Black holes vs Wormholes: Scattering, absorption and quasibound states

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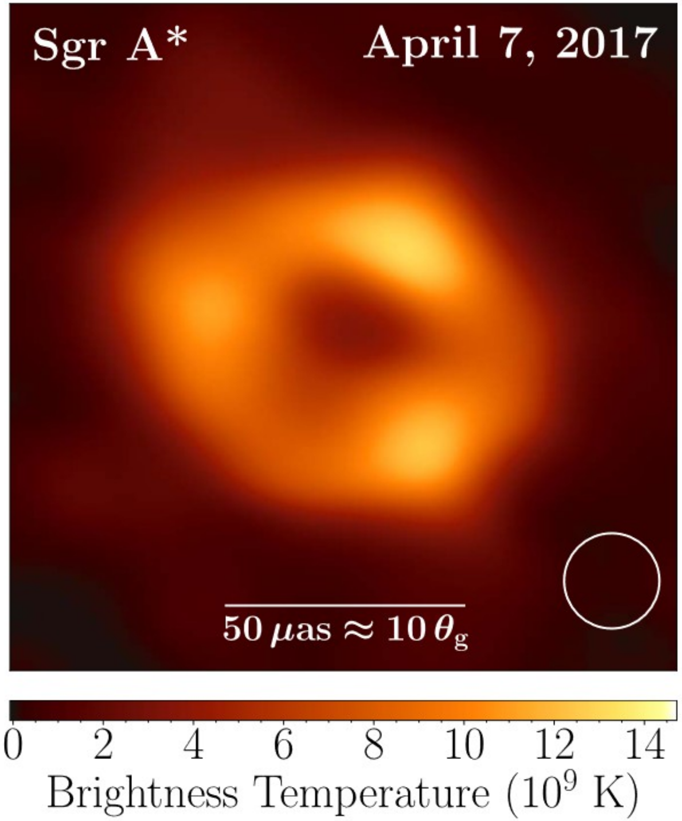
Universidade Federal do Pará

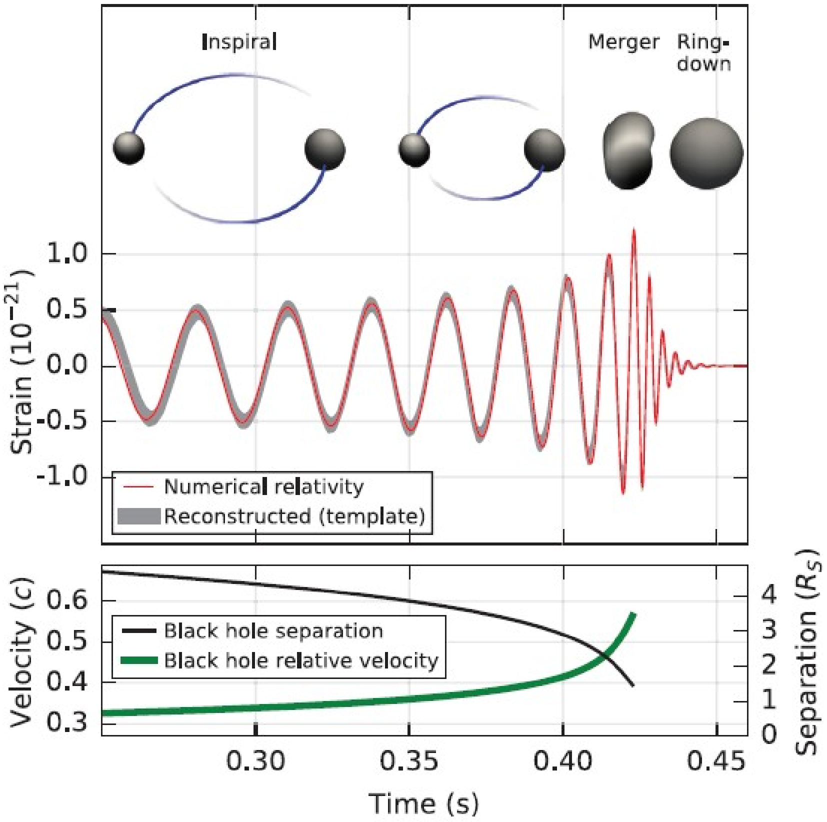
PRD 101, 124009 (2020); EPJC 82, 638 (2022)
[arXiv:2006.03967; arXiv:2211.09886]

Introduction



EHT Collab. (2022)

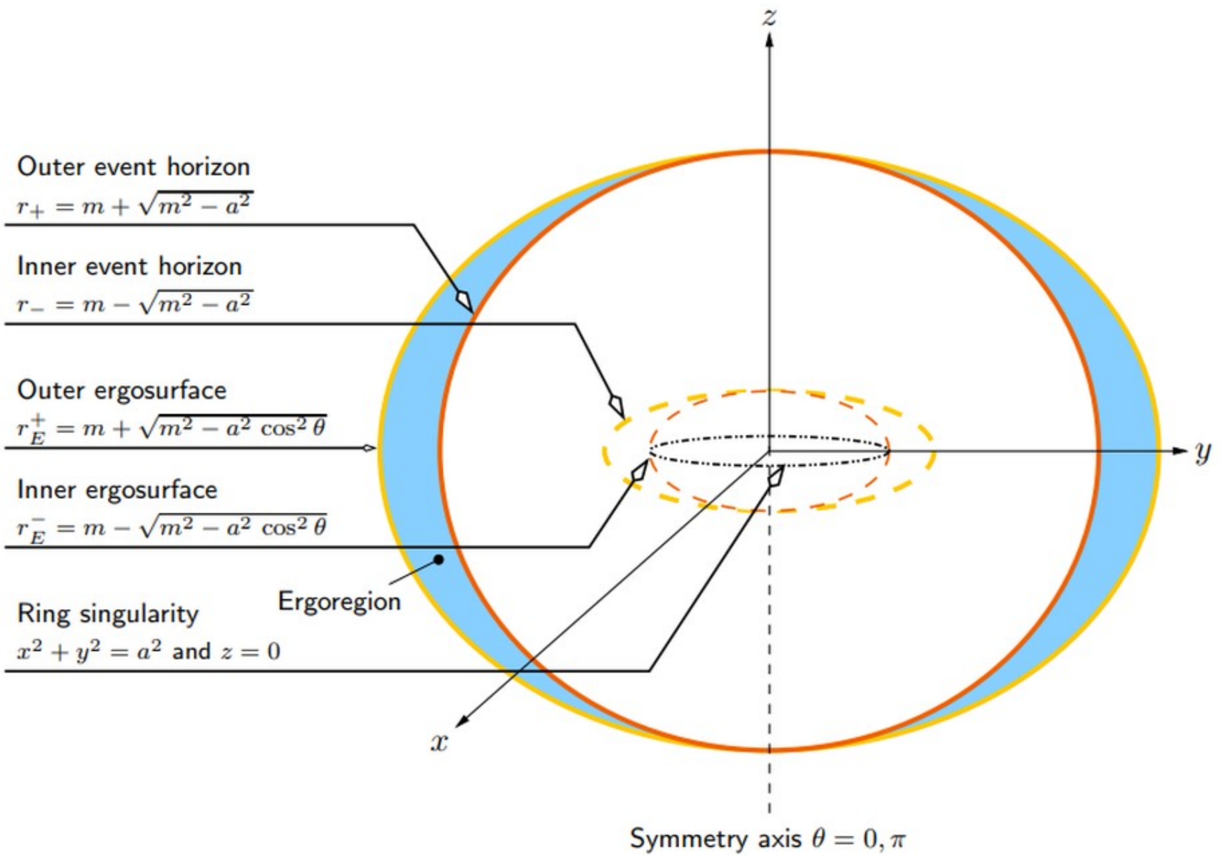
Abbot et al (2016)



Introduction

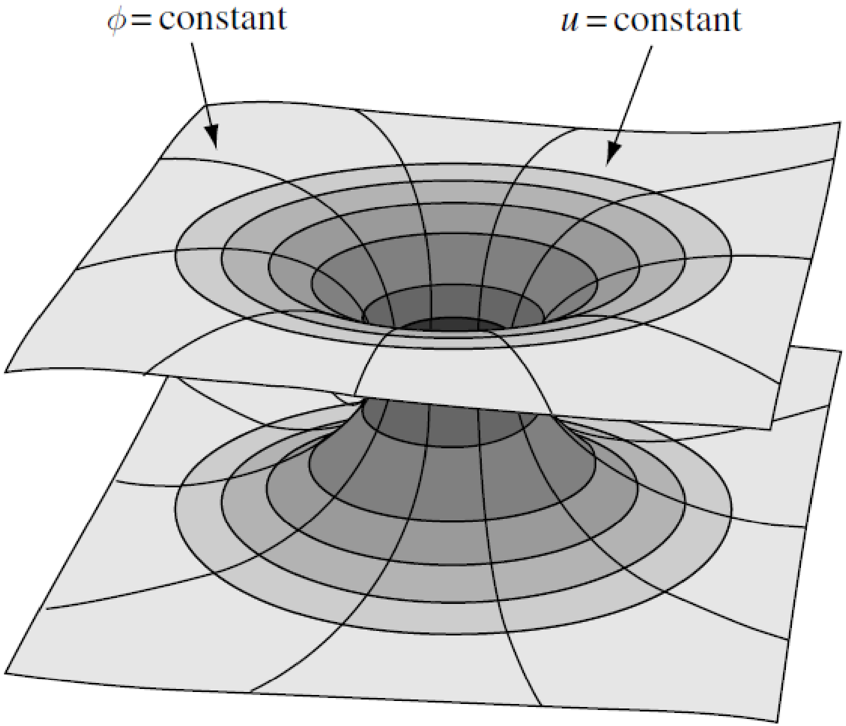
Penrose (1965)

