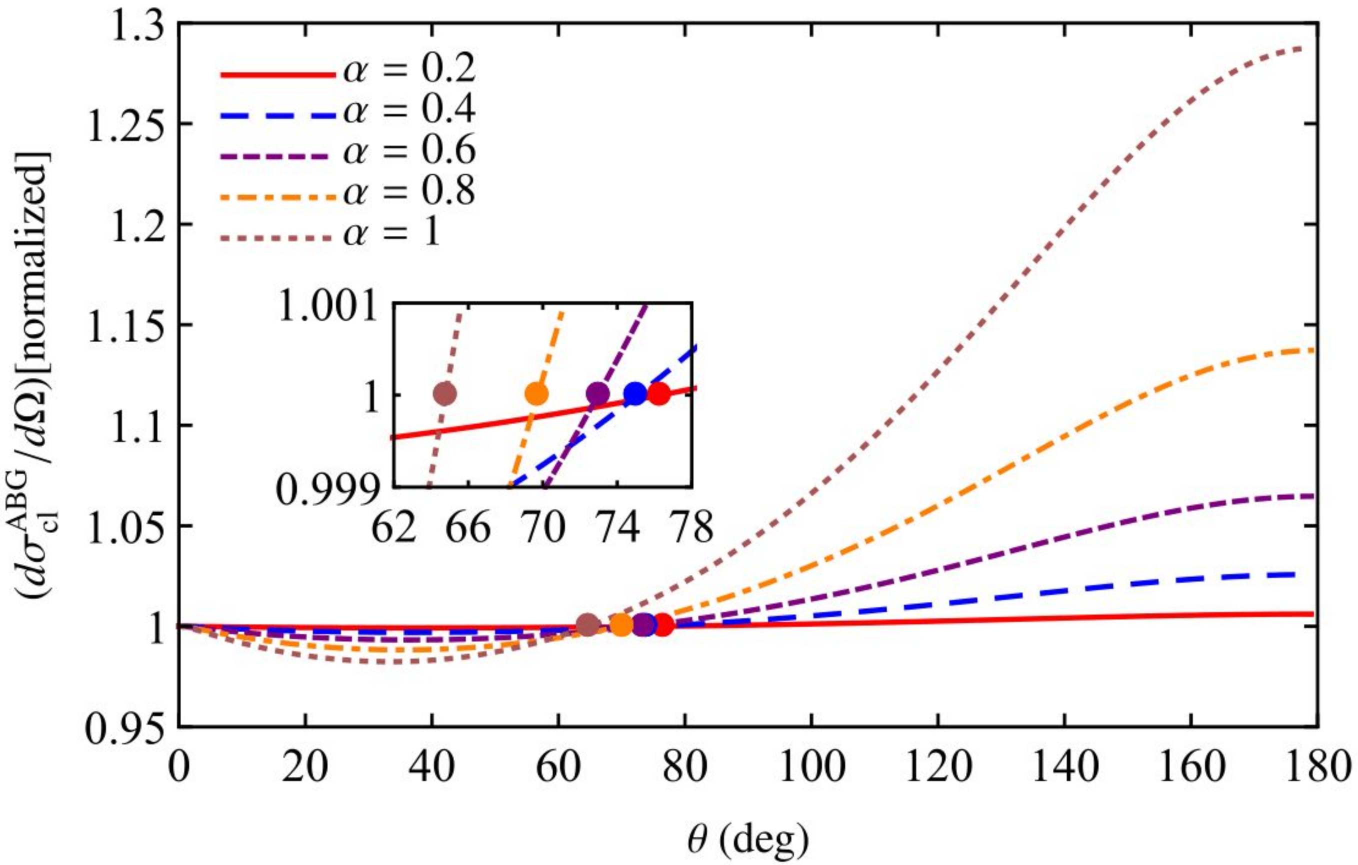


Figure 2: Classical differential SCS of the ABG RBH normalized by the Schwarzschild result, for different values of α.

