# Bosonic Dark Matter

Lorenzo Annulli

Amazonian Workshop on Gravity and Analogue Models

Universidade Federal do Pará, Belem, November 2022

# This short-course

## Aim

Introduce the audience to bosonic fields
and their role as dark matter candidates

## How

Is there a problem?

Which are the possible solutions?

Which model we will explore?

How to test them?

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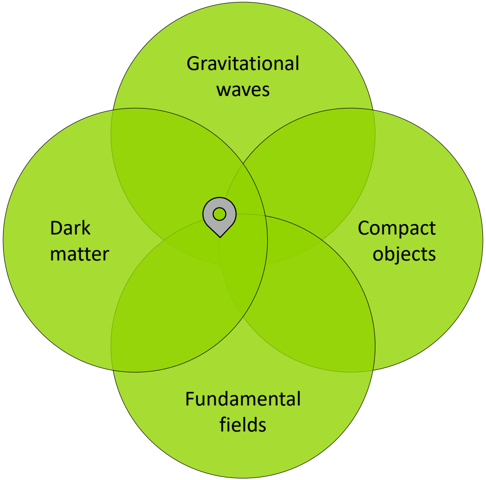
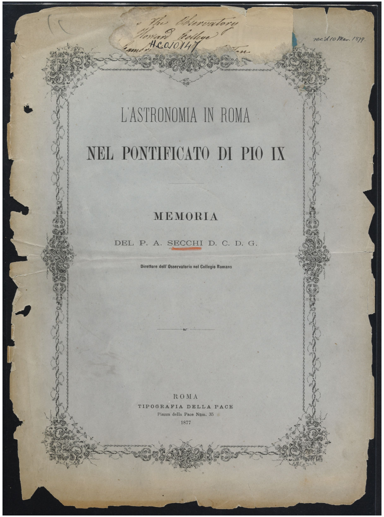
Thanks to the organizers for the warm hospitality!


Gravitational waves
Dark matter
Compact objects
Fundamental fields


ASIA
AFRICA

GIO DELLA LONGA INC.

VEDUTA ESTERNA DELL' OSSERVATORIO DEL COLLEGIO ROMANO
NELLA CHIESA DI S. IGNAZIO




L'ASTRONOMIA IN ROMA

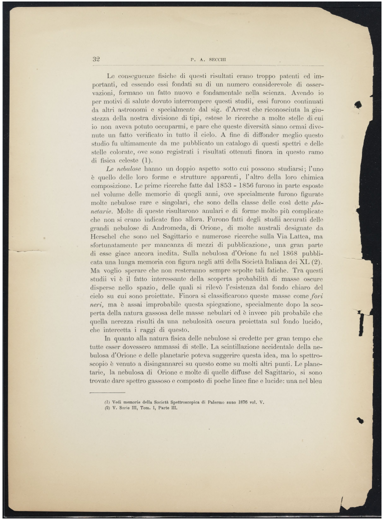
NEL PONTIFICATO DI PIO IX

MEMORIA

DEL P. A. SECCHI D. C. D. G.

Direttore dell'Osservatorio nel Collegio Romano

ROMA
TIPOGRAFIA DELLA PACE
Piazza della Pace Num. 35
1877


32 P. A. SECCHI

Le conseguenze fisiche di questi risultati erano troppo patenti ed importanti, ed essendo essi fondati su di un numero considerevole di osservazioni, formano un fatto nuovo e fondamentale nella scienza. Avendo io per motivi di salute dovuto interrompere questi studii, essi furono continuati da altri astronomi e specialmente dal sig. d'Arrest che riconosciuta la giustezza della nostra divisione di tipi, estese le ricerche a molte stelle di cui io non aveva potuto occuparmi, e pare che queste diversità siano ormai divenute un fatto verificato in tutto il cielo. A fine di diffonder meglio questo studio fu ultimamente da me pubblicato un catalogo di questi spettri e delle stelle colorate, ove sono registrati i risultati ottenuti finora in questo ramo di fisica celeste (1).

*Le nebulose* hanno un doppio aspetto sotto cui possono studiarsi; l'uno è quello delle loro forme e strutture apparenti, l'altro della loro chimica composizione. Le prime ricerche fatte dal 1853 - 1856 furono in parte esposte nel volume delle memorie di quegli anni, ove specialmente furono figurate molte nebulose rare e singolari, che sono della classe delle così dette *planetarie*. Molte di queste risultarono anulari e di forme molto più complicate che non si erano indicate fino allora. Furono fatti degli studii accurati delle grandi nebulose di Andromeda, di Orione, di molte australi designate da Herschel che sono nel Sagittario e numerose ricerche sulla Via Lattea, ma sfortunatamente per mancanza di mezzi di pubblicazione, una gran parte di esse giace ancora inedita. Sulla nebulosa d'Orione fu nel 1868 pubblicata una lunga memoria con figura negli atti della Società Italiana dei XL (2). Ma voglio sperare che non resteranno sempre sepolte tali fatiche. Tra questi studii vi è il fatto interessante della scoperta probabilità di masse oscure disperse nello spazio, delle quali si rilevò l'esistenza dal fondo chiaro del cielo su cui sono proiettate. Finora si classificarono queste masse come *fori neri*, ma è assai improbabile questa spiegazione, specialmente dopo la scoperta della natura gassosa delle masse nebulari ed è invece più probabile che quella nerezza risulti da una nebulosità oscura proiettata sul fondo lucido, che intercetta i raggi di questo.

In quanto alla natura fisica delle nebulose si credette per gran tempo che tutte esser dovessero ammassi di stelle. La scintillazione accidentale della nebulosa d'Orione e delle planetarie poteva suggerire questa idea, ma lo spettroscopio è venuto a disingannarci su questo come su molti altri punti. Le planetarie, la nebulosa di Orione e molte di quelle diffuse del Sagittario, si sono trovate dare spettro gassoso e composto di poche linee fine e lucide: una nel bleu

(1) Vedi memorie della Società Spettroscopica di Palermo anno 1876 vol. V.
(2) V. Serie III, Tom. I, Parte III.

*"[...] there is the interesting probable discovery of dark masses scattered in space [...] they were classified as black cavities, but this explanation is highly improbable [...]"*

# What's the matter?

**1904 and 1906, Lord Kelvin and Henri Poincaré**

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## 1933, Fritz Zwicky

$$M_{\text{Coma}} = \#_{\text{obs galaxies}} \times \bar{M}_{galaxy} \sim 800 \cdot 10^9 \, M_\odot, \text{(Hubble)}, \, \ell = 10^6 \, ly \sim 306000 \, pc \sim 9.4 \cdot 10^{18} \, km$$

Average kinetic energy and velocity dispersion: $80 \, km/s$ vs $1000 \, km/s$ observed one!