Bardeen (1968)



Visser (2017)
[arXiv:0706.0622v3]

Introduction



Morris&Thorne (1987)

Konoplya&Zhidenko (2021)
[arXiv:2106.05034v3]

Simpson-Visser spacetime

Simpson&Visser (2019)
[arXiv:1812.07114v3]

$$ds^2 = -\left(1 - \frac{2M}{\sqrt{r^2 + a^2}}\right)dt^2 + \left(1 - \frac{2M}{\sqrt{r^2 + a^2}}\right)^{-1} dr^2 + (r^2 + a^2)(d\theta^2 + \sin^2\theta d\varphi^2)$$

$a = 0$ ⟶ Schwarzschild BH

$0 < a < 2M$ ⟶ Regular BH

$a = 2M$ ⟶ One-way traversable wormhole

$a > 2M$ ⟶ Two-way traversable wormhole

Absorption in a static spacetime

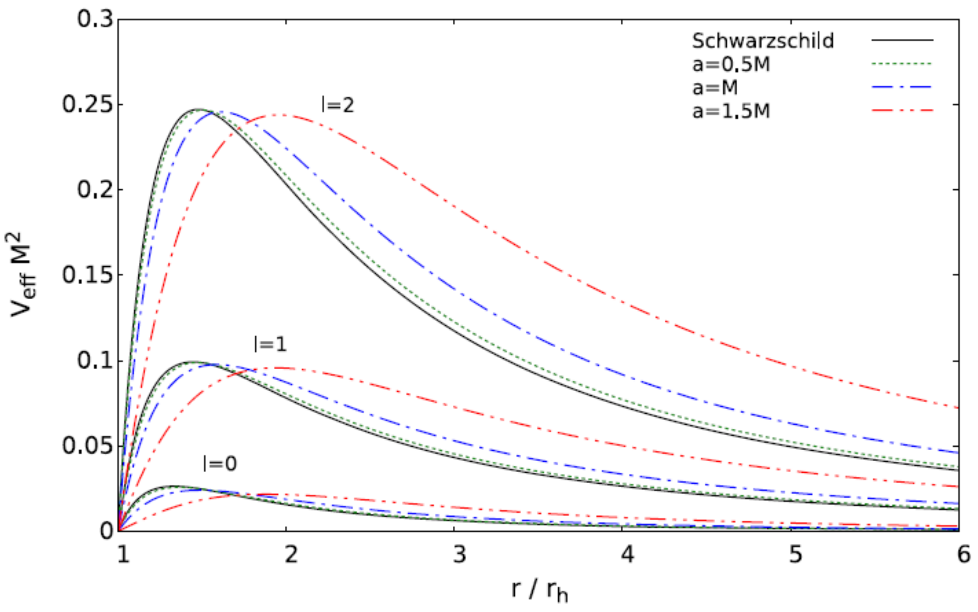
$$\nabla^\mu \nabla_\mu \Phi = 0$$

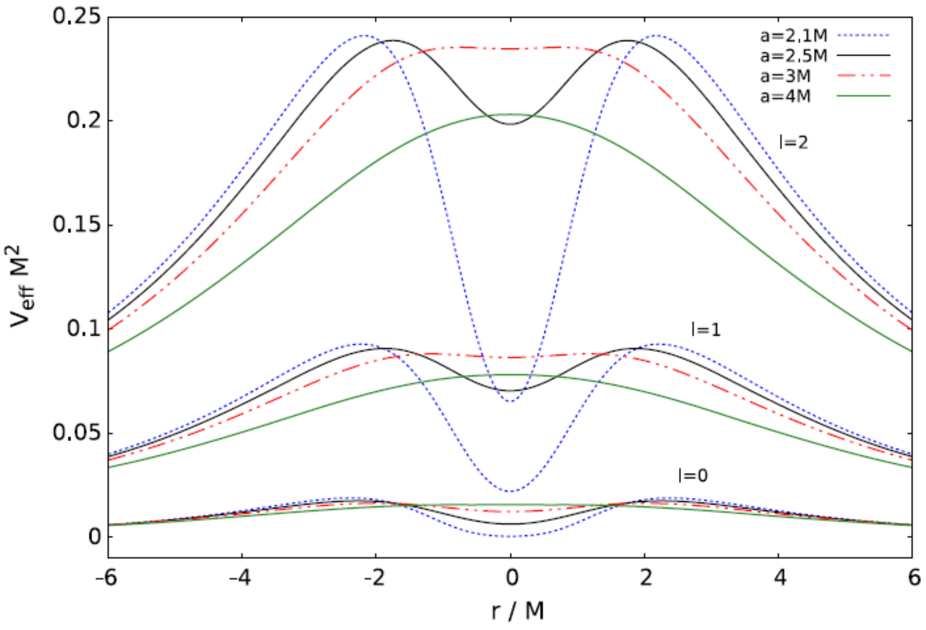
$$\Phi = \sum_{l,m} \frac{\phi(r)}{(r^2 + a^2)^{\frac{1}{2}}} Y_{lm}(\theta, \varphi) e^{-i\omega t}$$

$$f(r)\frac{d}{dr}\left(f(r)\frac{d\phi(r)}{dr}\right) + [\omega^2 - V_{\rm eff}]\phi(r) = 0$$

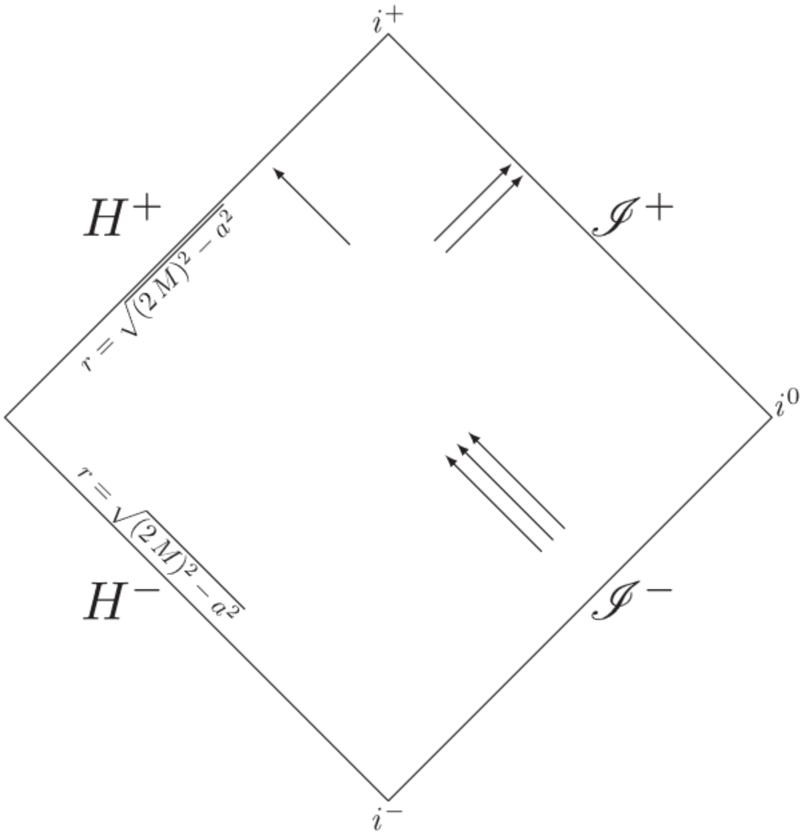
$$V_{\rm eff} \equiv f(r)\left[\frac{f'(r)r}{(r^2 + a^2)} + \frac{a^2 f(r)}{(r^2 + a^2)^2} + \frac{l(l+1)}{(r^2 + a^2)}\right]$$

Absorption in a static spacetime





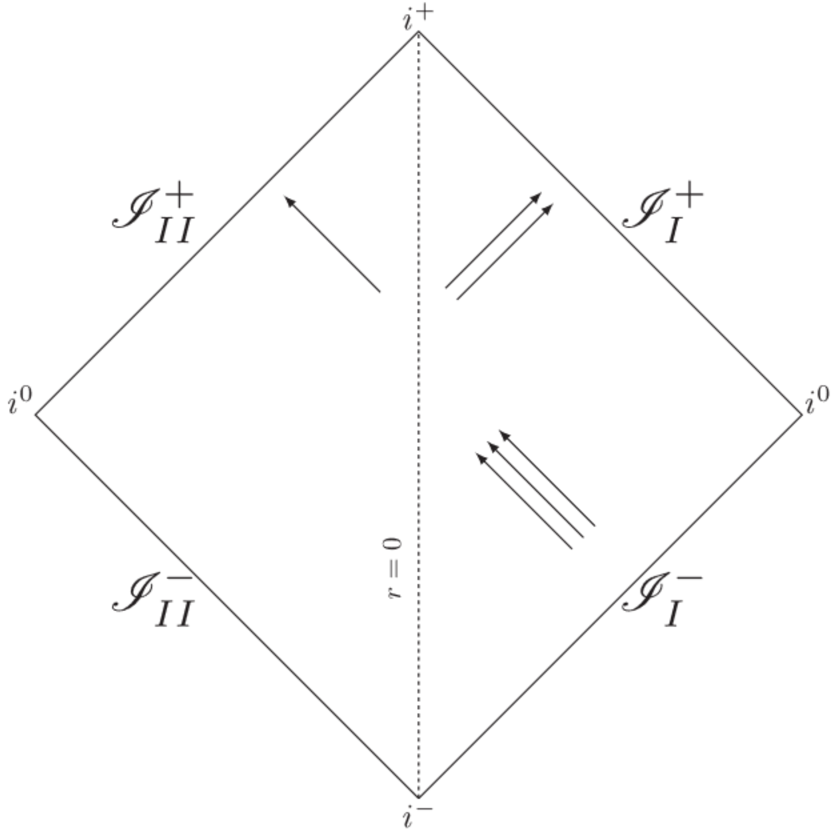
Absorption in a static spacetime



$$\phi(r) \approx \begin{cases} e^{-i\omega x} + e^{i\omega x} R_{\omega l}, & r \to +\infty (x \to +\infty), \\ T_{\omega l} e^{-i\omega x}, & r \to r_h (x \to -\infty). \end{cases}$$

