Equations of Motion for the Kerr problem

Clear Variables

This command clears every variable that is in memory. It will clear variables in all open notebooks.

In[ ]:=
```
ClearAll["Global`*"]
```

Introduction of the *Ansatz* for the metric

In[ ]:=
```
dim = 4;
coord = {t, x, θ, φ};
```

In[ ]:=
```
gmet =
```

$$\begin{pmatrix} -\text{Exp}[2\,\text{F0}[x,\theta]]\,\frac{x^2}{\text{S}[x]}\,\text{H}[x] + \text{Exp}[2\,\text{F2}[x,\theta]]\,\text{S}[x]\,\text{Sin}[\theta]^2\,\text{W}[x,\theta]^2 & 0 & 0 & -\text{E} \\ 0 & \frac{\text{Exp}[2\,\text{F1}[x,\theta]]}{\text{H}[x]} & 0 & \\ 0 & 0 & \text{Exp}[2\,\text{F1}[x,\theta]]\,\text{S}[x] & \\ -\text{Exp}[2\,\text{F2}[x,\theta]]\,\text{S}[x]\,\text{Sin}[\theta]^2\,\text{W}[x,\theta] & 0 & 0 & \end{pmatrix};$$

In[ ]:=
```
gi = FullSimplify[Inverse[gmet]];
detg = FullSimplify[Det[gmet]];
sqrtdetg =
  FullSimplify[√-detg, x > 0 && F1[x, θ] > 0 && F2[x, θ] > 0 && F0[x, θ] > 0 && Sin[θ] > 0];
```

The next line is only done to be a bit easier to write the computations

In[ ]:=
```
Do[gx[μ, ν] = gmet[[μ, ν]], {μ, 1, dim}, {ν, 1, dim}];
Do[gxi[μ, ν] = gi[[μ, ν]], {μ, 1, dim}, {ν, 1, dim}];
```

In[◦]:=
```
Print["Metric: ", gmet // MatrixForm]
Print["Inverse Metric: ", gi // MatrixForm]
Print["Determinant: ", detg]
Print["SquareRoot of the Determinant: ", sqrtdetg]
```

Metric:
$$\begin{pmatrix} -\frac{e^{2\,F0[x,\theta]}\,x^2\,H[x]}{S[x]} + e^{2\,F2[x,\theta]}\,S[x]\,Sin[\theta]^2\,W[x,\,\theta]^2 & 0 & 0 & -e^{2\,F2[x,\theta]}\,S[x]\,Sin[\theta]^2\,W[x,\,\theta] \\ 0 & \frac{e^{2\,F1[x,\theta]}}{H[x]} & 0 & 0 \\ 0 & 0 & e^{2\,F1[x,\theta]}\,S[x] & 0 \\ -e^{2\,F2[x,\theta]}\,S[x]\,Sin[\theta]^2\,W[x,\,\theta] & 0 & 0 & e^{2\,F2[x,\theta]}\,S[x]\,Sin[\theta]^2 \end{pmatrix}$$

Inverse Metric:
$$\begin{pmatrix} -\frac{e^{-2\,F0[x,\theta]}\,S[x]}{x^2\,H[x]} & 0 & 0 & -\frac{e^{-2\,F0[x,\theta]}\,S[x]\,W[x,\theta]}{x^2\,H[x]} \\ 0 & e^{-2\,F1[x,\theta]}\,H[x] & 0 & 0 \\ 0 & 0 & \frac{e^{-2\,F1[x,\theta]}}{S[x]} & 0 \\ -\frac{e^{-2\,F0[x,\theta]}\,S[x]\,W[x,\theta]}{x^2\,H[x]} & 0 & 0 & \frac{e^{-2\,F2[x,\theta]}\,Csc[\theta]^2}{S[x]} - \frac{e^{-2\,F0[x,\theta]}\,S[x]\,W[x,\theta]^2}{x^2\,H[x]} \end{pmatrix}$$

Determinant: $-e^{2\,(F0[x,\theta]+2\,F1[x,\theta]+F2[x,\theta])}\,x^2\,S[x]\,Sin[\theta]^2$

SquareRoot of the Determinant: $e^{F0[x,\theta]+2\,F1[x,\theta]+F2[x,\theta]}\,x\,\sqrt{S[x]}\,Sin[\theta]$

Christoffel Symbols: $\Gamma^{\mu}{}_{\nu\alpha} = \frac{1}{2}\,g^{\mu\beta}(\partial_\alpha g_{\nu\beta} + \partial_\nu g_{\beta\alpha} - \partial_\beta g_{\nu\alpha})$