*"If this would be confirmed, we would get the surprising result that dark matter is present in much greater amount than luminous matter."*

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Smith: $\bar{M}_{galaxy} \sim 2 \cdot 10^{11} M_\odot \sim 10^{14} M_\odot / 500 = M_{\text{Virgo}} / \#_{\text{obs galaxies}}$ (from orbiting galaxies)

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## 1937, Zwicky

if $\sigma_{\text{disp.}} = 700\, km/s$, using $\#_{\text{o. g.}} = 1000$ and $\ell = 2 \cdot 10^6\, ly$: $M_{\text{Coma}} = 4.5 \cdot 10^{13} M_\odot$ and $M/\gamma = 500$

## 1937 and 1941, Zwicky and Chandrasekhar

Zwicky:*"It is not possible to derive the masses of galaxies from observed rotations, without the use of additional information."*

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# Rotation curves (RC) of galaxies

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Image tube spectrograph (by Ford) to perform observations of Andromeda (with Rubin)

Photometric RC vs 21 cm RC (exponential disk)

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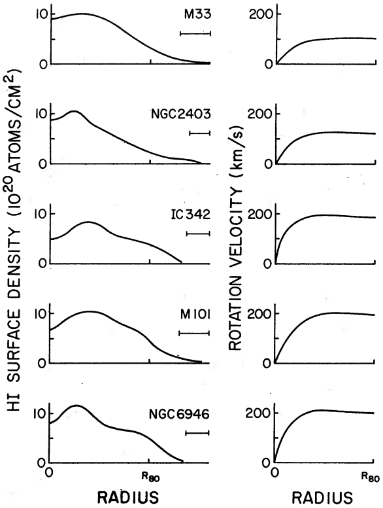
## 1978, Rubin, Ford and Norbert Thonnard

Optical RC for ten high-luminosity spiral galaxies found flat out to the outermost measured radius




*Credit: The Astrophysical Journal, 176, 1972, D. H. Rogstad and G. S. Shostak*

**1974, Einasto, Kaasik and Saar; J. Ostriker, Peebles and Yahil**

April: *"Dynamic evidence on massive coronas of galaxies"*

- ◇ Galactic rotation curves
- ◇ Total vs stellar mass discrepancy → *"[...] corona of unrecognised massive population"*

May: *"The size and mass of galaxies, and the mass of the universe"*

- ◇ Compiled existing mass estimates of mostly giant spiral galaxies
- ◇ Mass estimates from galaxy pairs; dynamics of dwarf galaxies,...

# Dark matter candidates

Cold - velocity wrt $\gamma$ after decoupling - dark - not emitting light nor interacting if not gravitationally

On large scales ($> 10\,kpc$) the predictions of $\Lambda$CDM have been amply tested

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## Non-baryonic CDM

WIMPs

- Neutrinos: not viable, but very long lived and without electromagnetic or strong interactions
- Motivation of supersymmetric particles does not rely on the dark matter problem (electroweak hierarchy problem, enabling gauge coupling unification)
- A zoo of WIMPs? Not too heavy ($1 - 100\,keV$); annihilation cross section similar to weak scale, etc...

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Axions: the QCD solvers?

- $\mathcal{L}_{\rm QCD} \sim \bar{\Theta}\frac{g^2}{32\pi^2}G^{a\mu\nu}\tilde{G}_{a\mu\nu}$, if $\bar{\Theta} \sim \mathcal{O}\left(1\right) \rightarrow$ strong CP violation: $\mu_n \gg$ observed
- Peccei–Quinn mechanism: new global (spontaneously broken at low energies) symmetry that give rise to a pseudo-Goldstone boson, with mass $\sim \lambda^2_{\rm QCD}/f_{\rm PQ}$

# Dark matter candidates

**Baryonic, Black holes, Modified gravity, etc...**

Massive astrophysical compact halo objects

- Brown/red /white dwarfs, neutron stars, and BHs...
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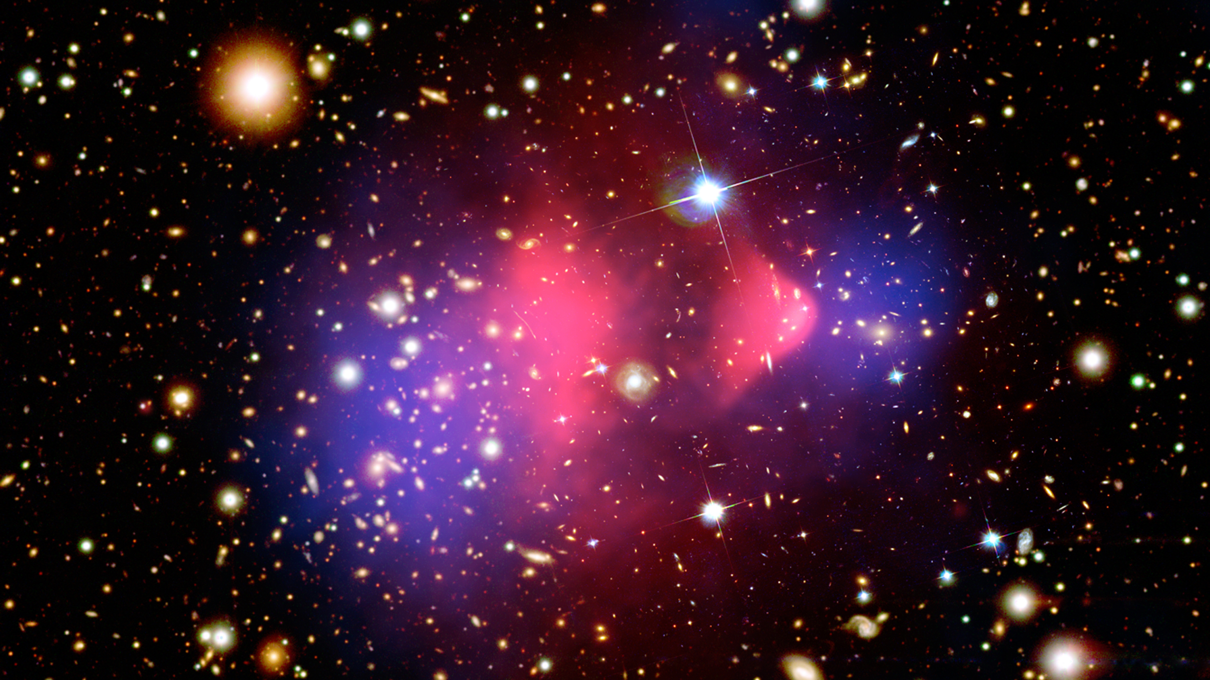
MOND: Modified Newtonian Dynamics, aka Tensor-Vector-Scalar gravity

- Two additional fields, three free parameters, one free function
- Explaining: dynamics of spiral and elliptical galaxies; low surface brightness; observed RC of hundreds of spiral galaxies
- Not explaining: dark matter abundance in galaxy clusters, ...
- "A direct empirical proof of the existence of dark matter" $(2006)$



## Cusp-halo problem

Expected DM density cusps in the centers of galaxies: gravitational stirring due to supernovae?

## Dwarf galaxies problem aka missing satellite

$$dn(M_*) \sim M_*^{-1/2} dM_*$$
$$dn(M_h) \sim M_h^{-2} dM_h$$

Lack of efficiency of baryon physics? Tidal disruption?