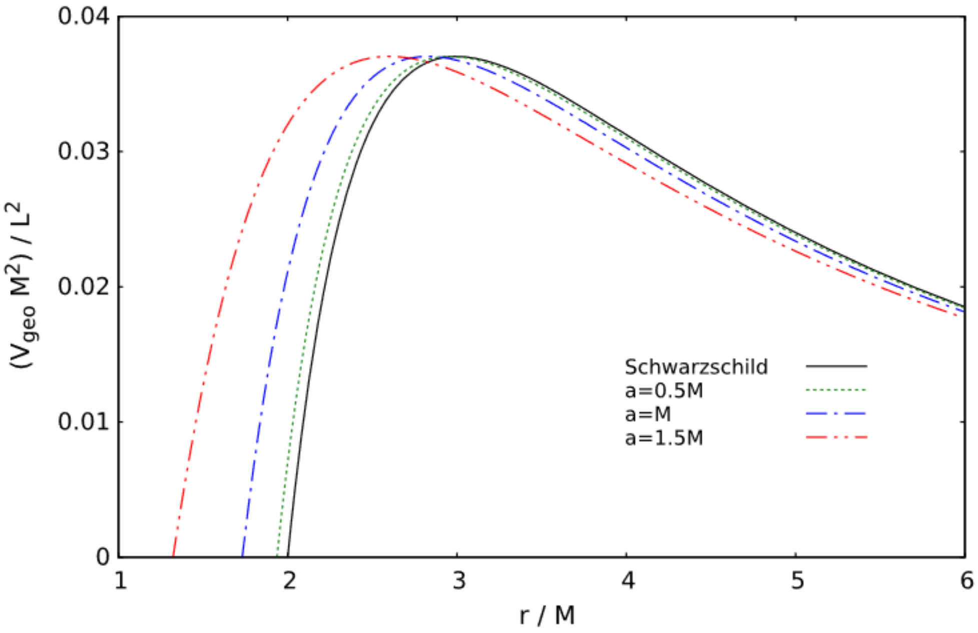
Absorption in a static spacetime

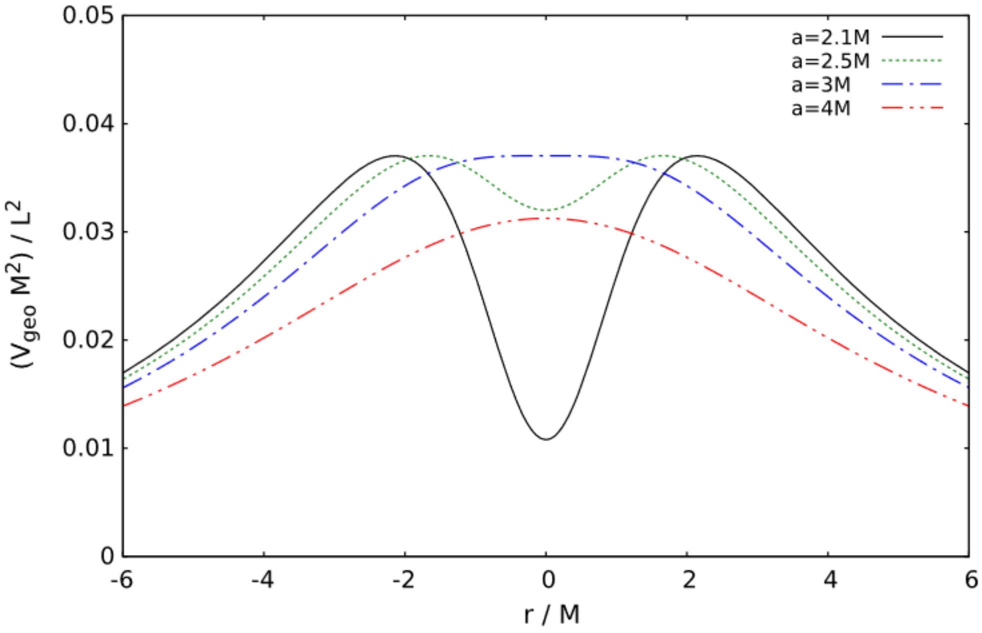
$$\phi(r) \approx \begin{cases} e^{-i\omega x} + e^{i\omega x} R_{\omega l}, & r \to +\infty (x \to +\infty), \\ T_{\omega l} e^{-i\omega x}, & r \to r_h (x \to -\infty). \end{cases}$$



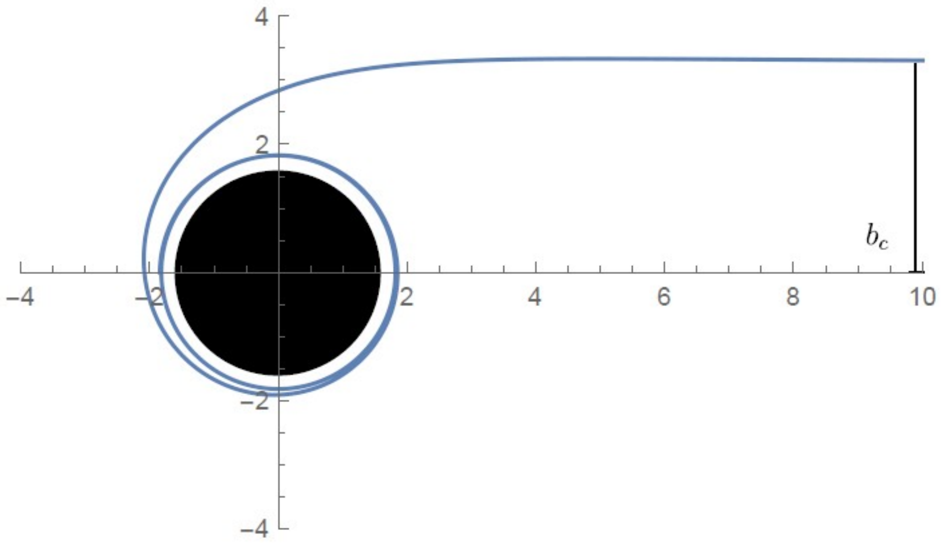
Absorption: Asymptotic limits

$$V_{\text{geo}} \equiv \left(1 - \frac{2M}{\sqrt{r^2 + a^2}}\right) \frac{L^2}{(r^2 + a^2)}$$





Absorption: Asymptotic limits



$$\sigma_{\rm geo} = \pi b_c^2$$

$$r_{\rm ph} = \begin{cases} \sqrt{9M^2 - a^2}, & \text{if } 0 \le a < 2M, \\ \pm\sqrt{9M^2 - a^2}, & \text{if } 2M \le a \le 3M, \\ 0, & \text{if } a > 3M. \end{cases}$$

$$\sigma_{\rm geo} = \begin{cases} \pi b_1^2 = 27\pi M^2, & \text{if } 0 \le a \le 3M \\ \pi b_0^2 = \pi\left(\frac{a^3}{a-2M}\right), & \text{if } a > 3M. \end{cases}$$