In[◦]:=
```
Do[Christoffel[μ, ν, α] = 1/2 Sum[gxi[μ, β] (D[gx[ν, β], coord[[α]]] + D[gx[β, α], coord[[ν]]] -
        D[gx[ν, α], coord[[β]]]), {β, 1, dim}], {μ, 1, dim}, {ν, 1, dim}, {α, 1, ν}];
Do[Christoffel[μ, α, ν] = Christoffel[μ, ν, α], {μ, 1, dim}, {ν, 1, dim}, {α, 1, ν}];
```

If you want to see the several components of the Christoffel symbols, uncomment the next lines

In[◦]:=
```
(*Print[
  "\n     Nonzero Components of the Christoffel Symbols: \n(Attention to the
    Christoffel Symbols Symmetry Γ^μ_να = Γ^μ_αν)\n"]
 Do[
  If[UnsameQ[Christoffel[μ,ν,α],0],
   Print["Γ[",coord[[μ]],",",coord[[ν]],",",coord[[α]],"]=",Christoffel[μ,ν,α]]]
  ,{μ,1,dim},{ν,1,dim},{α,1,ν}]*)
```

Riemann Tensor

$$R^{\mu}{}_{\nu\alpha\beta} = \partial_\alpha \Gamma^{\mu}{}_{\nu\beta} - \partial_\beta \Gamma^{\mu}{}_{\nu\alpha} + \Gamma^{\mu}{}_{\alpha\gamma} \Gamma^{\gamma}{}_{\nu\beta} - \Gamma^{\mu}{}_{\beta\gamma} \Gamma^{\gamma}{}_{\nu\alpha}$$

```
In[ ]:= Do[RiemannTensor[μ, ν, α, β] =
          D[Christoffel[μ, ν, β], coord[[α]]] - D[Christoffel[μ, ν, α], coord[[β]]] +
           Sum[Christoffel[μ, α, γ] × Christoffel[γ, ν, β], {γ, 1, dim}] -
           Sum[Christoffel[μ, β, γ] × Christoffel[γ, ν, α],
            {γ, 1, dim}], {μ, 1, dim}, {ν, 1, dim}, {α, 1, dim}, {β, 1, α}];
        Do[RiemannTensor[μ, ν, β, α] = -RiemannTensor[μ, ν, α, β],
          {μ, 1, dim}, {ν, 1, dim}, {α, 1, dim}, {β, 1, α}];
```

If you want to see the several components of the Riemann tensor, uncomment the next lines

```
In[ ]:= (*Print["     Nonzero Components of the Riemann Tensor: \n(Atention
            to the antisymmetry of the Riemann tensor R^μ_να β=-R^μ_νβα)"]
         Do[
          If[UnsameQ[RiemannTensor[μ,ν,α,β],0],
           Print["R^μ_ναβ[",coord[[μ]],",",coord[[ν]],
            ",",coord[[α]],",",coord[[β]],"]=",RiemannTensor[μ,ν,α,β]]]
          ,{μ,1,dim},{ν,1,dim},{α,1,dim},{β,1,α}]*)
```

Ricci Tensor

$$R_{\nu\beta} = R^{\mu}{}_{\nu\mu\beta} = \partial_\mu \Gamma^{\mu}{}_{\nu\beta} - \partial_\beta \Gamma^{\mu}{}_{\nu\mu} + \Gamma^{\mu}{}_{\mu\gamma} \Gamma^{\gamma}{}_{\nu\beta} - \Gamma^{\mu}{}_{\beta\gamma} \Gamma^{\gamma}{}_{\nu\mu}$$

```
In[ ]:= Do[RicciTensor[ν, β] = Sum[RiemannTensor[μ, ν, μ, β], {μ, 1, dim}], {ν, 1, dim}, {β, 1, ν}];
        Do[RicciTensor[β, ν] = RicciTensor[ν, β], {ν, 1, dim}, {β, 1, ν}];
```

If you want to see the several components of the Ricci tensor, uncomment the next lines

```
In[ ]:= (*Print["     Nonzero Components of the Ricci
            Tensor: \n(Attention to the Ricci Tensor Symmetry R_μν=R_νμ)"]
         Do[
          If[UnsameQ[RicciTensor[ν,β],0],
           Print["R_μν[",coord[[ν]],",",coord[[β]],"]=",RicciTensor[ν,β]]]
          ,{ν,1,dim},{β,1,ν}]*)
```

Ricci Scalar $R = g^{\mu\nu} R_{\mu\nu}$

```
In[ ]:= Ricci = Sum[gxi[μ, ν] × RicciTensor[μ, ν], {μ, 1, dim}, {ν, 1, dim}];
```