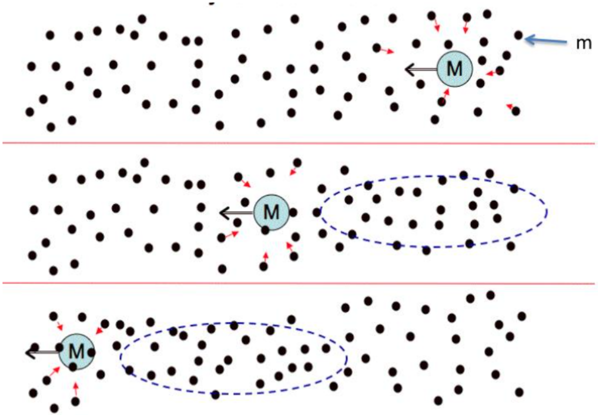
## Dwarf galaxies positioning

Dwarf galaxies *not* clustering at the center: drag from dynamical friction?

## And much more

Satellite disk problem, Galaxy morphology problem ...

# Dynamical friction




Credit: Alice Quillen

# Massive scalar fields

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Approximate shift symmetry [Hui+, 2016]

DM with large $\lambda_{\rm deBroglie} \sim 1$kpc for $\mu \sim 10^{-22}$eV

Absence of DM cusps

Weakness of dynamical friction in dwarf galaxies

# Ultra-light scalars DM

Stability of DM structures

*Which are the proper modes of these objects?*

*Stable or unstable modes? Can we detect them?*

Interaction with surrounding bodies

*How a BH changes the local DM density?*

*And a moving star?*

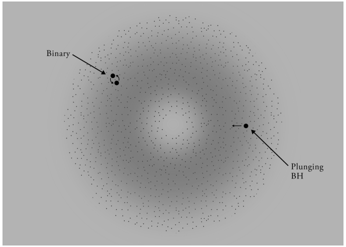
*Effect of dynamical friction on "kicked" black holes?*

Impact on scalar and GW emission

*Nontrivial environmental effect on GW phase?*

*Constraints from an Hulse & Taylor-like experiments?*

*M 87* black hole. Credit: EHT Collaboration*




*Credit: Ana Carvalho*

# Theoretical framework

Derrick's theorem [Derrick, 1964] : "no stable, time-independent, localized scalar field solutions to non-linear wave equation in three (spatial) dimensional flat space exist"

In GR, time-periodic bosonic fields can form self-gravitating structures: Klein-Gordon geons [Kaup, 1968] and Boson stars [Ruffini&Bonazzola, 1969] (see [Liebling&Palenzuela, 2012] )

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## The theory

$$\int d^4x\sqrt{-g}\left(\frac{R}{16\pi}-\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi^*-\frac{\mu^2}{2}|\Phi|^2\right),\quad T^S_{\mu\nu}=\partial_{(\mu}\Phi^*\partial_{\nu)}\Phi-\frac{1}{2}g_{\mu\nu}\left(\partial_\alpha\Phi^*\partial^\alpha\Phi+\frac{\mu^2}{2}|\Phi|^2\right)$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi\right)=\mu^2\Phi,\quad R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=8\pi T^S_{\mu\nu}$$

$$j_\mu=-\frac{i}{2}\left(\Phi^*\partial_\mu\Phi-\Phi\partial_\mu\Phi^*\right),\quad Q=-\int d^3x\sqrt{h}\,j_t,\quad E=\int d^3x\sqrt{h}\,T^S_{tt}$$

## Einstein - Klein Gordon

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi\right)=\mu^2\Phi,\quad R_{\mu\nu}=8\pi\widetilde{T}^S_{\mu\nu}$$

where the trace-reversed stress-energy tensor of the scalar field

$$\widetilde{T}^S_{\mu\nu}\equiv T^S_{\mu\nu}-\frac{1}{2}T^S g_{\mu\nu}=\partial_{(\mu}\Phi^*\partial_{\nu)}\Phi+\frac{1}{2}g_{\mu\nu}\mu^2|\Phi|^2\,.$$

we consider that $\Phi\sim\mathcal{O}(\epsilon)$, with $\epsilon\ll 1$.

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we consider that $\Phi \sim \mathcal{O}(\epsilon)$, with $\epsilon \ll 1$.

In the Newtonian limit, we consider the spacetime metric ansatz

$$g_{tt} = -1 - 2U + \mathcal{O}(\epsilon^4), \quad g_{tj} = \mathcal{O}(\epsilon^3), \; g_{jk} = \mathcal{O}(\epsilon^2)$$

with $j,k = \{x,y,z\}$ and where $U(t,x,y,z) \sim \mathcal{O}(\epsilon^2)$

## Ricci tensor

$$R_{tt} = \nabla^2 U + \mathcal{O}(\epsilon^4), \;\; R_{tj} = \mathcal{O}(\epsilon^3), \; R_{jk} = \mathcal{O}(\epsilon^2)$$

where

$$\partial_t U \sim \mathcal{O}(\epsilon^3), \; \partial_t^2 U \sim \mathcal{O}(\epsilon^4)$$

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## Non-relativistic scalar

The non-relativistic limit of the scalar field $\Phi$ is incorporated in our perturbation scheme by considering that (rigorously by doing an expansion in powers of $(1/c)$)

$$\partial_j \Phi \sim \mathcal{O}(\epsilon^2), \; \partial_t \widetilde{\Phi} \sim \mathcal{O}(\epsilon^3)$$

where we introduced an auxiliary scalar field $\widetilde{\Phi}$ such $\Phi = e^{-i\mu t}\widetilde{\Phi}/\sqrt{\mu}$

It corresponds to the assertion that, in the non-relativistic limit, the energy-momentum relation is $E \sim \mu + \frac{1}{2\mu}p^2 + \mu U$, with $p^2 \ll \mu^2$ and $|U| \ll 1$.

Then

$$\widetilde{T}^S_{tt} = \frac{1}{2}\mu|\widetilde{\Phi}|^2 + \mathcal{O}(\epsilon^4),\ \ \widetilde{T}^S_{tj} = \mathcal{O}(\epsilon^3),\ \widetilde{T}^S_{jk} = \mathcal{O}(\epsilon^2)$$

Therefore, at Newtonian order, the Einstein equations reduce to the Poisson equation

$$\nabla^2 U = 4\pi\mu|\widetilde{\Phi}|^2$$

At leading order $\mathcal{O}(\epsilon^3)$, the Klein-Gordon equation reduces to the Schrödinger equation

$$i\partial_t\widetilde{\Phi} = -\frac{1}{2\mu}\nabla^2\widetilde{\Phi} + \mu U\widetilde{\Phi}$$

# The objects

Spherically symmetric, time-periodic, localized solutions: new DM stars or the core of DM halos.