Absorption: Asymptotic limits

Décanini+ (2011)
[arXiv:1101.0781v2]

$$\sigma_{\rm hf} \approx \sigma_{\rm geo} + \sigma_{\rm osc}$$

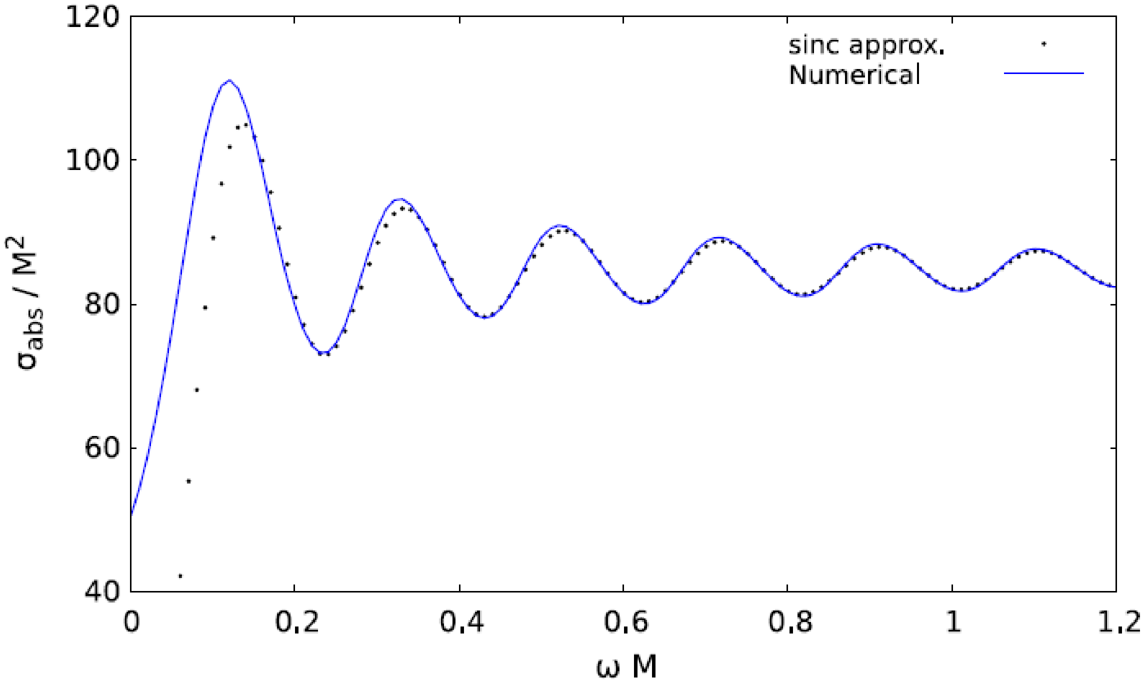
$$\sigma_{\rm osc} = -\frac{8\pi\lambda_l}{\Omega_l} e^{-\frac{\pi\lambda_l}{\Omega_l}} \operatorname{sinc}\left(\frac{2\pi\omega}{\Omega_l}\right) \sigma_{\rm geo}$$

Das+ (1997)
[arXiv:hep-th/9609052]

$$\sigma_{lf} = A_h = 16\pi M^2$$

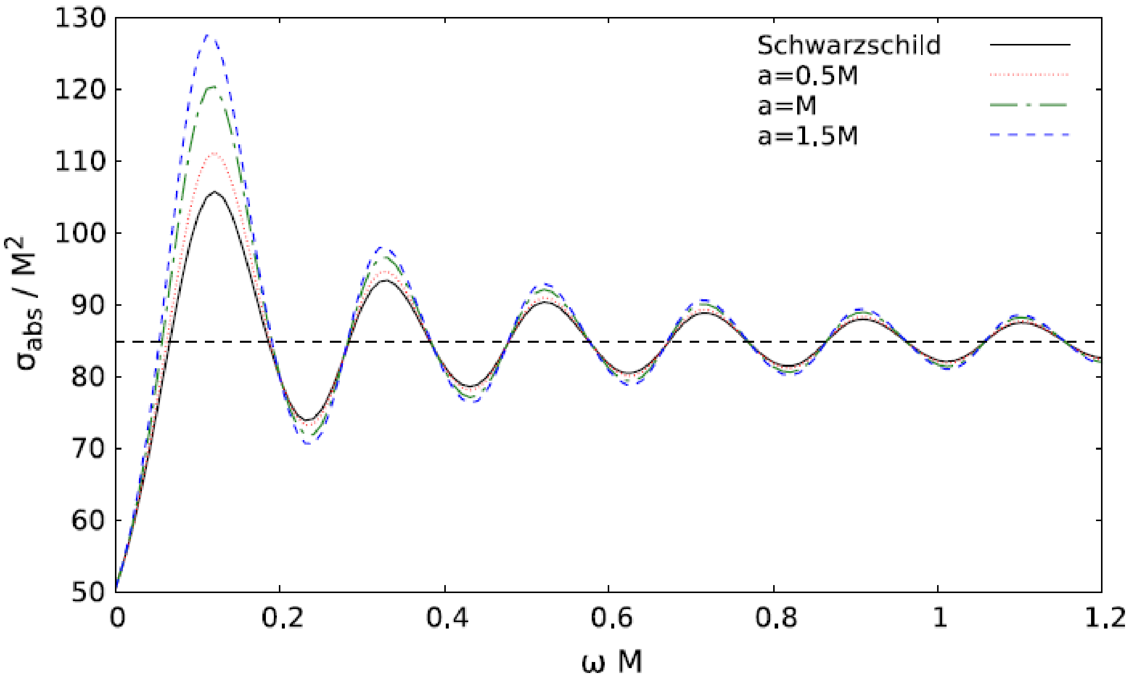
$$\sigma_{\rm abs} = \sum_{l=0}^{\infty} \sigma_l \qquad \sigma_l = \frac{\pi}{\omega^2}(2l+1)|T_{\omega l}|^2$$

Absorption: Numerical results



Junior+ (2020)
[arXiv:2006.03967]

Absorption: Numerical results



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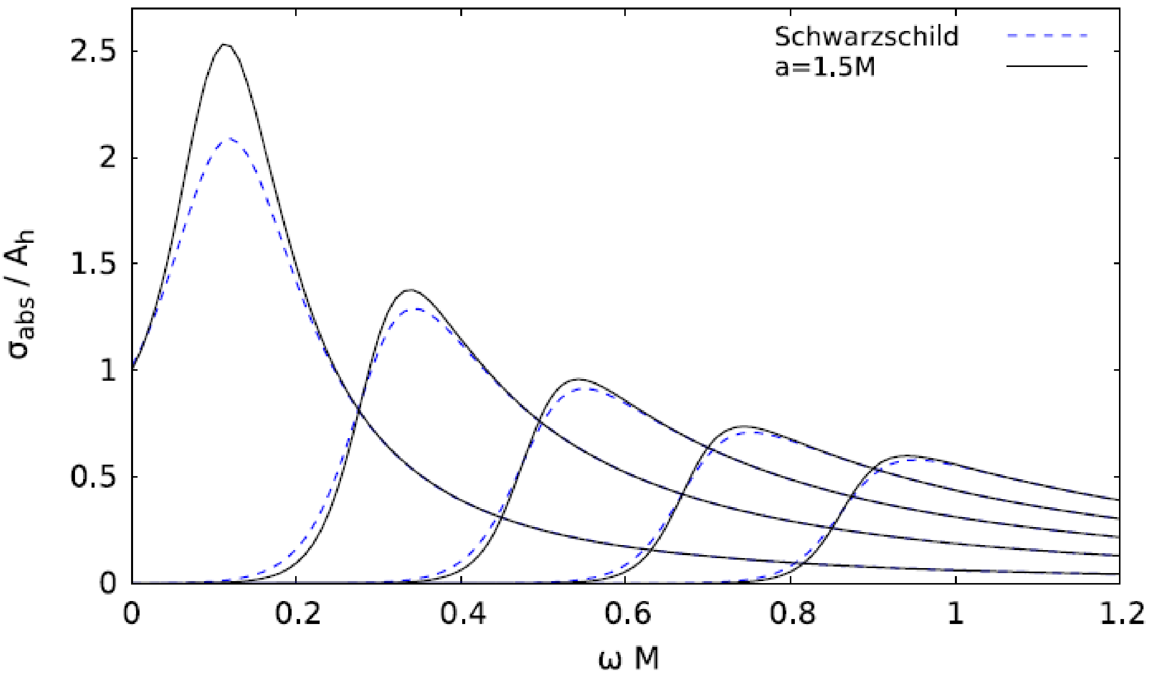
Absorption: Numerical results



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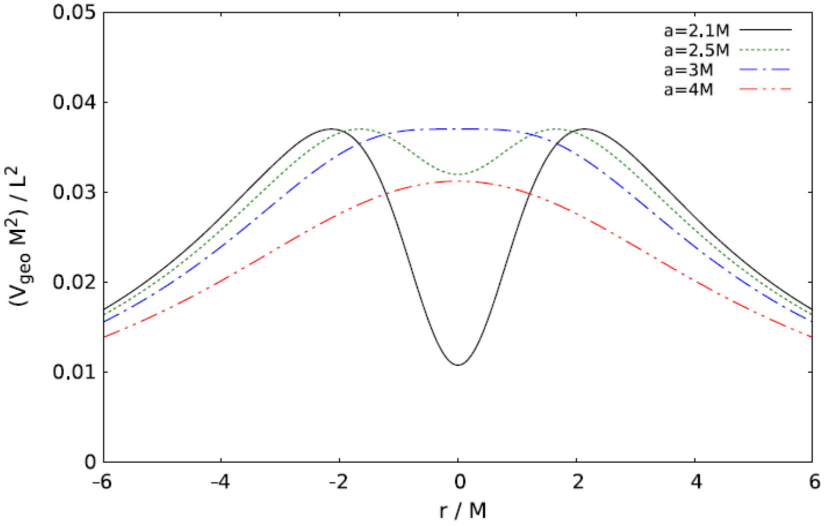
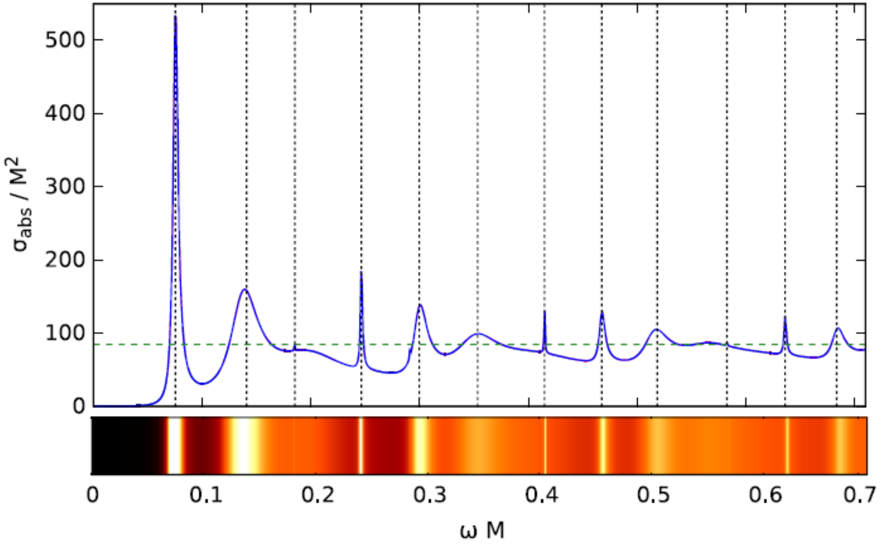
Absorption: Numerical results