If you want to see the Ricci scalar, uncomment the next lines

```
In[ ]:= (*Print[" Ricci Scalar : "]
         Print["R = ",Ricci]*)
```

Einstein Tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

```
In[ ]:= Do[EinsteinTensor[μ, ν] = RicciTensor[μ, ν] - 1/2 gx[μ, ν] Ricci, {μ, 1, dim}, {ν, 1, μ}];
        Do[EinsteinTensor[ν, μ] = EinsteinTensor[μ, ν], {μ, 1, dim}, {ν, 1, μ}];
```

If you want to see the several components of the Einstein tensor, uncomment the next lines

```
In[ ]:= (*Print["     Nonzero Components of the Einstein
            Tensor: \n(Attention to the Einstein Tensor Symmetry Eμν=Eνμ)"]
         Do[
          If[UnsameQ[EinsteinTensor[μ,ν],0],
           Print["Gμν[",coord[[μ]],",",coord[[ν]],"]=",EinsteinTensor[μ,ν]]]
          ,{μ,1,dim},{ν,1,μ}]*)
```

Mixed Einstein Tensor $G_{\mu}{}^{\nu} = g^{\alpha\nu} G_{\mu\alpha} = R_{\mu}{}^{\nu} - \frac{1}{2} \delta_{\mu}{}^{\nu} R$

```
In[ ]:= Do[MixedEinsteinTensor[μ, ν] = Sum[gxi[α, ν] × EinsteinTensor[μ, α], {α, 1, dim}],
          {μ, 1, dim}, {ν, 1, dim}];
```

If you want to see the several components of the mixed Einstein tensor, uncomment the next lines

```
In[ ]:= (*Print["     Nonzero Components of the Mixed Einstein Tensor: "]
         Do[
          If[UnsameQ[MixedEinsteinTensor[μ,ν],0],
           Print["Gμ^ν[",coord[[μ]],",",coord[[ν]],"]=",MixedEinsteinTensor[μ,ν]]]
          ,{μ,1,dim},{ν,1,dim}]*)
```

Kretschmann Scalar K = $R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} = g_{\mu\gamma} R^{\gamma}{}_{\nu\alpha\beta} g^{\sigma\nu} g^{\lambda\alpha} g^{\epsilon\beta} R^{\mu}{}_{\sigma\lambda\epsilon}$

ATTENTION!! These lines will take a lot of time and memory do to the Simplify function. It may "crash" Mathematica and clear all memory used by it, cleaning all variables. Thus, be careful.

```
In[ ]:= Do[CovariantRiemannTensor[μ, ν, α, β] =
            Simplify[Sum[RiemannTensor[ρ, ν, α, β] × gx[μ, ρ], {ρ, 1, dim}]],
          {μ, 1, dim}, {ν, 1, dim}, {α, 1, dim}, {β, 1, dim}];
```

```
In[ ]:= Do[ContravariantRiemannTensor[μ, ν, α, β] =
            Simplify[Sum[RiemannTensor[μ, ρ, σ, δ] × gxi[ν, ρ] × gxi[α, σ] × gxi[β, δ],
              {ρ, 1, dim}, {σ, 1, dim}, {δ, 1, dim}]],
          {μ, 1, dim}, {ν, 1, dim}, {α, 1, dim}, {β, 1, dim}];
```

```
In[∘]:= KretschmanScalar = ExpandAll[
           Sum[CovariantRiemannTensor[μ, ν, α, β] × ContravariantRiemannTensor[μ, ν, α, β],
            {μ, 1, dim}, {ν, 1, dim}, {α, 1, dim}, {β, 1, dim}]];

In[∘]:= (*Print[" Kretschman Scalar : "]
         Print["K = ",KretschmanScalar]*)
```

Energy-Momentum Tensor

For the Kerr solution, the energy-momentum tensor is zero.