In the limit $\theta \to \pi$, we can use the glory approximation (Matzner *et al.*, 1985):

$$\frac{d\sigma_{\mathrm{g}}}{d\Omega} = 2\pi\omega b_g^2 \left|\frac{db}{d\theta}\right|_{\theta=\pi} J_0^2(\omega b_g \sin\theta). \tag{11}$$

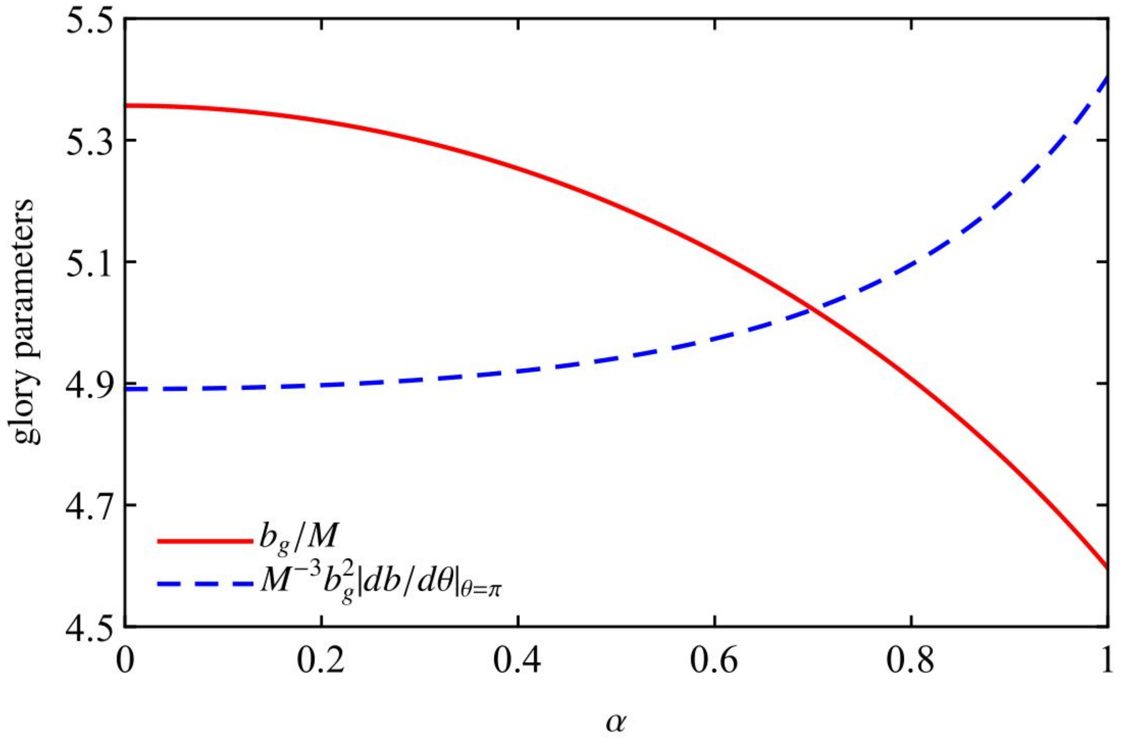


Figure 3: Glory parameters b_g and $b_g^2|db/d\theta|_{\theta=\pi}$.

The Klein-Gordon equation can be written as

[redacted] (12)

We can decompose the scalar field as

$$\Phi \equiv \sum_{l}^{\infty} C_{\omega l} \Phi_{\omega l}$$ [redacted] (13)

The radial function satisfies

[redacted] (14)

where $dr = f(r) dr_{\star}$, with

[redacted] (15)

For the BH scattering problem, we assume the following boundary conditions

[redacted] (16)

From the conservation of the flux, one can show that

$$|T_{\omega l}|^2 + |R_{\omega l}|^2 = 1. \tag{17}$$

The differential scattering cross sections is given by (Futterman *et al.*, 1988)

[redacted] (18)

where

[redacted] (19)

with

$$e^{2i\delta_l(\omega)} \equiv (-1)^{l+1} R_{\omega l}. \tag{20}$$

We need to obtain the coefficients $R_{\omega l}$.

We applied the stiffness switching integration method (Press *et al.*, 2007) to solve Eq. (14), namely

[redacted] (14)

Then we match the numerical solutions of Eq. (14) with the boundary conditions given by Eq. (16):

[redacted] (16)

We also adopt the convergence method developed by Yenne *et al.* (1954) and Dolan *et al.* (2006).

We perform the summations in the angular momentum up to $l = 40$.