l	ω_R	$-i\omega_I$	l	ω_R	$-i\omega_I$
0	0.0753	2.673×10^{-3}	1	0.1852	4.249×10^{-5}
0	0.1408	8.304×10^{-3}	1	0.2464	1.011×10^{-3}
			1	0.2996	7.578×10^{-3}
			1	0.3532	9.417×10^{-3}
2	0.4147	3.432×10^{-4}	3	0.6360	1.222×10^{-3}
2	0.4674	3.154×10^{-3}	3	0.6834	6.861×10^{-3}
2	0.5184	9.281×10^{-3}	3	0.5825	1.096×10^{-4}



Junior+ (2020)
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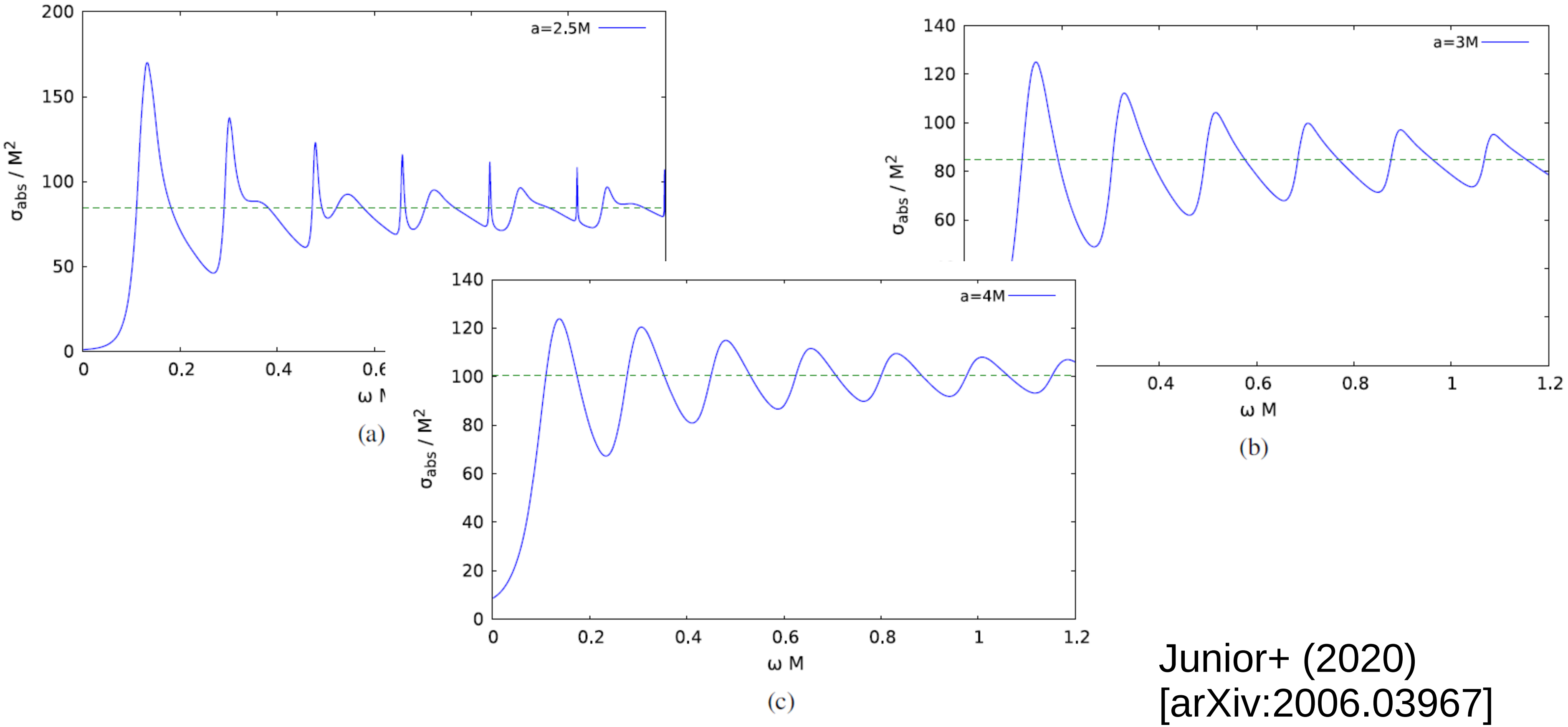
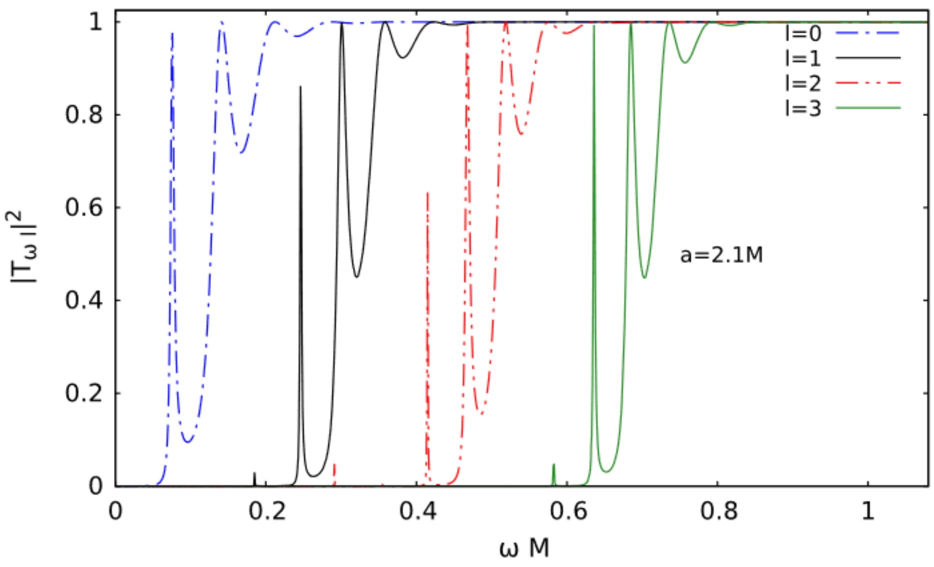
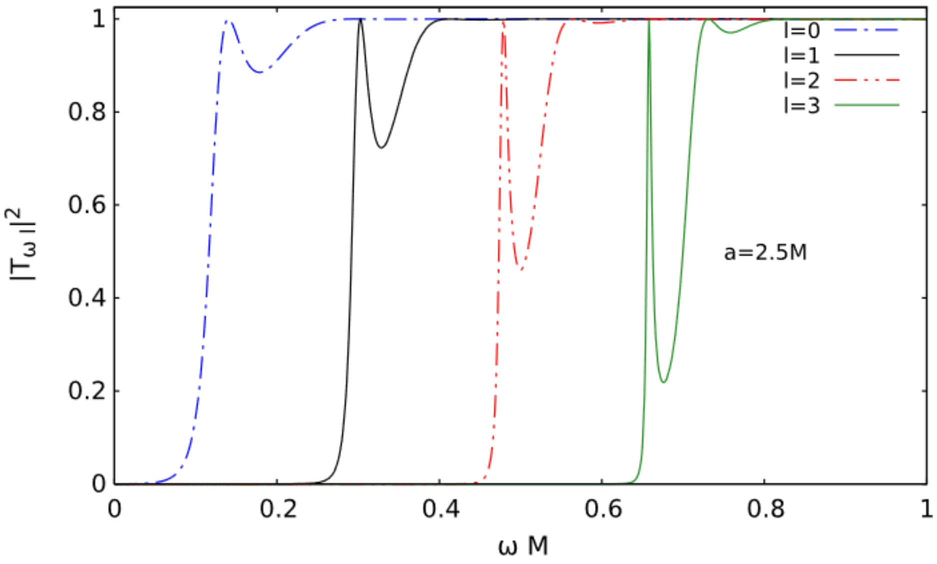
Absorption: Numerical results



Junior+ (2020)
[arXiv:2006.03967]