Full Equations

```
In[∘]:= Do[Equations[μ, ν] = ExpandAll[MixedEinsteinTensor[μ, ν]], {μ, 1, dim}, {ν, 1, dim}]
```

1st Equation: $-G_t{}^t + G_r{}^r + G_\theta{}^\theta - G_\varphi{}^\varphi = 0$

```
In[∘]:= Eq1 = ExpandAll[-(e^(2 F1[x,θ]) x Sin[θ]^2)/(2 H[x]) (-Equations[1, 1] + Equations[2, 2] + Equations[3, 3]
            - Equations[4, 4])];
```

2nd Equation: $G_t{}^t + G_r{}^r + G_\theta{}^\theta - G_\varphi{}^\varphi + 2\,W\,G_\varphi{}^t = 0$

```
In[∘]:= Eq2 = ExpandAll[(e^(2 F1[x,θ]) Sin[θ]^2)/(2 H[x]) (Equations[1, 1] + Equations[2, 2] + Equations[3, 3]
            - Equations[4, 4] + 2 W[x, θ] × Equations[4, 1])];
```

3rd Equation: $-G_t{}^t + G_r{}^r + G_\theta{}^\theta + G_\varphi{}^\varphi - 2\,W\,G_\varphi{}^t = 0$

```
In[∘]:= Eq3 = ExpandAll[(e^(2 F1[x,θ]) x Sin[θ])/(H[x]) (-Equations[1, 1] + Equations[2, 2] + Equations[3, 3]
            + Equations[4, 4] - 2 W[x, θ] × Equations[4, 1])];
```

4th Equation: $G_\varphi{}^t = 0$

```
In[∘]:= Eq4 = ExpandAll[-(2 e^(2 (F0[x,θ]+F1[x,θ]-F2[x,θ])) x^4)/(S[x]^2) (Equations[4, 1])];
```

1st Constrain Equation: $G_r{}^r - G_\theta{}^\theta = 0$

```
In[∘]:= ConstrainEq1 = ExpandAll[Equations[2, 2] - Equations[3, 3]];
```

2^{nd} Constrain Equation: $G_r^{\;\theta} = 0$

In[∘]:=
```
ConstrainEq2 = ExpandAll[Equations[2, 3]];
```

Output for the “Expansion” file

In[∘]:=
```
Eq1
```

Out[∘]=

$$-\frac{\text{Sin}[\theta]^2\,\text{S}'[x]}{2\,\text{S}[x]}+\frac{x\,\text{Sin}[\theta]^2\,\text{S}''[x]}{2\,\text{S}[x]}-\frac{x\,\text{Cos}[\theta]\,\text{Sin}[\theta]\,\text{F0}^{(0,1)}[x,\theta]}{\text{H}[x]\times\text{S}[x]}-\frac{x\,\text{Sin}[\theta]^2\,\text{F0}^{(0,1)}[x,\theta]\,\text{F2}^{(0,1)}[x,\theta]}{\text{H}[x]\times\text{S}[x]}-\frac{e^{-2\,\text{F0}[x,\theta]+2\,\text{F2}[x,\theta]}\,\text{S}[x]\,\text{Sin}[\theta]^4\,\text{W}^{(0,1)}[x,\theta]^2}{4\,x\,\text{H}[x]^2}+\frac{x\,\text{Sin}[\theta]^2\,\text{F1}^{(0,2)}[x,\theta]}{\text{H}[x]\times\text{S}[x]}-\frac{x\,\text{Sin}[\theta]^2\,\text{S}'[x]\,\text{F0}^{(1,0)}[x,\theta]}{2\,\text{S}[x]}+\frac{x\,\text{Sin}[\theta]^2\,\text{H}'[x]\,\text{F1}^{(1,0)}[x,\theta]}{2\,\text{H}[x]}+\frac{x\,\text{Sin}[\theta]^2\,\text{S}'[x]\,\text{F1}^{(1,0)}[x,\theta]}{2\,\text{S}[x]}-\text{Sin}[\theta]^2\,\text{F2}^{(1,0)}[x,\theta]-\frac{x\,\text{Sin}[\theta]^2\,\text{H}'[x]\,\text{F2}^{(1,0)}[x,\theta]}{2\,\text{H}[x]}+\frac{x\,\text{Sin}[\theta]^2\,\text{S}'[x]\,\text{F2}^{(1,0)}[x,\theta]}{2\,\text{S}[x]}-x\,\text{Sin}[\theta]^2\,\text{F0}^{(1,0)}[x,\theta]\,\text{F2}^{(1,0)}[x,\theta]-\frac{e^{-2\,\text{F0}[x,\theta]+2\,\text{F2}[x,\theta]}\,\text{S}[x]^2\,\text{Sin}[\theta]^4\,\text{W}^{(1,0)}[x,\theta]^2}{4\,x\,\text{H}[x]}+x\,\text{Sin}[\theta]^2\,\text{F1}^{(2,0)}[x,\theta]$$