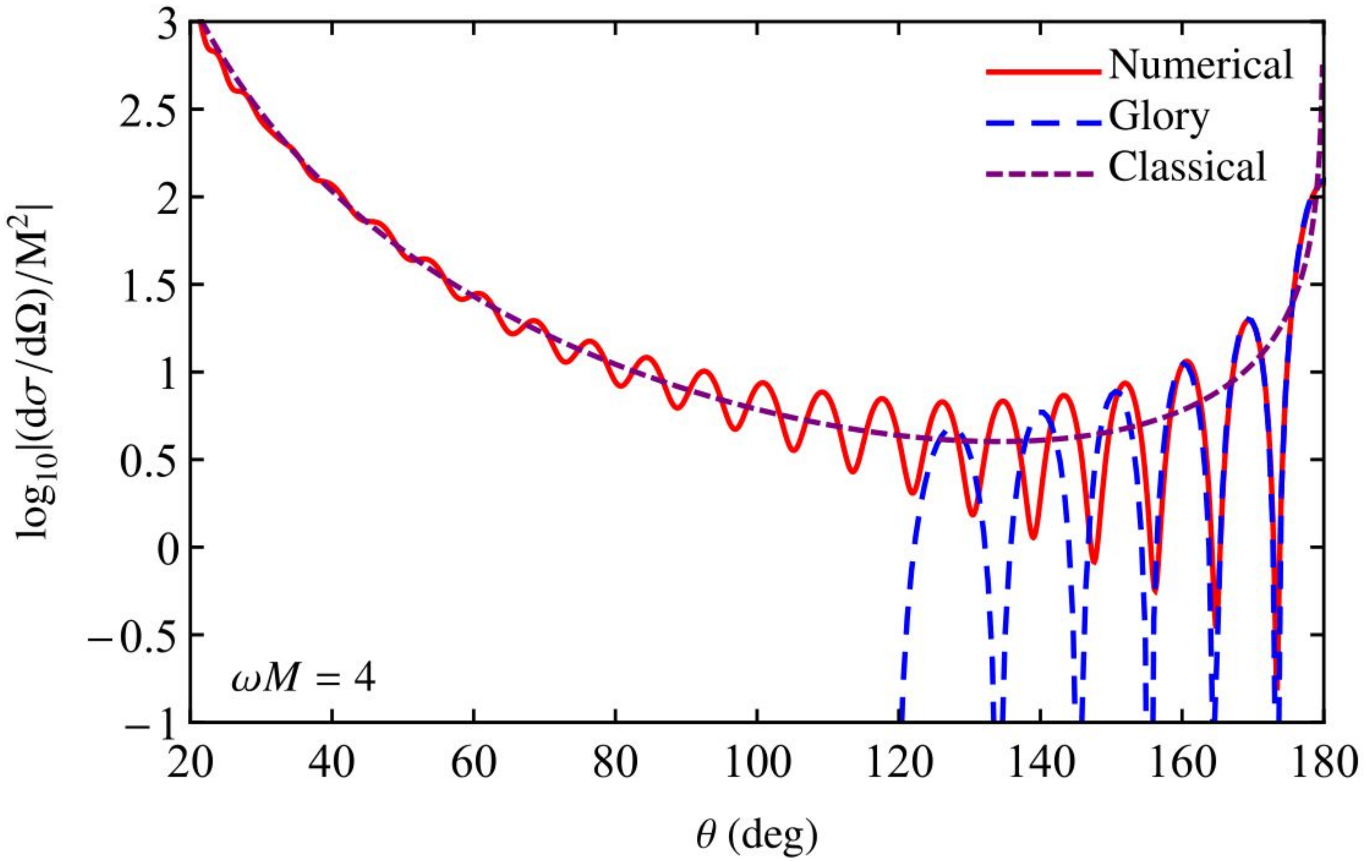


Figure 4: Differential SCS of the ABG RBH, with $\alpha = 0.5$, for $\omega M = 4$ .We also plot the corresponding classical and glory approximations.

$$\frac{d\sigma_{\rm g}}{d\Omega} = 2\pi\omega b_g^2 \left|\frac{db}{d\theta}\right|_{\theta=\pi} J_0^2(\omega b_g \sin\theta).$$