Absorption: Numerical results

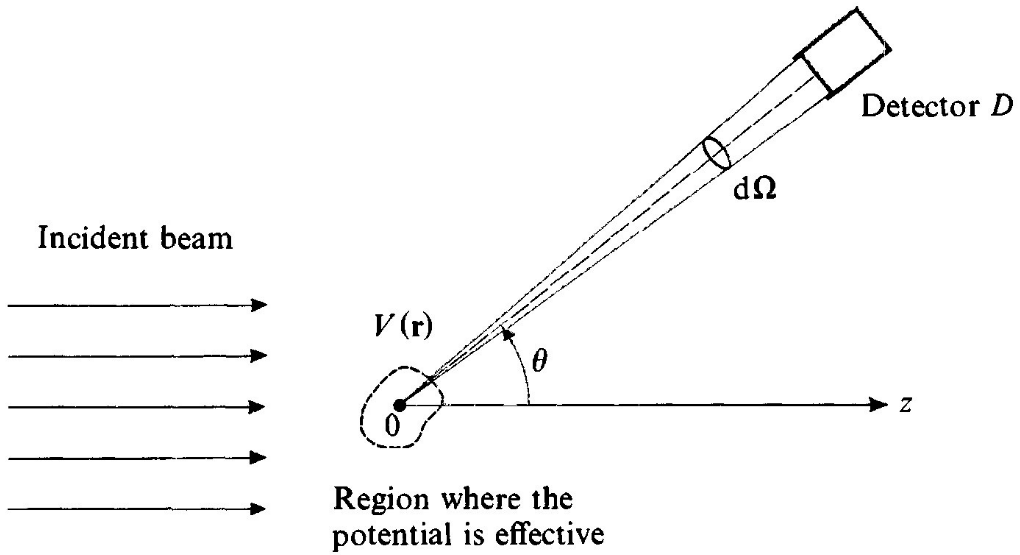




$$|T_{\omega l}|^2 = \frac{A_{\omega l}}{(\omega - \omega_R)^2 + \omega_I^2}$$

Breit&Wigner(1936)

Scattering



$$\frac{d\sigma_{sc}}{d\Omega} = |f(\theta)|^2$$

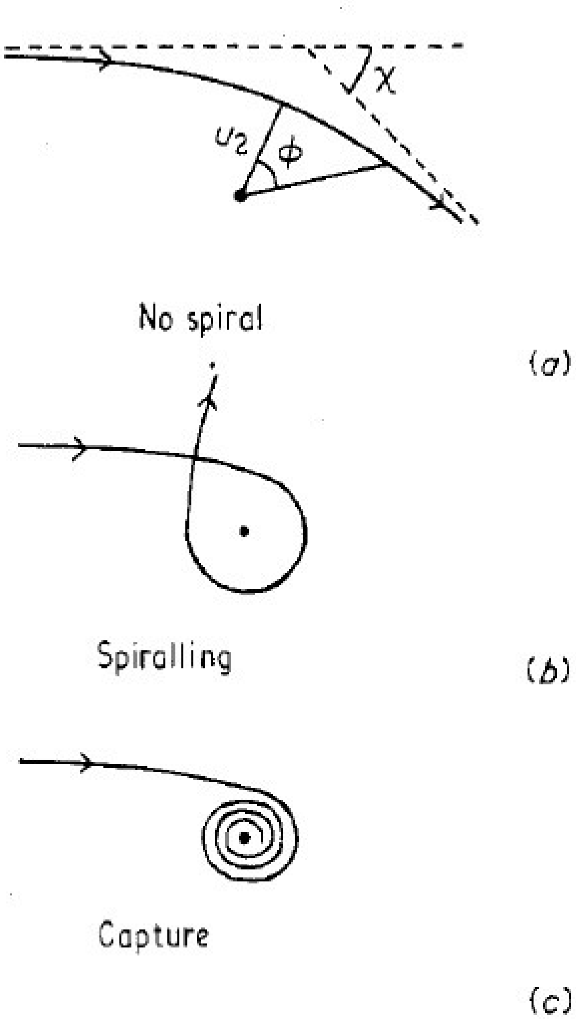
$$f(\theta) = \frac{1}{2i\omega} \sum_{l=0}^{\infty} (2l+1) \left[e^{2i\delta_l(\omega)} - 1 \right] P_l(\cos\theta)$$

$$f(\theta) = \frac{1}{(1-\cos\theta)^n} \sum_{j=0}^{+\infty} F_{\omega j}^{(n)} Y_{j0}(\theta)$$

Yennie+ (1954)

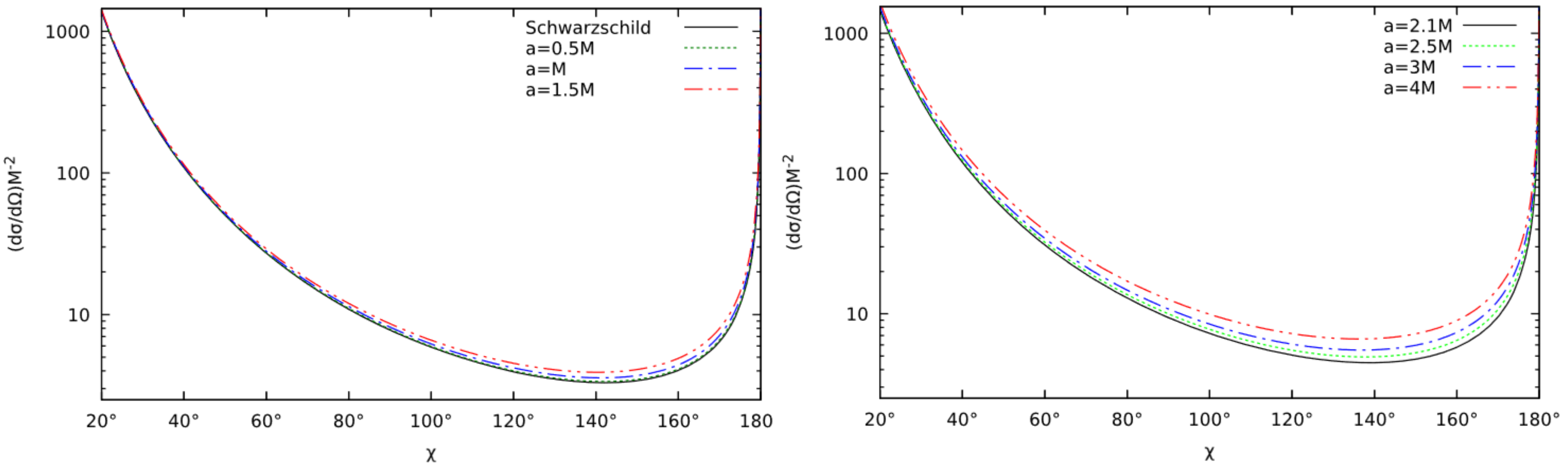
Scattering

$$\frac{d\sigma_{cl}}{d\Omega} = \frac{1}{\sin\chi} \sum b(\chi) \left| \frac{db(\chi)}{d\chi} \right|$$



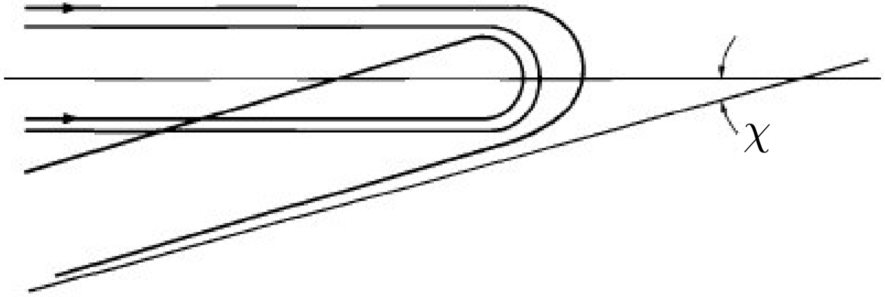
Collins+ (1973)

Scattering



Schwarzschild
a=0.5M
a=M
a=1.5M
1000
100
10
(dσ/dΩ)M-2
20°
40°
60°
80°
100°
120°
140°
160°
180
χ
a=2.1M
a=2.5M
a=3M
a=4M
1000
100
10
(dσ/dΩ)M-2
20°
40°
60°
80°
100°
120°
140°
160°
180°
χ

Scattering

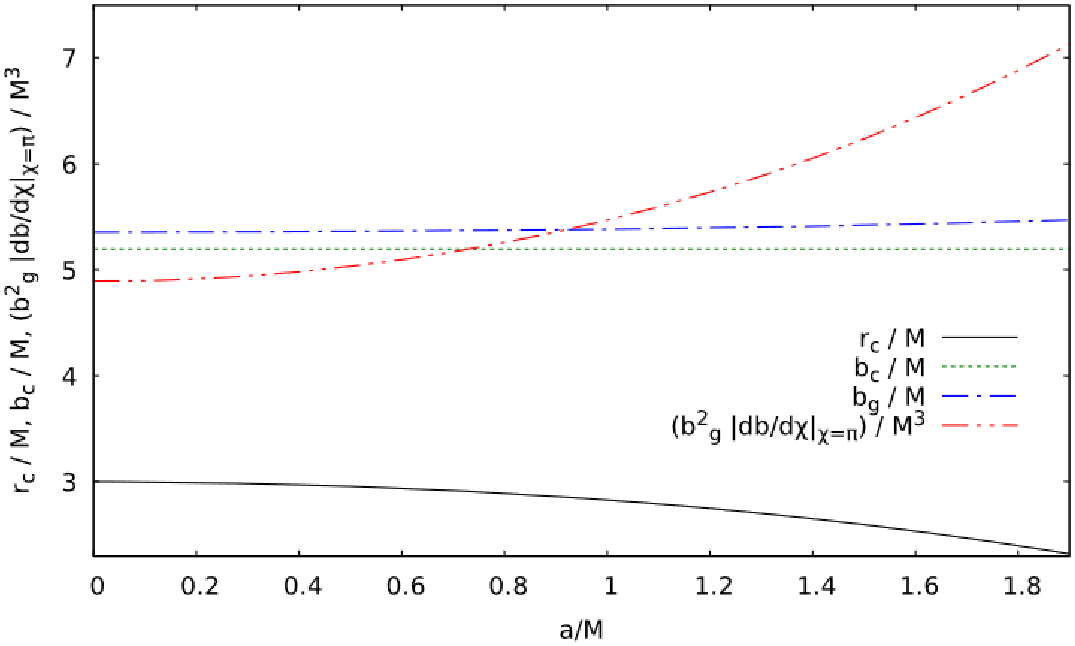


Ford&Wheeler (1959)
Nelson&DeWitt-Morette (1984)