In[∘]:=
```
Eq2
```

Out[∘]=

$$-\frac{\text{Sin}[\theta]^2}{\text{H}[x]\times\text{S}[x]}+\frac{\text{Sin}[\theta]^2\,\text{S}'[x]}{2\,x\,\text{S}[x]}+\frac{\text{Sin}[\theta]^2\,\text{H}'[x]\,\text{S}'[x]}{2\,\text{H}[x]\times\text{S}[x]}-\frac{\text{Sin}[\theta]^2\,\text{S}'[x]^2}{4\,\text{S}[x]^2}+\frac{\text{Sin}[\theta]^2\,\text{S}''[x]}{2\,\text{S}[x]}+\frac{\text{Cos}[\theta]\,\text{Sin}[\theta]\,\text{F0}^{(0,1)}[x,\theta]}{\text{H}[x]\times\text{S}[x]}+\frac{2\,\text{Cos}[\theta]\,\text{Sin}[\theta]\,\text{F2}^{(0,1)}[x,\theta]}{\text{H}[x]\times\text{S}[x]}+\frac{\text{Sin}[\theta]^2\,\text{F0}^{(0,1)}[x,\theta]\,\text{F2}^{(0,1)}[x,\theta]}{\text{H}[x]\times\text{S}[x]}+\frac{\text{Sin}[\theta]^2\,\text{F2}^{(0,1)}[x,\theta]^2}{\text{H}[x]\times\text{S}[x]}+\frac{e^{-2\,\text{F0}[x,\theta]+2\,\text{F2}[x,\theta]}\,\text{S}[x]\,\text{Sin}[\theta]^4\,\text{W}^{(0,1)}[x,\theta]^2}{2\,x^2\,\text{H}[x]^2}+\frac{\text{Sin}[\theta]^2\,\text{F2}^{(0,2)}[x,\theta]}{\text{H}[x]\times\text{S}[x]}+\frac{\text{Sin}[\theta]^2\,\text{S}'[x]\,\text{F0}^{(1,0)}[x,\theta]}{2\,\text{S}[x]}+\frac{\text{Sin}[\theta]^2\,\text{F2}^{(1,0)}[x,\theta]}{x}+\frac{\text{Sin}[\theta]^2\,\text{H}'[x]\,\text{F2}^{(1,0)}[x,\theta]}{\text{H}[x]}+\frac{\text{Sin}[\theta]^2\,\text{S}'[x]\,\text{F2}^{(1,0)}[x,\theta]}{\text{S}[x]}+\text{Sin}[\theta]^2\,\text{F0}^{(1,0)}[x,\theta]\,\text{F2}^{(1,0)}[x,\theta]+\text{Sin}[\theta]^2\,\text{F2}^{(1,0)}[x,\theta]^2+\frac{e^{-2\,\text{F0}[x,\theta]+2\,\text{F2}[x,\theta]}\,\text{S}[x]^2\,\text{Sin}[\theta]^4\,\text{W}^{(1,0)}[x,\theta]^2}{2\,x^2\,\text{H}[x]}+\text{Sin}[\theta]^2\,\text{F2}^{(2,0)}[x,\theta]$$

In[ ]:= **Eq3**

Out[ ]=

$$\frac{3\,\mathrm{Sin}[\theta]\,H'[x]}{H[x]} - \frac{x\,\mathrm{Sin}[\theta]\,H'[x]\,S'[x]}{2\,H[x]\times S[x]} + \frac{x\,\mathrm{Sin}[\theta]\,S'[x]^2}{2\,S[x]^2} + \frac{x\,\mathrm{Sin}[\theta]\,H''[x]}{H[x]} - \frac{x\,\mathrm{Sin}[\theta]\,S''[x]}{S[x]} +$$
$$\frac{2\,x\,\mathrm{Cos}[\theta]\,F0^{(0,1)}[x,\theta]}{H[x]\times S[x]} + \frac{2\,x\,\mathrm{Sin}[\theta]\,F0^{(0,1)}[x,\theta]^2}{H[x]\times S[x]} + \frac{2\,x\,\mathrm{Sin}[\theta]\,F0^{(0,1)}[x,\theta]\,F2^{(0,1)}[x,\theta]}{H[x]\times S[x]} -$$
$$\frac{e^{-2\,F0[x,\theta]+2\,F2[x,\theta]}\,S[x]\,\mathrm{Sin}[\theta]^3\,W^{(0,1)}[x,\theta]^2}{x\,H[x]^2} + \frac{2\,x\,\mathrm{Sin}[\theta]\,F0^{(0,2)}[x,\theta]}{H[x]\times S[x]} + 4\,\mathrm{Sin}[\theta]\,F0^{(1,0)}[x,\theta] +$$
$$\frac{3\,x\,\mathrm{Sin}[\theta]\,H'[x]\,F0^{(1,0)}[x,\theta]}{H[x]} + 2\,x\,\mathrm{Sin}[\theta]\,F0^{(1,0)}[x,\theta]^2 + 2\,\mathrm{Sin}[\theta]\,F2^{(1,0)}[x,\theta] +$$
$$\frac{x\,\mathrm{Sin}[\theta]\,H'[x]\,F2^{(1,0)}[x,\theta]}{H[x]} - \frac{x\,\mathrm{Sin}[\theta]\,S'[x]\,F2^{(1,0)}[x,\theta]}{S[x]} + 2\,x\,\mathrm{Sin}[\theta]\,F0^{(1,0)}[x,\theta]\,F2^{(1,0)}[x,\theta] -$$
$$\frac{e^{-2\,F0[x,\theta]+2\,F2[x,\theta]}\,S[x]^2\,\mathrm{Sin}[\theta]^3\,W^{(1,0)}[x,\theta]^2}{x\,H[x]} + 2\,x\,\mathrm{Sin}[\theta]\,F0^{(2,0)}[x,\theta]$$