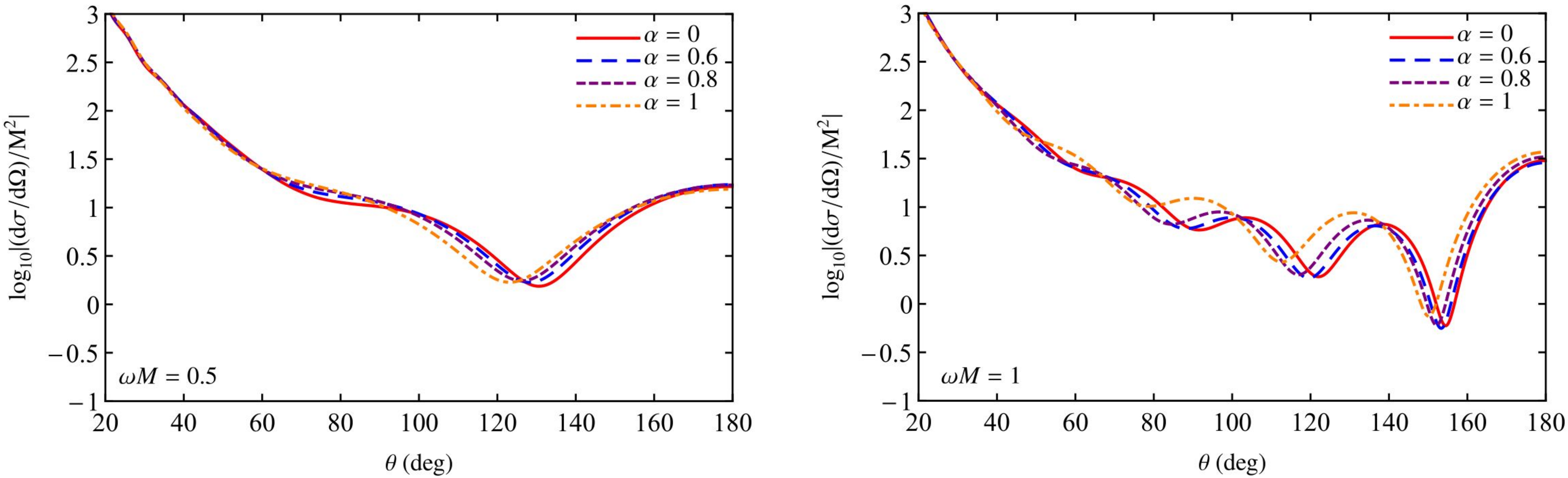


Figure 5: Differential SCS of the ABG RBH for distinct values of α and ωM.

[redacted]