$$\frac{d\sigma}{d\Omega} = \mathcal{A} J_0^2(\omega b_g \sin\chi)$$

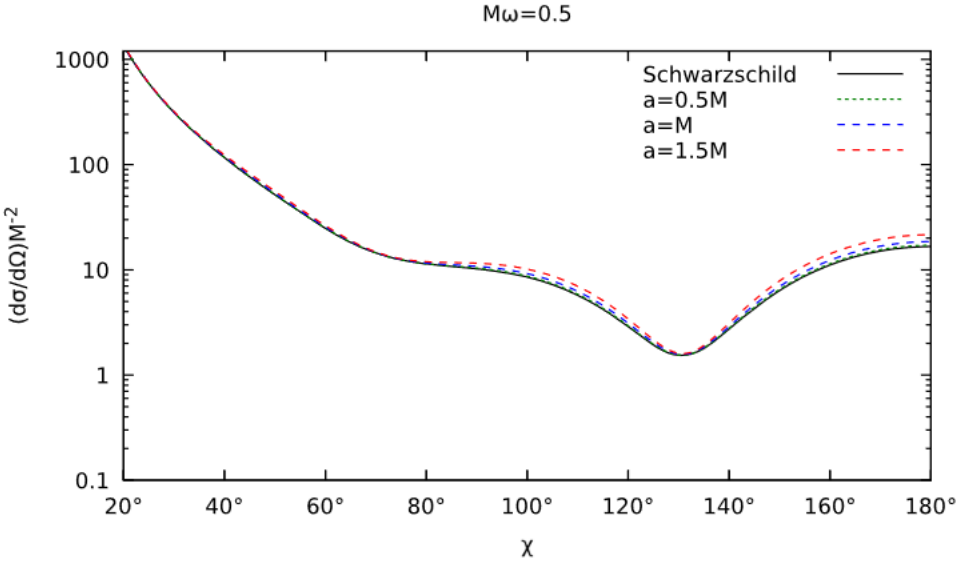
$$\mathcal{A} = 2\pi\omega b_g^2 \left|\frac{db}{d\chi}\right|_{\chi=\pi}$$

Scattering

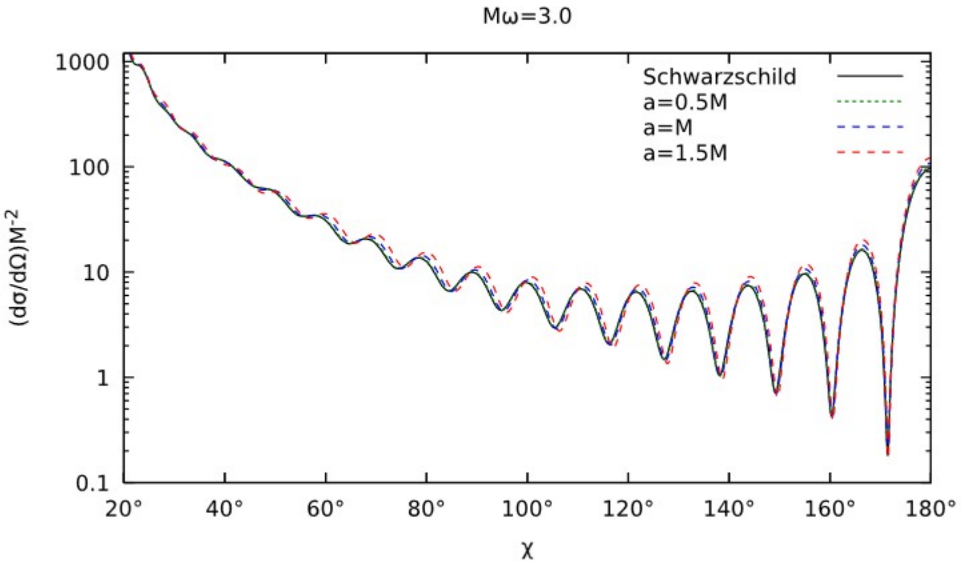


r_c / M, b_c / M, $(b^2_g\ |db/d\chi|_{\chi=\pi})$ / M^3
7
6
5
4
3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
a/M
r_c / M
b_c / M
b_g / M
$(b^2_g\ |db/d\chi|_{\chi=\pi})$ / M^3

Scattering

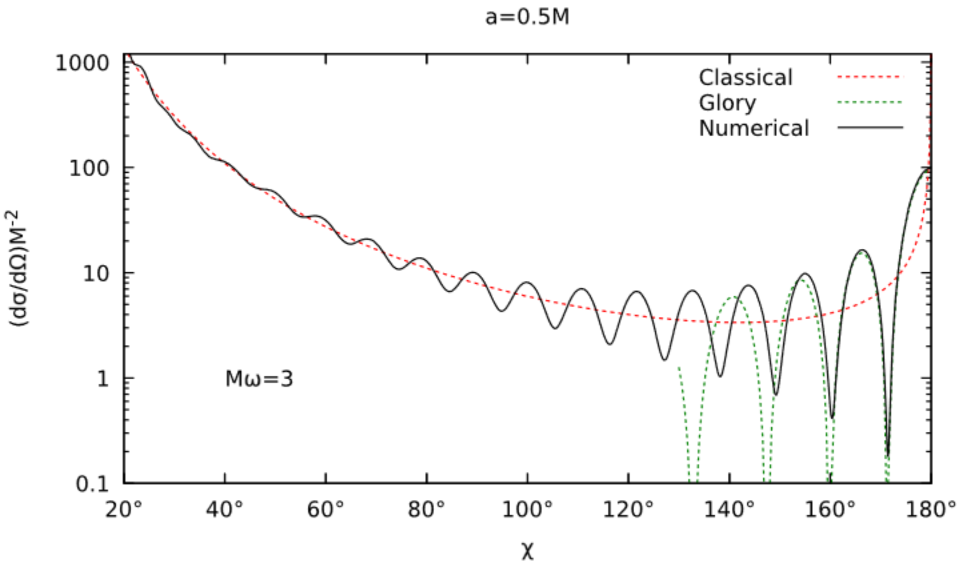


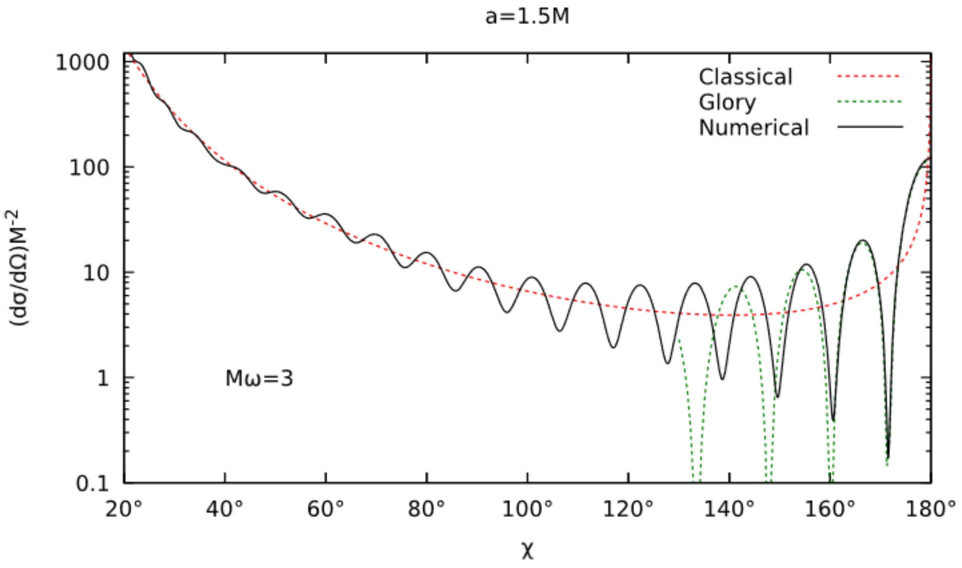
Mω=0.5
Schwarzschild
a=0.5M
a=M
a=1.5M
(dσ/dΩ)M-2
1000
100
10
1
0.1
20°
40°
60°
80°
100°
120°
140°
160°
180°
χ