$$\Phi_0 = \Psi_0(r)e^{-i\Omega t} \quad (\Omega = \mu - \gamma)$$

where $\Psi_0$ is a real-function satisfying

$$\partial_r \Psi_0(0) = 0 \text{ and } \lim_{r\to\infty} \Psi_0 = 0$$

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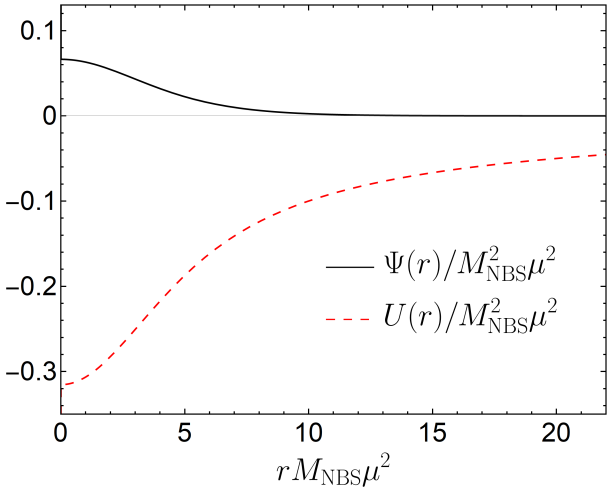
## Newtonian self-gravitating solutions

Consider minimal boson stars – self-gravitating configurations of scalar field in curved spacetime with a simple mass term potential

$$\mathcal{U}_{\mathrm{NBS}} = \frac{\mu^2}{2}|\Phi|^2$$

...gravity is not very strong: NBSs have compactness $M/R \sim 10^{-5}$ (in geometrized units)

# Newtonian Boson star



## “Unus pro omnibus, omnes pro uno!”

Using $\Phi = \Psi(r)e^{-i\Omega t}$

$$\partial_r^2\Psi + \frac{2}{r}\partial_r\Psi - 2\mu\left(\mu U + \gamma\right)\Psi = 0\,,$$
$$\partial_r^2 U + \frac{2}{r}\partial_r U - 4\pi\mu^2\Psi^2 = 0$$

with $0 < \gamma \ll \mu$, $|U| \ll 1$ and $|\Psi| \ll 1$.

SP is invariant for $(\Psi, U, \gamma) \to \lambda^2(\Psi, U, \gamma)$, $r \to r/\lambda$, $\quad M_{\rm NBS} \rightsquigarrow \lambda M_{\rm NBS}$

## “Unus pro omnibus, omnes pro uno!”

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Fundamental (zero-node) NBSs satisfy the scale-invariant relation $\frac{M_{\rm NBS}}{M_\odot} = 9 \times 10^9\,\frac{100\,{\rm pc}}{R}\,\left(\frac{10^{-22}\,{\rm eV}}{\mu}\right)^2$

The number of particles contained in an NBS is $Q_{\rm NBS} = 4\pi\mu\int_0^\infty dr\, r^2\,|\Psi|^2 \rightsquigarrow M_{\rm NBS} \sim \mu\, Q_{\rm NBS}$

# Perturbative framework

Dynamical response to external perturbers

$$\Phi = \left[\Psi_0(r) + \delta\Psi(t,r,\theta,\varphi)\right] e^{-i(\mu-\gamma)t}, \qquad U = U_0(r) + \delta U(t,r,\theta,\varphi)$$

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Linearized Schrödinger-Poisson (SP) system

$$i\partial_t \delta\Psi = -\frac{1}{2\mu}\nabla^2 \delta\Psi + (\mu U_0 + \gamma)\,\delta\Psi + \mu\Psi_0 \delta U \quad + \text{ suitable boundary conditions (BCs)}$$

$$\nabla^2 \delta U = 4\pi\left[\mu^2 \Psi_0 \left(\delta\Psi + \delta\Psi^*\right) + P\right]$$

External point-like perturber $P \equiv m_p \frac{\delta\left(r - r_p(t)\right)}{r^2} \frac{\delta\left(\theta - \theta_p(t)\right)}{\sin\theta} \delta\left(\varphi - \varphi_p(t)\right)$

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From $\delta\Psi$: energy, linear and angular momenta radiated from the flux of certain currents

Eternal process (i.e. *circular motion*)

$$\left(\dot{E}^{\rm rad}, \dot{P}_i^{\rm rad}, \dot{L}_z^{\rm rad}, \dot{Q}^{\rm rad}\right)$$

Finite time interaction (i.e. *plunge*)

$$\left(dE^{\rm rad}/d\omega, dP^{\rm rad}/d\omega, dL^{\rm rad}/\omega, dQ^{\rm rad}/d\omega\right)$$

# Perturbative framework - details 1

$$\delta\Psi = \sum_{l,m} \int \frac{d\omega}{\sqrt{2\pi} r} \left[ Z_1^{\omega l m} Y_l^m e^{-i\omega t} + \left( Z_2^{\omega l m} \right)^* \left( Y_l^m \right)^* e^{i\omega t} \right] \ldots \text{ and similar for } \delta U \text{ and } P$$

$$\partial_r \boldsymbol{X} - V_{\mathrm{B}}(r) \boldsymbol{X} = \boldsymbol{P}$$

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with the vector $\boldsymbol{X} \equiv (Z_1, Z_2, u, \partial_r Z_1, \partial_r Z_2, \partial_r u)^T$ and the matrix $V_B$ given by

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ V - 2\mu(\omega - \gamma) & 0 & 2\mu^2 \Psi_0 & 0 & 0 & 0 \\ 0 & V + 2\mu(\omega + \gamma) & 2\mu^2 \Psi_0 & 0 & 0 & 0 \\ 4\pi\mu^2 \Psi_0 & 4\pi\mu^2 \Psi_0 & V - 2\mu^2 U_0 & 0 & 0 & 0 \end{pmatrix} .$$

$$V(r) \equiv \frac{l(l+1)}{r^2} + 2\mu^2 U_0, \quad \boldsymbol{P}(r) \equiv (0, 0, 0, 0, 0, 4\pi p)^T$$

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$$V(r) \equiv \frac{l(l+1)}{r^2} + 2\mu^2 U_0, \quad \boldsymbol{P}(r) \equiv (0, 0, 0, 0, 0, 4\pi p)^T$$

The condition of non-relativistic fluctuations translates into the simple inequality $|\omega| \ll \mu$.

# Perturbative framework - details 2

Regularity at the origin,

$$\boldsymbol{X}(r \to 0) \sim \left(ar^{l+1}, br^{l+1}, cr^{l+1}, a(l+1)r^{l}, b(l+1)r^{l}, c(l+1)r^{l}\right)^{T}$$

with complex constants $a$, $b$ and $c$, and the Sommerfeld radiation condition at infinity,

$$\boldsymbol{X}(r \to \infty) \sim \left(Z_1^{\infty} e^{ik_1 r}, Z_2^{\infty} e^{ik_2 r}, u^{\infty}, ik_1 Z_1^{\infty} e^{ik_1 r}, ik_2 Z_2^{\infty} e^{ik_2 r}, 0\right)^{T}$$

with

$$\begin{aligned} k_1 &\equiv \sqrt{2\mu\left(\omega-\gamma\right)} \\ k_2 &\equiv -\left(\sqrt{-2\mu\left(\omega+\gamma\right)}\right)^{*} \end{aligned}$$

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with

$$\begin{aligned} k_1 &\equiv \sqrt{2\mu\,(\omega-\gamma)} \\ k_2 &\equiv -\left(\sqrt{-2\mu\,(\omega+\gamma)}\right)^* \end{aligned}$$