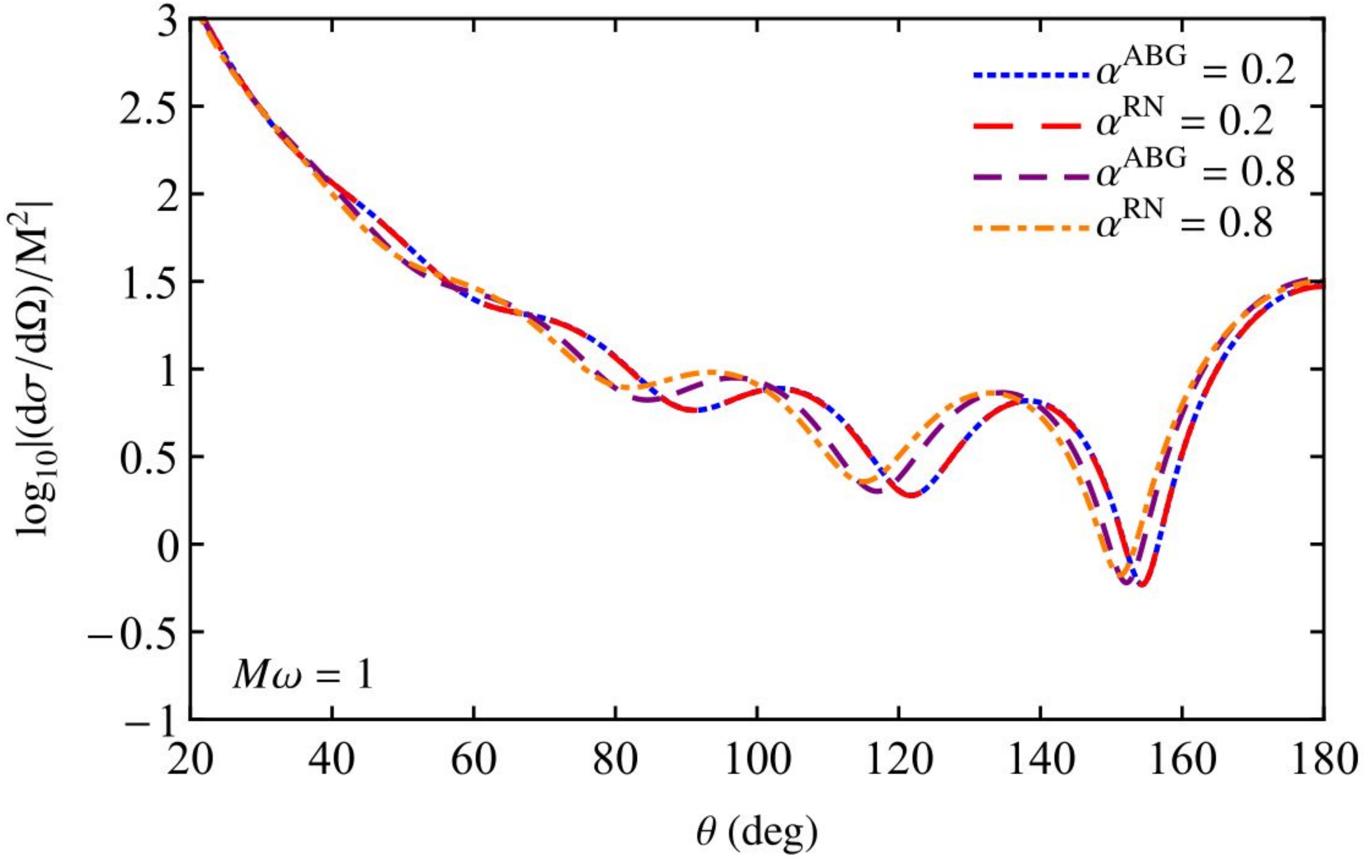


Figure 6: Differential SCS of ABG and RN BHs for two distinct values of α, considering $\omega M = 1$ in both cases.

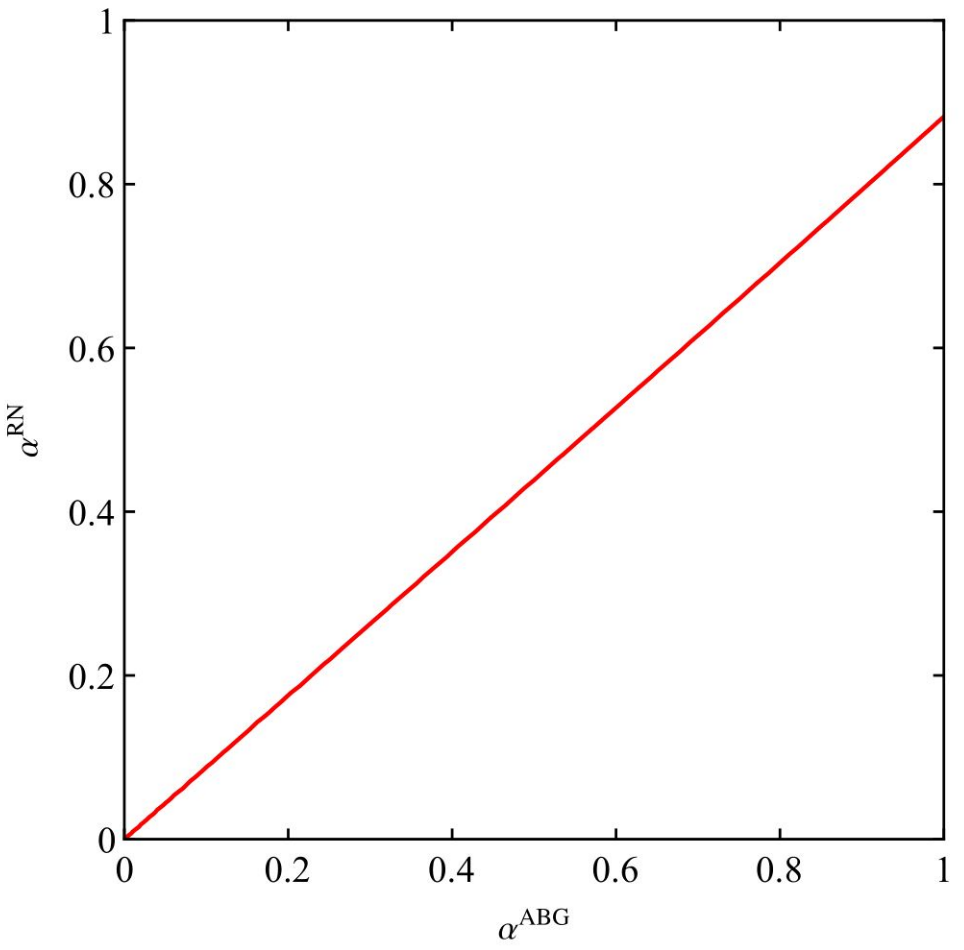


Figure 7: Values of the pair (α^{ABG}, α^{RN}) for which their impact parameter of backscattered rays coincide.