In[ ]:= **Eq4**

Out[ ]=

$$\frac{3\,x^2\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\,W^{(0,1)}[x,\theta]}{H[x]\times S[x]} - \frac{x^2\,\mathrm{Sin}[\theta]^2\,F0^{(0,1)}[x,\theta]\,W^{(0,1)}[x,\theta]}{H[x]\times S[x]} +$$
$$\frac{3\,x^2\,\mathrm{Sin}[\theta]^2\,F2^{(0,1)}[x,\theta]\,W^{(0,1)}[x,\theta]}{H[x]\times S[x]} + \frac{x^2\,\mathrm{Sin}[\theta]^2\,W^{(0,2)}[x,\theta]}{H[x]\times S[x]} - x\,\mathrm{Sin}[\theta]^2\,W^{(1,0)}[x,\theta] +$$
$$\frac{5\,x^2\,\mathrm{Sin}[\theta]^2\,S'[x]\,W^{(1,0)}[x,\theta]}{2\,S[x]} - x^2\,\mathrm{Sin}[\theta]^2\,F0^{(1,0)}[x,\theta]\,W^{(1,0)}[x,\theta] +$$
$$3\,x^2\,\mathrm{Sin}[\theta]^2\,F2^{(1,0)}[x,\theta]\,W^{(1,0)}[x,\theta] + x^2\,\mathrm{Sin}[\theta]^2\,W^{(2,0)}[x,\theta]$$

In[ ]:= ConstrainEq1

Out[ ]=

$$-\frac{e^{-2\,F1[x,\theta]}}{S[x]}-\frac{3\,e^{-2\,F1[x,\theta]}\,H'[x]}{2\,x}+\frac{3\,e^{-2\,F1[x,\theta]}\,H[x]\,S'[x]}{2\,x\,S[x]}+\frac{3\,e^{-2\,F1[x,\theta]}\,H'[x]\,S'[x]}{4\,S[x]}-$$

$$\frac{e^{-2\,F1[x,\theta]}\,H[x]\,S'[x]^2}{2\,S[x]^2}-\frac{1}{2}\,e^{-2\,F1[x,\theta]}\,H''[x]+\frac{e^{-2\,F1[x,\theta]}\,F0^{(0,1)}[x,\,\theta]^2}{S[x]}-$$

$$\frac{2\,e^{-2\,F1[x,\theta]}\,Cot[\theta]\,F1^{(0,1)}[x,\,\theta]}{S[x]}-\frac{2\,e^{-2\,F1[x,\theta]}\,F0^{(0,1)}[x,\,\theta]\,F1^{(0,1)}[x,\,\theta]}{S[x]}+$$

$$\frac{2\,e^{-2\,F1[x,\theta]}\,Cot[\theta]\,F2^{(0,1)}[x,\,\theta]}{S[x]}-\frac{2\,e^{-2\,F1[x,\theta]}\,F1^{(0,1)}[x,\,\theta]\,F2^{(0,1)}[x,\,\theta]}{S[x]}+\frac{e^{-2\,F1[x,\theta]}\,F2^{(0,1)}[x,\,\theta]^2}{S[x]}-$$

$$\frac{e^{-2\,F0[x,\theta]-2\,F1[x,\theta]+2\,F2[x,\theta]}\,S[x]\,Sin[\theta]^2\,W^{(0,1)}[x,\,\theta]^2}{2\,x^2\,H[x]}+\frac{e^{-2\,F1[x,\theta]}\,F0^{(0,2)}[x,\,\theta]}{S[x]}+$$