To calculate the fluctuations we will make use of the set of independent homogeneous solutions $\{\boldsymbol{Z}_{(1)}, \boldsymbol{Z}_{(2)}, \boldsymbol{Z}_{(3)}, \boldsymbol{Z}_{(4)}, \boldsymbol{Z}_{(5)}, \boldsymbol{Z}_{(6)}\}$

Then, the matrix

$$F(r) \equiv \left(\boldsymbol{Z}_{(1)}, \boldsymbol{Z}_{(2)}, \boldsymbol{Z}_{(3)}, \boldsymbol{Z}_{(4)}, \boldsymbol{Z}_{(5)}, \boldsymbol{Z}_{(6)}\right)$$

is known as the fundamental matrix of system

A solution of the system which is regular at the origin and satisfies the Sommerfeld condition at infinity can be obtained through the method of variation of parameters

$$Z_1(r) = 4\pi \left[ \sum_{n=1}^{3} F_{1,n}(r) \int_{\infty}^{r} dr' F^{-1}_{n,6}(r') p(r') + \sum_{n=4}^{6} F_{1,n}(r) \int_{0}^{r} dr' F^{-1}_{n,6}(r') p(r') \right],$$

$$Z_2(r) = 4\pi \left[ \sum_{n=1}^{3} F_{2,n}(r) \int_{\infty}^{r} dr' F^{-1}_{n,6}(r') p(r') + \sum_{n=4}^{6} F_{2,n}(r) \int_{0}^{r} dr' F^{-1}_{n,6}(r') p(r') \right],$$

$$u(r) = 4\pi \left[ \sum_{n=1}^{3} F_{3,n}(r) \int_{\infty}^{r} dr' F^{-1}_{n,6}(r') p(r') + \sum_{n=4}^{6} F_{3,n}(r) \int_{0}^{r} dr' F^{-1}_{n,6}(r') p(r') \right],$$

where $F_{i,j}$ is the $(i,j)$-component of the fundamental matrix

# How accurate is our treatment?

## Point-like particle

Black hole (BH) perturbations [Zerilli,1970] , drag in perfect fluids [Ostriker, 1999]

Small-scale information is lost, but light fields' Compton wavelength $\gg R_{\rm star}/R_{\rm BH}$ etc.

## Perturbative scheme

$m_p$ as small as necessary? Background neglects higher-order post-Newtonian (PN) terms

$$U_0^2 \ll \delta U \Longrightarrow m_p \gtrsim 10^4 M_\odot \left(\frac{M_{\rm NBS}}{10^{10} M_\odot}\right)^3 \left(\frac{\mu}{10^{-22}\,{\rm eV}}\right)^2$$ but dynamics is OK!

Non-relativistic sources: $v \lesssim R\mu$ for plunges and $\omega_{\rm orb} \lesssim 2 \times 10^{-8}$Hz $\left(\mu/10^{-22}{\rm eV}\right)$
Relativistic sources: LIGO and LISA binaries

# Gross fluxes

Scalar field fluctuations cause a perturbation to its stress-energy tensor, which, at leading order and asymptotically, is given by

$$\delta T^S_{\mu\nu}(r\to\infty)\sim\partial_{(\mu}\delta\Phi^*\partial_{\nu)}\delta\Phi-\frac{1}{2}\eta_{\mu\nu}\left[\partial_\alpha\delta\Phi^*\partial^\alpha\delta\Phi+\mu^2|\delta\Phi|^2\right],\quad \delta\Phi\equiv e^{-i\Omega t}\delta\Psi$$

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The outgoing fluxes $t$ through a 2-sphere at infinity are

$$\dot{E}^{\rm rad}=\lim_{r\to\infty}r^2\int d\theta d\varphi\sin\theta\,\delta T^S_{r\mu}\xi^\mu_t\,,\ \dot{Q}^{\rm rad},\ \dot{L}^{\rm rad}_z$$

with the timelike Killing vector field $\boldsymbol{\xi}_t=-\partial_t$.

$$\dot{E}^{\rm rad}=\int\frac{d\omega}{2\pi}|\omega+\Omega|\,{\rm Re}\left[\sqrt{(\omega+\Omega)^2-\mu^2}\right]\sum_{l,m}|Z_1^\infty(\omega,l,m)+(-1)^m Z_2^\infty(-\omega,l,-m)^*|^2$$

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For a process happening during a finite amount of time the change in the NBS energy is

$$\Delta E_{\rm NBS} = -\int_{t=+\infty} d^3x\sqrt{h}\,\delta T^S_{tt} + \int_{t=-\infty} d^3x\sqrt{h}\,\delta T^S_{tt} = \mu\Delta Q_{\rm NBS}$$

# Perturber's fluxes - 1

Both the energy and momenta of the scalar configuration may change due to the interaction

$$E^{\text{lost}} = \Delta E + E^{\text{rad}},\, P_z^{\text{lost}} = \Delta P_z + P_z^{\text{rad}},\, L_z^{\text{lost}} = \Delta L_z + L_z^{\text{rad}}\,,$$

$\Delta E$ etc. normally include terms mixing $\Phi_0$ with $\delta^2\Phi$

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$U(1)$-invariant action, Noether's theorem implies

$$\nabla_\mu\, \delta j^\mu = 0\,,$$

Using the divergence theorem, we obtain that the number of particles is conserved,

$$\Delta Q = -\int_{t=+\infty} d^3x\sqrt{h}\,\delta j_t + \int_{t=-\infty} d^3x\sqrt{h}\,\delta j_t = -Q^{\rm rad}\,,$$

... the number of particles lost by the configuration matches the number of radiated particles – no scalar particles are created!

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For NBS, $\Delta M$ from $Q^{\rm rad}$

# Outlook

**Free perturbations**

- NBS quasi normal modes

**Sourced perturbations**

Static

- BH in the center of a NBS
- BH eating its host NBS

Dynamics

- Massive objects plunging into NBSs
- A perturber oscillating at the center
- Low/high energy binaries within NBSs

*Image: M 87* black hole, EHT Collaboration*

# NBS proper modes

Quasi normal modes (QNMs) of NBSs are solutions of the sourceless SP with proper BCs

| $l$ | $\omega^n_{\rm QNM} M^{-2}_{\rm NBS} \mu^{-3}$ | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0.0682 | 0.121 | 0.138 | 0.146 | 0.151 | 0.154 | 0.159 |
| 1 | 0.111 | 0.134 | 0.144 | 0.149 | 0.153 | 0.157 | 0.162 |
| 2 | 0.106 | 0.131 | 0.143 | 0.149 | 0.153 | 0.156 | 0.161 |

Modes cluster around $\gamma \simeq 0.162712 M^2_{\rm NBS} \mu^3$

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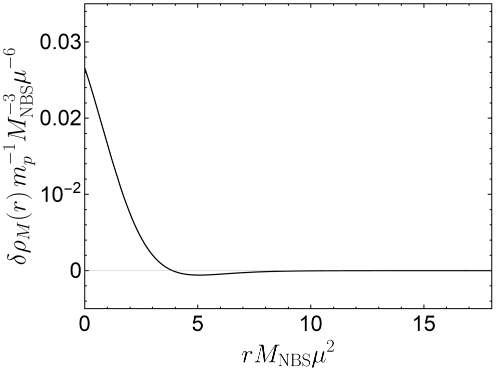
Agreement with [Guzman & Urena-Lopez, 2004] and with [Macedo, *private communication*] for rel. BS