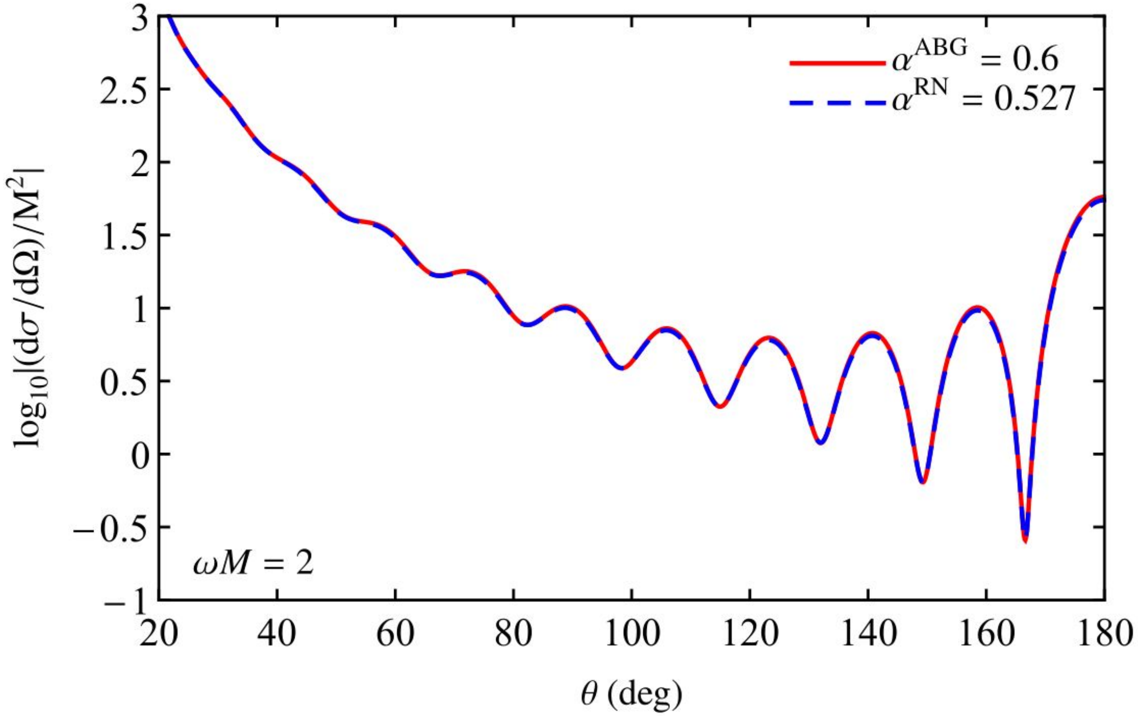


Figure 8: Differential SCS for $(\alpha^{\text{ABG}}, \alpha^{\text{RN}}) = (0.6, 0.527)$, considering $\omega M = 2$ in all cases.