$$\frac{e^{-2\,F1[x,\theta]}\,F2^{(0,2)}[x,\,\theta]}{S[x]}-\frac{2\,e^{-2\,F1[x,\theta]}\,H[x]\,F0^{(1,0)}[x,\,\theta]}{x}-\frac{3}{2}\,e^{-2\,F1[x,\theta]}\,H'[x]\,F0^{(1,0)}[x,\,\theta]+$$

$$\frac{3\,e^{-2\,F1[x,\theta]}\,H[x]\,S'[x]\,F0^{(1,0)}[x,\,\theta]}{2\,S[x]}-e^{-2\,F1[x,\theta]}\,H[x]\,F0^{(1,0)}[x,\,\theta]^2+\frac{2\,e^{-2\,F1[x,\theta]}\,H[x]\,F1^{(1,0)}[x,\,\theta]}{x}+$$

$$e^{-2\,F1[x,\theta]}\,H'[x]\,F1^{(1,0)}[x,\,\theta]+2\,e^{-2\,F1[x,\theta]}\,H[x]\,F0^{(1,0)}[x,\,\theta]\,F1^{(1,0)}[x,\,\theta]-\frac{1}{2}\,e^{-2\,F1[x,\theta]}\,H'[x]\,F2^{(1,0)}[x,\,\theta]-$$

$$\frac{e^{-2\,F1[x,\theta]}\,H[x]\,S'[x]\,F2^{(1,0)}[x,\,\theta]}{2\,S[x]}+2\,e^{-2\,F1[x,\theta]}\,H[x]\,F1^{(1,0)}[x,\,\theta]\,F2^{(1,0)}[x,\,\theta]-$$

$$e^{-2\,F1[x,\theta]}\,H[x]\,F2^{(1,0)}[x,\,\theta]^2+\frac{e^{-2\,F0[x,\theta]-2\,F1[x,\theta]+2\,F2[x,\theta]}\,S[x]^2\,Sin[\theta]^2\,W^{(1,0)}[x,\,\theta]^2}{2\,x^2}-$$

$$e^{-2\,F1[x,\theta]}\,H[x]\,F0^{(2,0)}[x,\,\theta]-e^{-2\,F1[x,\theta]}\,H[x]\,F2^{(2,0)}[x,\,\theta]$$

In[ ]:= **ConstrainEq2**

Out[ ]=

$$-\frac{e^{-2\,F1[x,\theta]}\,F0^{(0,1)}[x,\theta]}{x\,S[x]}-\frac{e^{-2\,F1[x,\theta]}\,H'[x]\,F0^{(0,1)}[x,\theta]}{2\,H[x]\times S[x]}+\frac{e^{-2\,F1[x,\theta]}\,S'[x]\,F0^{(0,1)}[x,\theta]}{S[x]^2}+\frac{e^{-2\,F1[x,\theta]}\,F1^{(0,1)}[x,\theta]}{x\,S[x]}+\frac{e^{-2\,F1[x,\theta]}\,H'[x]\,F1^{(0,1)}[x,\theta]}{2\,H[x]\times S[x]}-\frac{e^{-2\,F1[x,\theta]}\,F0^{(0,1)}[x,\theta]\,F0^{(1,0)}[x,\theta]}{S[x]}+\frac{e^{-2\,F1[x,\theta]}\,F1^{(0,1)}[x,\theta]\,F0^{(1,0)}[x,\theta]}{S[x]}+\frac{e^{-2\,F1[x,\theta]}\,Cot[\theta]\,F1^{(1,0)}[x,\theta]}{S[x]}+\frac{e^{-2\,F1[x,\theta]}\,F0^{(0,1)}[x,\theta]\,F1^{(1,0)}[x,\theta]}{S[x]}+\frac{e^{-2\,F1[x,\theta]}\,F2^{(0,1)}[x,\theta]\,F1^{(1,0)}[x,\theta]}{S[x]}-\frac{e^{-2\,F1[x,\theta]}\,Cot[\theta]\,F2^{(1,0)}[x,\theta]}{S[x]}+\frac{e^{-2\,F1[x,\theta]}\,F1^{(0,1)}[x,\theta]\,F2^{(1,0)}[x,\theta]}{S[x]}-\frac{e^{-2\,F1[x,\theta]}\,F2^{(0,1)}[x,\theta]\,F2^{(1,0)}[x,\theta]}{S[x]}+\frac{e^{-2\,F0[x,\theta]-2\,F1[x,\theta]+2\,F2[x,\theta]}\,S[x]\,Sin[\theta]^2\,W^{(0,1)}[x,\theta]\,W^{(1,0)}[x,\theta]}{2\,x^2\,H[x]}-\frac{e^{-2\,F1[x,\theta]}\,F0^{(1,1)}[x,\theta]}{S[x]}-\frac{e^{-2\,F1[x,\theta]}\,F2^{(1,1)}[x,\theta]}{S[x]}$$