# How DM is affected by "impurities"?

Compute $\delta\Psi_p$ and $\delta U_p$ induced by the central particle solving static SP system (adiabaticity)

$$\delta\rho_M(0)/\rho_M(0) \sim 10\, m_p/M_{\rm NBS}$$

Results consistent with [Bar+, 2018]



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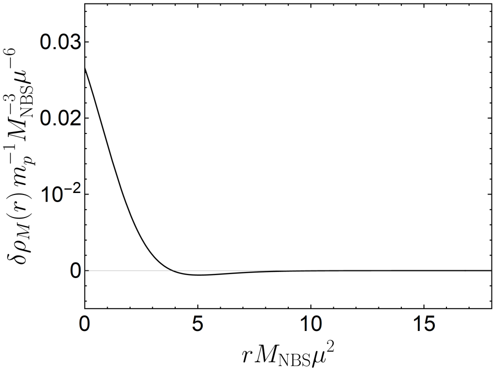


... but yet, the density close to supermassive BH increases significantly in particle-like DM models

[Gondolo&Silk, 2013] [Sadeghian+, 1999]

# Feeding a BH

No hair theorem: no stationary, spherically symm. sol. for non-spin BH with non-trivial scalars

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The entire NBS will be accreted by the BH: *what is the lifetime?*

(i) Sphere of radius $r_+ = 2M_{\rm BH}$, flux of energy $\dot{E}_{\rm in} \approx 10^{-3}\mu^7 r_+^2 M_{\rm NBS}^5$ in and out

(ii) Low-freq. waves ($\mu M_{\rm BH} \ll 1$) are poorly absorbed [Unruh, 1976]

$$\dot{E}_{\rm abs} = 32\pi \left(M_{\rm BH}\mu\right)^3 \dot{E}_{\rm in} = \frac{16\pi}{125}\frac{M_{\rm BH}^5}{M_{\rm NBS}^5}\left(M_{\rm NBS}\mu\right)^{10}$$

(iii) With $\dot{E}_{\rm abs} = \dot{M}_{\rm BH}$ and fixed NBS mass

$$\tau \sim \frac{1}{M_{\rm BH}^4 M_{\rm NBS}^5 \mu^{10}} = 10^{24}\,{\rm yr}\,\frac{M_{\rm NBS}}{10^{10}M_\odot}\left(\frac{\chi}{10^4}\right)^4\left(\frac{0.1}{M_{\rm NBS}\mu}\right)^{10}$$

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(iv) With rotation, superradiance kicks in [Herdeiro&Radu, 2016]

# Dynamical perurbations of NBSs

# Outlook

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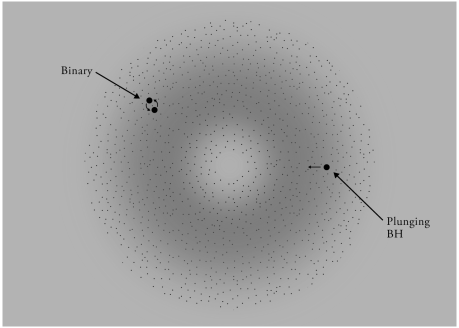



*Animation: Ana Carvalho*

# Dynamical friction

"Momentum loss by a massive moving object due to its gravitational interaction with its own gravitationally-induced wake" [Ostriker, 1999]

Collisionless [Chandrasekhar, 1943] , astrophysical consequences [Binney&Tremaine, 1987] , ultralight fields toymodel [Hui+, 2016] (non self-gravitating scalar)

## How do we compute this friction?

Total scalar flux ($E^{\rm rad}$), energy and momentum lost by the perturber, together with the change in the background configuration momentum:

$$P^{\rm lost} = -E^{\rm lost}/v_R \;\rightsquigarrow\; dP/dt \sim P^{\rm lost} v/(2R)$$

Key role: conservation of the number of particles

# Diving into dark matter

*Why is it interesting?*

Matter tends to accumulate near the center of a DM structure

Collapse can impart a recoil velocity to the BH $\sim 300\text{Km/s}$ [Bekenstein, 1973]

Stars crossing an NBS

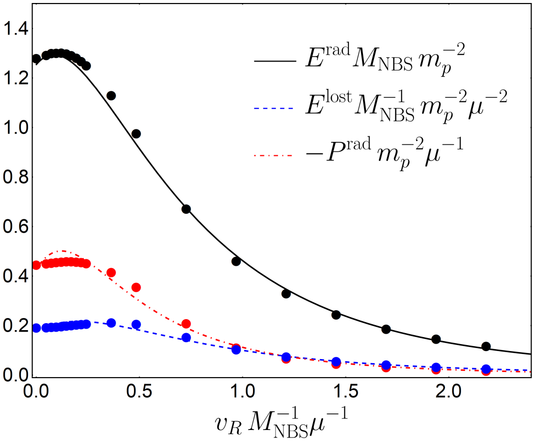
*How is it interesting?*

Escape from the surface of the NBS, $v_{esc} \sim 0.47 M_{\text{NBS}}\mu \rightsquigarrow \begin{cases} \text{Unbound} \\ \text{Oscillatory} \end{cases}$

*What is interesting?*

Spectral fluxes of energy, linear momentum and energy lost by the perturber

The energy and momentum fluxes converge exponentially with increasing values of $\ell$ and yield




Non-relativistic astrophysical relevant velocities: $0 \lesssim v_R[\mathrm{km/s}] \lesssim 6000$ (Milky Way DM core)

# BH oscillating - details

Perturber's motion $z_p(t) = -\sqrt{(3v_0^2)/(4\pi\rho_{\rm DM})}\,\sin\left(\sqrt{4\pi\rho_{\rm DM}/3}t\right)$

Using the low-energy limit $\gamma \ll \mu, \omega_{\rm osc} \ll \mu$, this approximate the numerical results:

$$\dot{E}^{\rm lost} = \frac{2\sqrt{2}}{\pi}(m_p\mu)^2 \left(\frac{2\omega_{\rm osc} - \gamma}{\mu}\right)^{\frac{3}{2}} \sum_l c_l \left(\frac{\mathcal{A}}{R}\right)^{2(l+1)}$$

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When the condition

$$\frac{\dot{E}^{\rm lost}\left(\frac{2\pi}{\omega_{\rm osc}}\right)}{\frac{1}{2}m_p\omega_{\rm osc}^2\mathcal{A}^2} \ll 1$$

is verified, the system is suited to an adiabatic approximation, and

$$m_p\omega_{\rm osc}^2\mathcal{A}\,\dot{\mathcal{A}} = -\dot{E}^{\rm lost}$$

*How long a BH settles down at the center of a DM halo purely due to dynamical friction?*