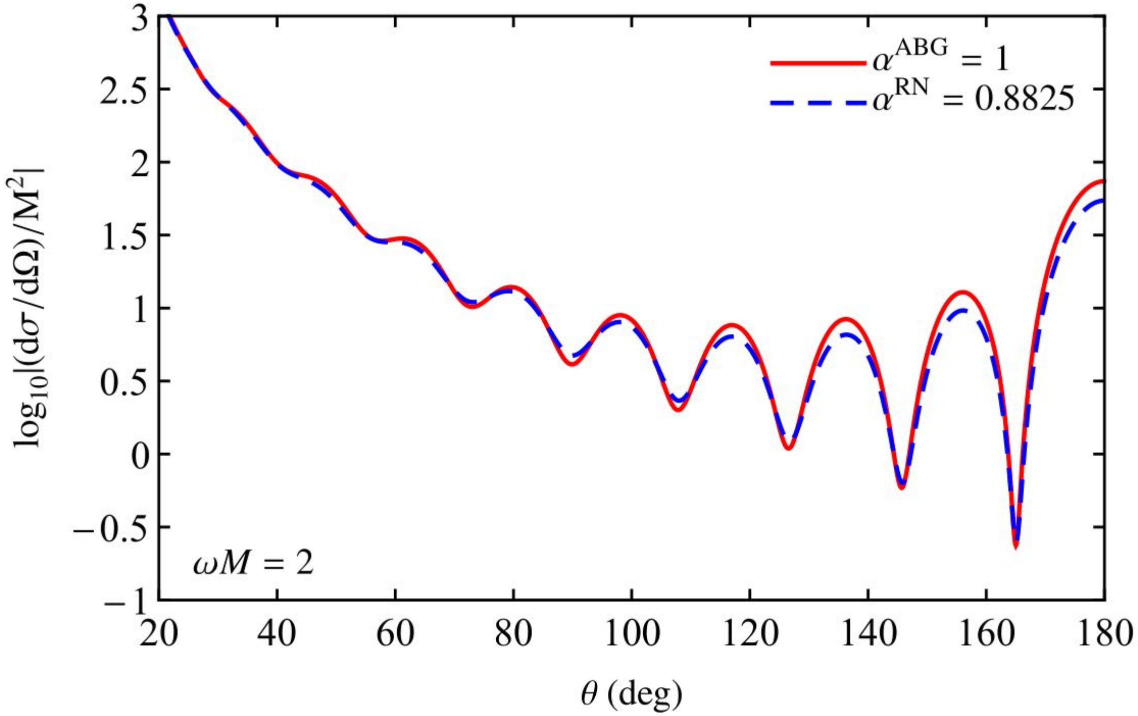


Figure 9: Differential SCS for $(\alpha^{\text{ABG}}, \alpha^{\text{RN}}) = (1, 0.8825)$, considering $\omega M = 2$ in all cases.