In[ ]:= **R = FullSimplify[Ricci]**

Out[ ]=

$$\frac{1}{2\,x^2\,H[x]\times S[x]}\,e^{-2\,(F0[x,\theta]+F1[x,\theta])}\Big(e^{2\,F2[x,\theta]}\,S[x]^2\,Sin[\theta]^2\,W^{(0,1)}[x,\theta]^2-H[x]\Big(e^{2\,F0[x,\theta]}\,x^2\,\Big(H'[x]\,S'[x]+4\,\big(-1+F0^{(0,1)}[x,\theta]\,(Cot[\theta]+F0^{(0,1)}[x,\theta])+\big(2\,Cot[\theta]+F0^{(0,1)}[x,\theta]\big)\,F2^{(0,1)}[x,\theta]+F2^{(0,1)}[x,\theta]^2+F0^{(0,2)}[x,\theta]+F1^{(0,2)}[x,\theta]+F2^{(0,2)}[x,\theta]\big)\Big)+2\,e^{2\,F0[x,\theta]}\,x\,S[x]\,\Big(x\,H''[x]+H'[x]\,\big(3+x\,(3\,F0^{(1,0)}[x,\theta]+F1^{(1,0)}[x,\theta]+2\,F2^{(1,0)}[x,\theta])\big)\Big)-e^{2\,F2[x,\theta]}\,S[x]^3\,Sin[\theta]^2\,W^{(1,0)}[x,\theta]^2\Big)-2\,e^{2\,F0[x,\theta]}\,x\,H[x]^2\,\Big(x\,S''[x]+x\,S'[x]\,\big(F1^{(1,0)}[x,\theta]+2\,F2^{(1,0)}[x,\theta]\big)+2\,S[x]\,\Big(x\,F0^{(1,0)}[x,\theta]^2+F2^{(1,0)}[x,\theta]+F0^{(1,0)}[x,\theta]\,\big(2+x\,F2^{(1,0)}[x,\theta]\big)+x\,\big(F2^{(1,0)}[x,\theta]^2+F0^{(2,0)}[x,\theta]+F1^{(2,0)}[x,\theta]+F2^{(2,0)}[x,\theta]\big)\Big)\Big)\Big)$$

In[ ]:= KretschmanScalar

Out[ ]=

$$\frac{4 e^{-4 F1[x,\theta]}}{S[x]^2} + \frac{9 e^{-4 F1[x,\theta]} H'[x]^2}{x^2} - \frac{12 e^{-4 F1[x,\theta]} H[x] H'[x] S'[x]}{x^2 S[x]} - \frac{9 e^{-4 F1[x,\theta]} H'[x]^2 S'[x]}{x S[x]} - \frac{2 e^{-4 F1[x,\theta]} H[x] S'[x]^2}{S[x]^3} + \frac{6 e^{-4 F1[x,\theta]} H[x]^2 S'[x]^2}{x^2 S[x]^2} +$$

$$\cdots 574 \cdots + \frac{1}{x^2} 4 e^{-2 F0[x,\theta]-4 F1[x,\theta]+2 F2[x,\theta]} H[x] S[x]^2 \mathrm{Sin}[\theta]^2 F0^{(1,0)}[x,\theta] W^{(1,0)}[x,\theta] W^{(2,0)}[x,\theta] +$$

$$\frac{1}{x^2} 4 e^{-2 F0[x,\theta]-4 F1[x,\theta]+2 F2[x,\theta]} H[x] S[x]^2 \mathrm{Sin}[\theta]^2 F1^{(1,0)}[x,\theta] W^{(1,0)}[x,\theta] W^{(2,0)}[x,\theta] -$$

$$\frac{1}{x^2} 12 e^{-2 F0[x,\theta]-4 F1[x,\theta]+2 F2[x,\theta]} H[x] S[x]^2 \mathrm{Sin}[\theta]^2 F2^{(1,0)}[x,\theta] W^{(1,0)}[x,\theta] W^{(2,0)}[x,\theta] -$$

$$\frac{2 e^{-2 F0[x,\theta]-4 F1[x,\theta]+2 F2[x,\theta]} H[x] S[x]^2 \mathrm{Sin}[\theta]^2 W^{(2,0)}[x,\theta]^2}{x^2}$$

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