At the center of a NBS the energy density is approx constant $\rho_E \simeq 4 \times 10^{-3} M_{\rm NBS}^4 \mu^6$

$$\tau_s \sim \frac{M_{\rm NBS}}{m_p}\left(\frac{2\pi}{\omega_{\rm osc}}\right) \sim 10^{10}{\rm yr}\left(\frac{10^{-22}\,{\rm eV}}{\mu}\right)^2\left(\frac{10^5 M_\odot}{m_p}\right)\left(\frac{0.01}{M_{\rm NBS}\mu}\right)$$

obtained for velocities up to $300$Km/s (Milky Way DM core)

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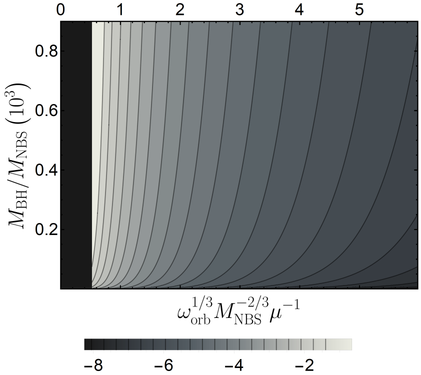
Dynamical friction on BHs ejected from galaxy cores due to stellar distribution (N-body) *[Gualandris & Merritt, 2008]*

$$\tau^* \sim 0.1\,\frac{M_{\rm c}}{m_p}\left(\frac{2\pi}{\omega_{\rm osc}}\right), \text{ if } M_{\rm c} = M_{\rm NBS} \Longrightarrow \tau^* \sim 0.1\,\tau_s$$

Source modelled: $P = \frac{m_p}{r_{\rm orb}^2}\delta(r - r_{\rm orb})\delta\left(\theta - \frac{\pi}{2}\right)\left[\delta(\varphi - \omega_{\rm orb}t) + \delta(\varphi + \pi - \omega_{\rm orb}t)\right]$

Alternatively, applying the transformation $m_p(1 + (-1)^m) \to m_p$ for EMRIs




Logarithm of the universal rate of scalar energy radiated by an EMRI: $\log_{10}\left[\dot{E}^{\rm rad}_{\rm EMRI}\left(m_p^2 M_{\rm NBS}\mu^3\right)^{-1}\right]$.

At high, but still non-relativistic freqs we find the following analytic solution

$$\dot{E}^{\rm lost} \simeq 0.28\,\pi^3\,(\mu m_p)^2\,(\mu M_{\rm NBS})^4 \sum_{m=1}^{+\infty} \left[1+(-1)^m\right]^2$$
$$\times \left( \frac{Y_m^m\left(\frac{\pi}{2},0\right)}{\Gamma\left(m+\frac{3}{2}\right)} \frac{m^{\left(\frac{m}{2}-\frac{1}{4}\right)}(M\omega_{\rm orb})^{\frac{m}{3}}}{2^{\left(\frac{7}{4}+\frac{m}{2}\right)}(\omega_{\rm orb}/\mu)^{\left(\frac{1}{4}+\frac{m}{2}\right)}} \right)^2 \Theta\left[m\omega_{\rm orb}-\gamma\right]$$

Flux converges exponentially in $l$ and it agrees with full numerical solution

Coupling between gravity and scalar implies large frequency sources radiate less

Depletion time by BH binaries $>$ Hubble time

## Hulse & Taylor

Energy emitted in scalar waves by equal-mass binaries, with respect to their own quadrupole

$$\dot{E}^{\rm GW} = (32/5) m_p^2 M^{-2} \left(M\omega_{\rm orb}\right)^{10/3}$$

$$\frac{\dot{E}^{\rm lost}}{\dot{E}^{\rm GW}} \sim 10^{-5} \left[\frac{M_{\rm NBS}\mu}{0.01}\right]^4 \left[\frac{\mu}{10^{-22}\,{\rm eV}}\right]^{\frac{9}{2}} \left[\frac{T_{\rm orb}}{16\,{\rm yrs}}\right]^{\frac{9}{2}}$$

## LIGO/LISA sources - Fourier space

Use $\dot{E}^{\rm lost} + \dot{E}^{\rm GW}$ for the correction to the GW phase [Flanagan &Hughes, 1997]

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Use $\dot{E}^{\rm lost} + \dot{E}^{\rm GW}$ for the correction to the GW phase [Flanagan &Hughes, 1997]

Decompose the phase of the GW signal

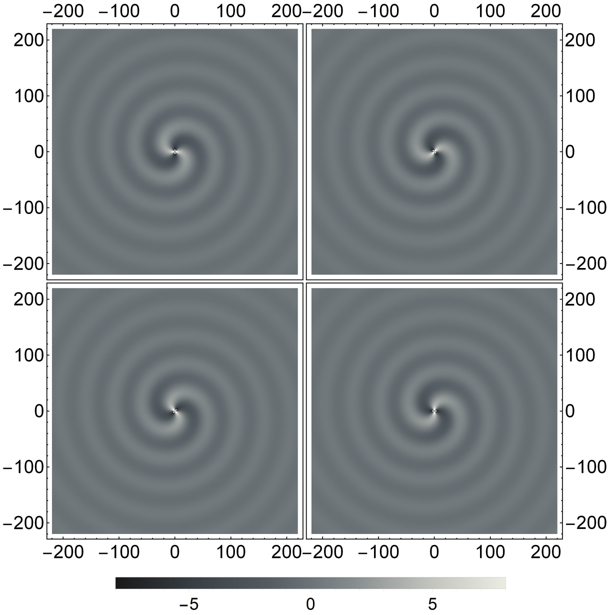
$$\tilde{h}(f) = \mathcal{A} e^{i\Upsilon(f)}, \;\; \Upsilon(f) = \Upsilon^{(0)}[1 + \delta_\Upsilon]$$

where $\Upsilon^{(0)} = 3/128(\mathcal{M}\pi f)^{-5/3}$ is the leading term of the phase's PN expansion, and $f = \omega_{\rm orb}/\pi$

$$\delta_\Upsilon = \frac{16\mu^4\Psi_0^2}{51\pi^3 f^4} \sim 10^{-24}\left[\frac{\mu}{10^{-22}\,{\rm eV}}\right]^4 \left[\frac{10^{-4}}{f}\right]^4 \left[\frac{M_{\rm NBS}\mu}{0.01}\right]^4$$

$$\delta_\Upsilon = \frac{16\mu^4\Psi_0^2}{51\pi^3 f^4} \sim 10^{-8}\left[\frac{\mu}{10^{-19}\,{\rm eV}}\right]^4 \left[\frac{10^{-4}{\rm Hz}}{f}\right]^4 \left[\frac{M_{\rm NBS}\mu}{0.1}\right]^4$$

for scalar contribution (equal-mass binaries) correction corresponds to a $-6PN$ order contribution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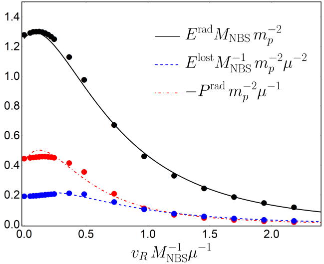
200
100
0
−100
−200
−5 0 5