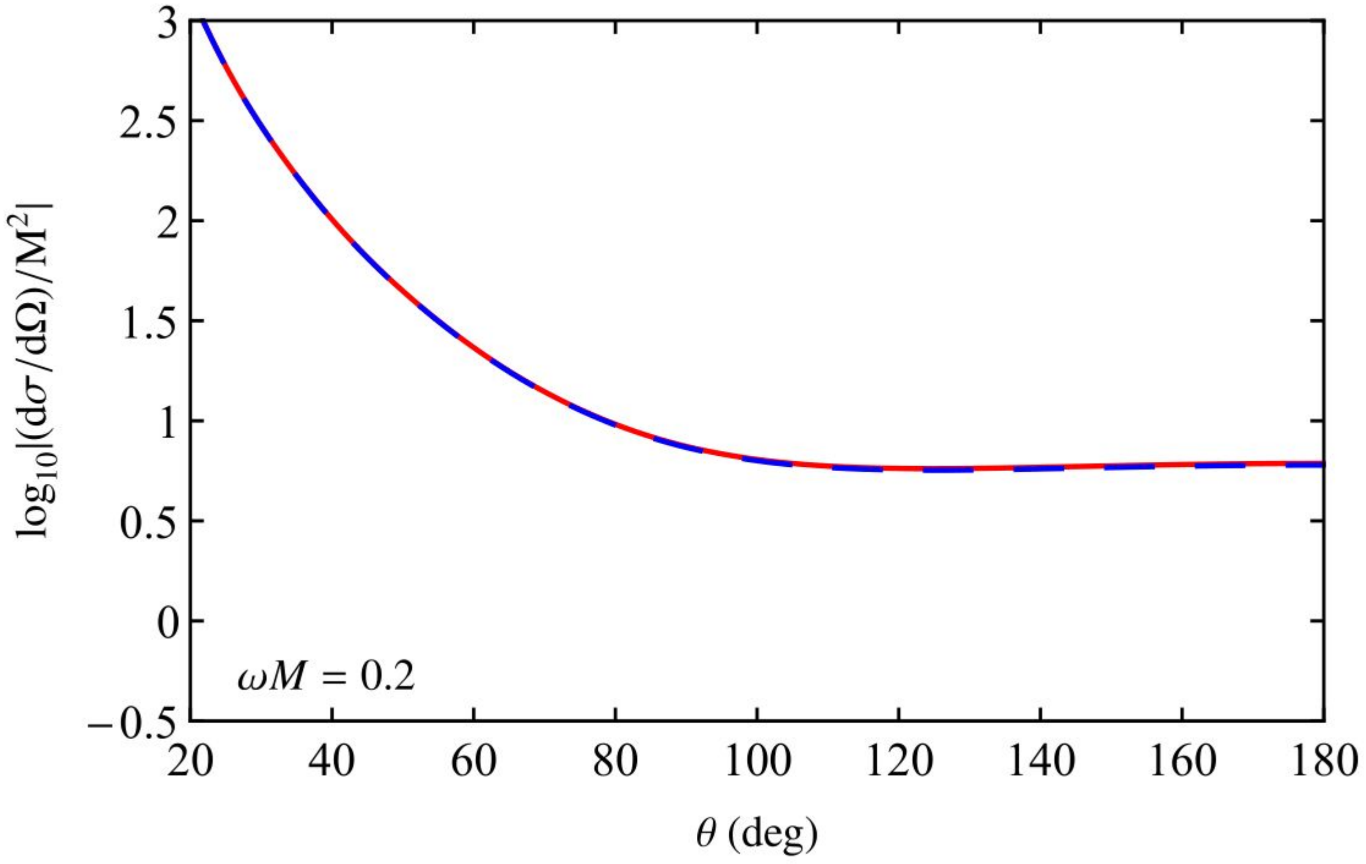
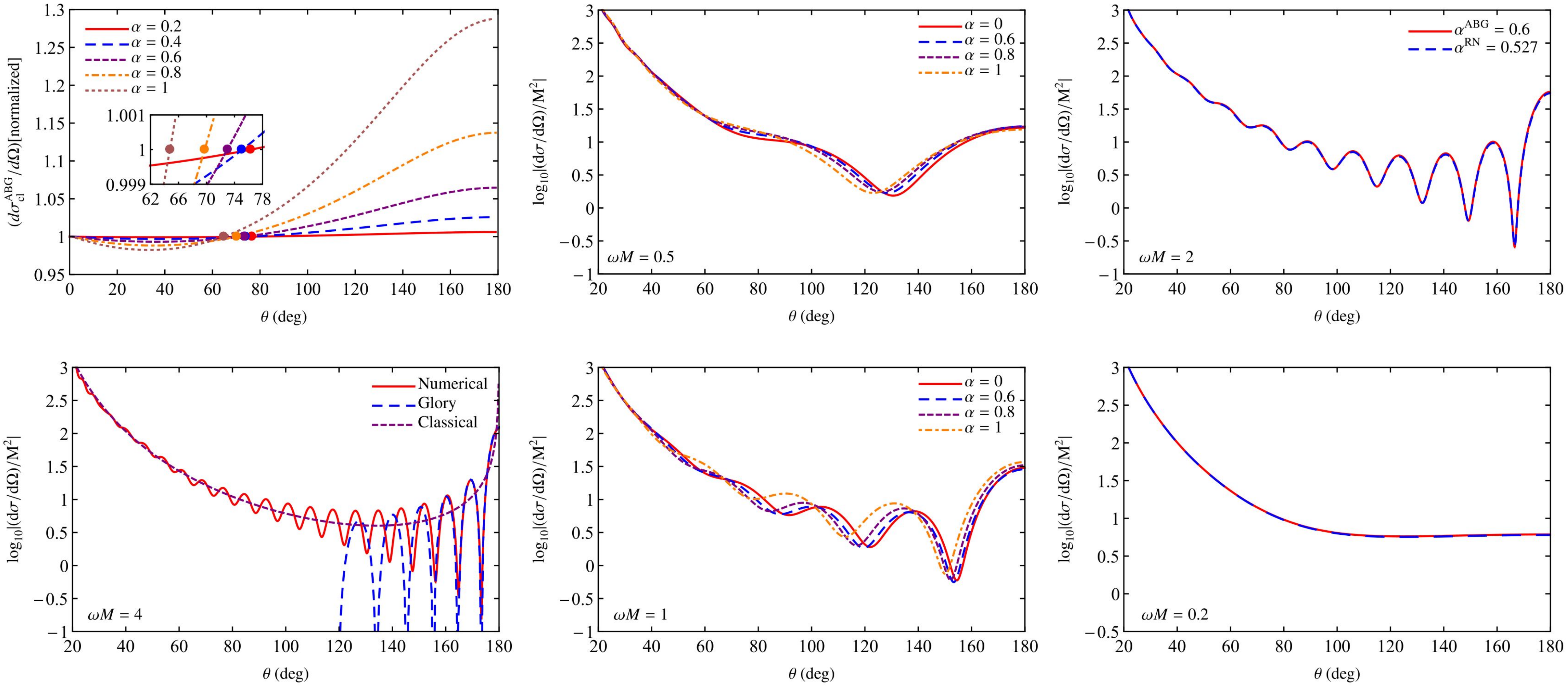


Figure 10: Differential SCS for the pair $\alpha^{\mathrm{ABG}} = 0.4$ and $\alpha^{\mathrm{RN}} = 0.3509$, which satisfies $b_g^{\mathrm{ABG}} = b_g^{\mathrm{RN}}$, considering $\omega M = 0.2$.

✔ We have investigated the scattering properties of a massless and chargeless test scalar field in the background of ABG RBHs, and compared our results with the RN ones.



As future perspectives, we can:

- Consider massive and/or scalar fields or fields with higher order spins;

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Quasinormal modes of charged black holes with corrections from nonlinear electrodynamics

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- Rotating spacetimes.

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Generating rotating regular black hole solutions without complexification

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Thank you all for your attention.