## Plunge




Scalar dynamical friction?

$$\tau_s \sim 10^{10}{\rm yr}\left[\frac{10^{-22}\,{\rm eV}}{\mu}\right]^2\left[\frac{10^5 M_\odot}{m_p}\right]\left[\frac{0.01}{M_{\rm NBS}\mu}\right]$$

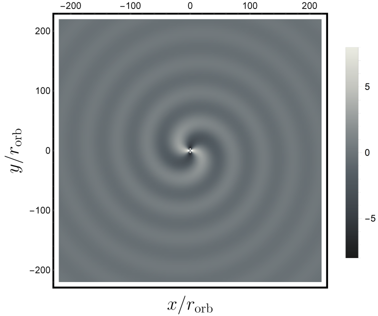
Stellar dynamical friction? [Gualandris&Merritt, 2008]

$$\tau^* \sim 0.1\,\frac{M_c}{m_p}\left(\frac{2\pi}{\omega_{\rm osc}}\right),\ \text{if } M_c = M_{\rm NBS} \Rightarrow \tau^* \sim 0.1\,\tau_s$$

## Binaries

$$\frac{\dot{E}^{\rm lost}}{\dot{E}^{\rm GW}} \sim 10^{-5}\left[\frac{M_{\rm NBS}\mu}{0.01}\right]^4\left[\frac{\mu}{10^{-22}\,{\rm eV}}\right]^{\frac{9}{2}}\left[\frac{T_{\rm orb}}{16\,{\rm yrs}}\right]^{\frac{9}{2}}$$

$$\delta_\Upsilon \sim 10^{-8}\left[\frac{\mu}{10^{-19}\,{\rm eV}}\right]^4\left[\frac{10^{-4}{\rm Hz}}{f}\right]^4\left[\frac{M_{\rm NBS}\mu}{0.1}\right]^4$$




Benchmark e.g. [Traykova, 2021]

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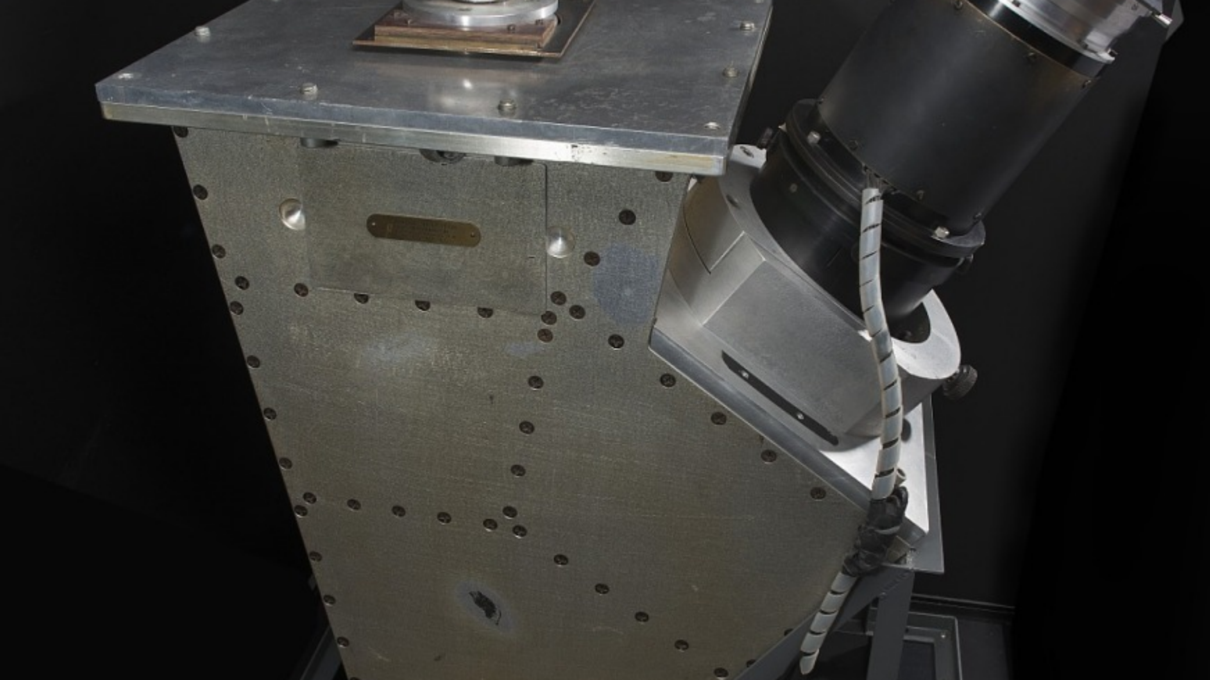


Até logo!

## 2. PROOF

If $\theta$ is a function of $\mathbf{r}$ only, we can replace (4) by $\delta E = 0$ with the energy $E$ given by

$$E = \int [(\nabla \theta)^2 + f(\theta)] \, d^3\mathbf{r}.$$

A necessary condition for the solution to be stable is that the second-order variation $\delta^2 E \geq 0$. Suppose $\theta(\mathbf{r})$ is a localized solution of $\delta E = 0$. Define $\theta_\lambda(\mathbf{r}) = \theta(\lambda \mathbf{r})$ where $\lambda$ is an arbitrary constant, and write $I_1 = \int (\nabla \theta)^2 \, d^3\mathbf{r}$, $I_2 = \int f(\theta) \, d^3\mathbf{r}$.
Then

$$E_\lambda = \int [(\nabla \theta_\lambda)^2 + f(\theta_\lambda)] \, d^3\mathbf{r}$$

$$= I_1/\lambda + I_2/\lambda^3$$

on changing the variable of integration from $\mathbf{r}$ to $\lambda\mathbf{r}$; whence

$$(dE_\lambda/d\lambda)_{\lambda=1} = -I_1 - 3I_2,$$

$$(d^2E_\lambda/d\lambda^2)_{\lambda=1} = 2I_1 + 12I_2.$$

Since $\theta_\lambda$ is a solution of $\delta E = 0$ for $\lambda = 1$, we must have

$$(dE_\lambda/d\lambda)_{\lambda=1} = 0, \qquad I_2 = -\tfrac{1}{3}I_1,$$

$$(d^2E_\lambda/d\lambda^2)_{\lambda=1} = -2I_1 < 0.$$

That is, $\delta^2 E < 0$ for a variation corresponding to a uniform stretching of the "particle." Hence the solution $\theta(\mathbf{r})$ is unstable, proving the theorem.

(ii) Adiabaticy (*no radiation*): sum a trivial homogeneous solution to enforce $\delta Q_{\rm NBS} = \delta M_{\rm NBS} = 0$ keeping $\delta\Psi = \delta\Psi_p + \delta\Psi_\epsilon +$ and $\delta U = \delta U_p + \delta U_\epsilon$

(iii) Which $\epsilon$? $\delta\rho_M = \delta T^S_{00} = \mu\,\delta\rho_Q = 2\mu^2\Psi_0\left(\delta\Psi_p + \frac{\epsilon}{2}\Psi_0\right)$ s.t. $4\pi\int_0^\infty dr\, r^2\delta\rho_M = 0$