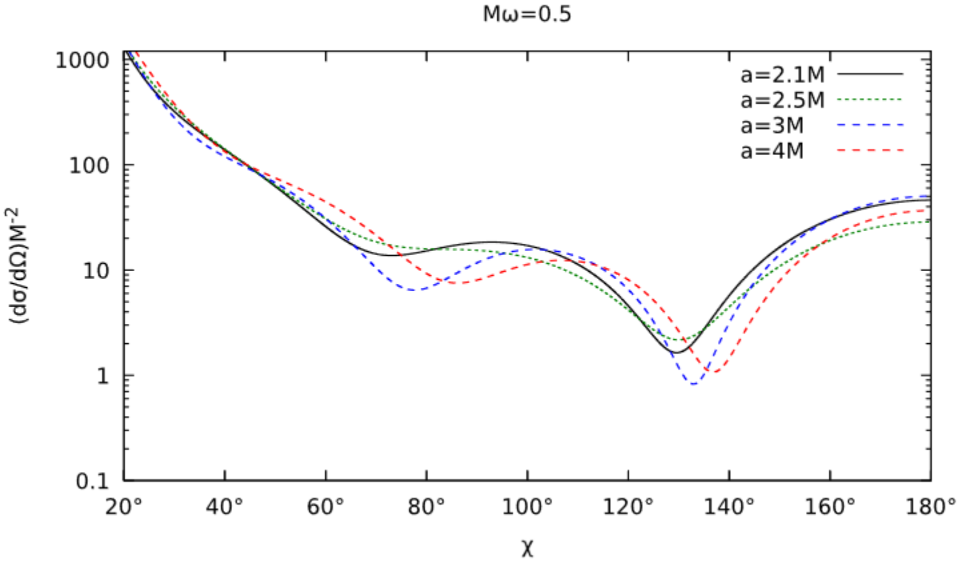
Mω=3.0
Schwarzschild
a=0.5M
a=M
a=1.5M
(dσ/dΩ)M-2
1000
100
10
1
0.1
20°
40°
60°
80°
100°
120°
140°
160°
180°
χ

Scattering

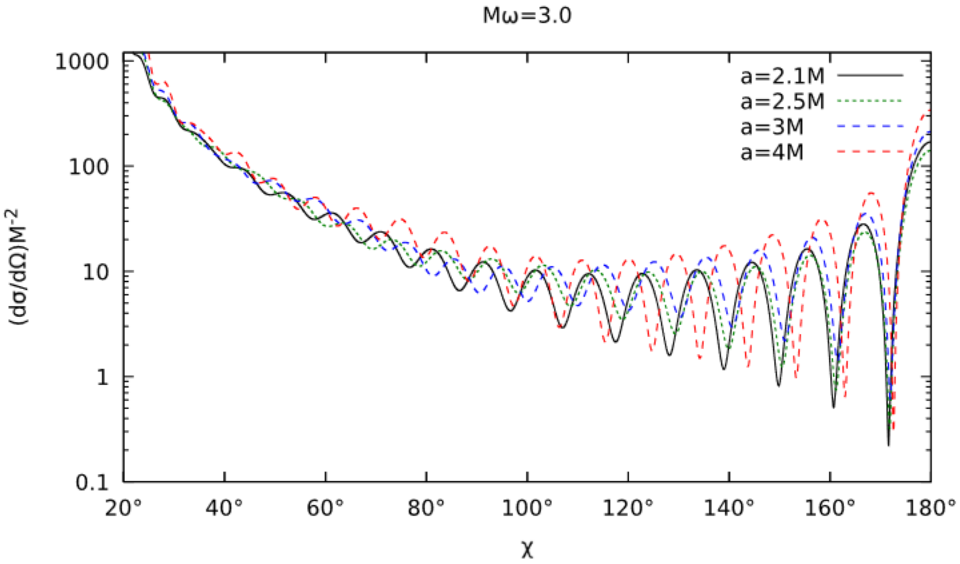

a=0.5M
Classical
Glory
Numerical
Mω=3
(dσ/dΩ)M-2
1000
100
10
1
0.1
20°
40°
60°
80°
100°
120°
140°
160°
180°
χ


a=1.5M
Classical
Glory
Numerical
Mω=3
(dσ/dΩ)M-2
1000
100
10
1
0.1
20°
40°
60°
80°
100°
120°
140°
160°
180°
χ

Scattering

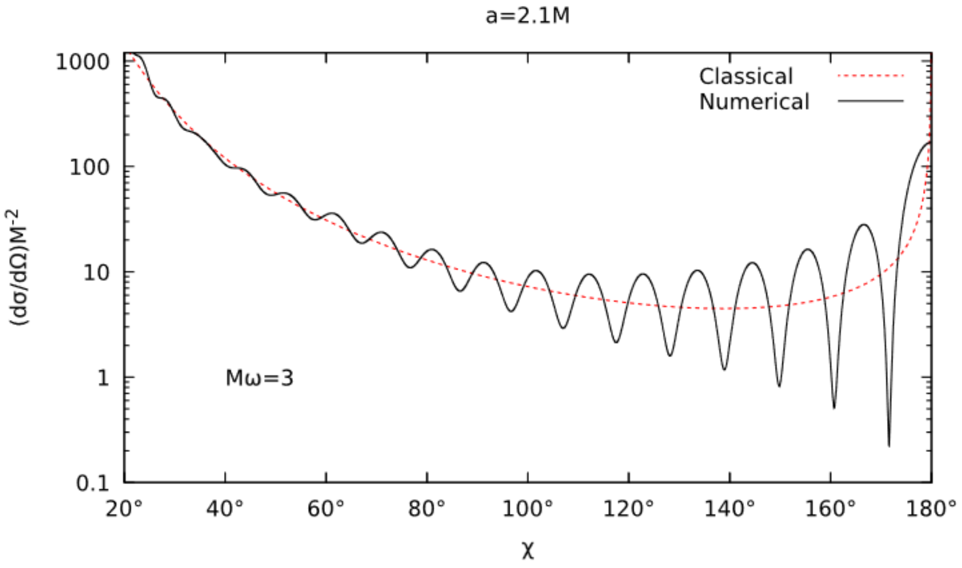


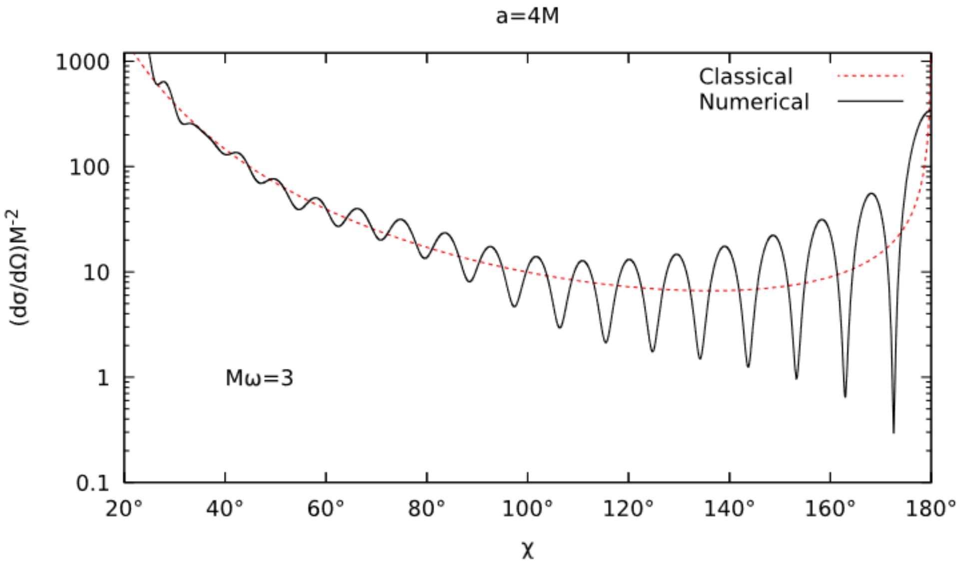
Mω=0.5
a=2.1M
a=2.5M
a=3M
a=4M
1000
100
10
1
0.1
(dσ/dΩ)M^-2
20°
40°
60°
80°
100°
120°
140°
160°
180°
χ



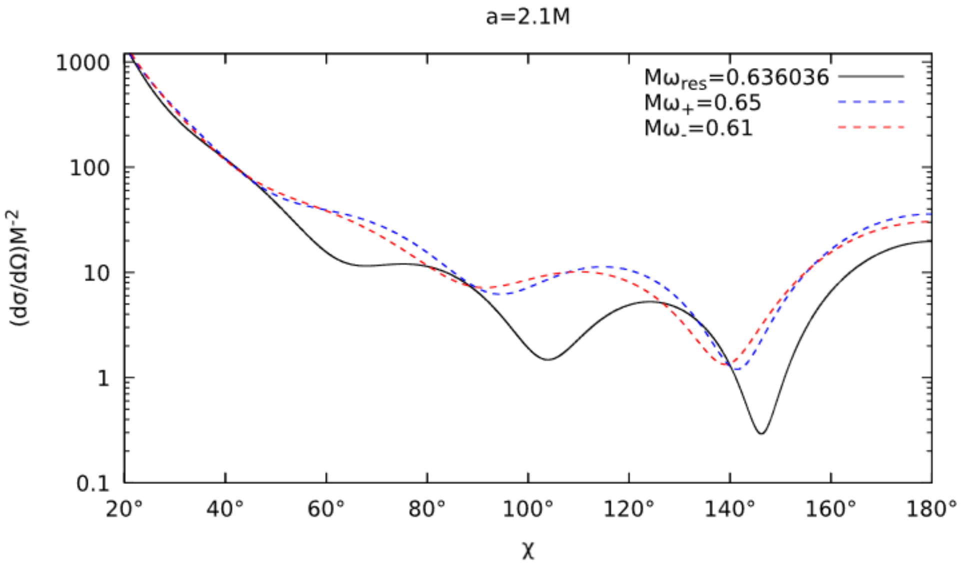
Mω=3.0
a=2.1M
a=2.5M
a=3M
a=4M
1000
100
10
1
0.1
(dσ/dΩ)M^-2
20°
40°
60°
80°
100°
120°
140°
160°
180°
χ

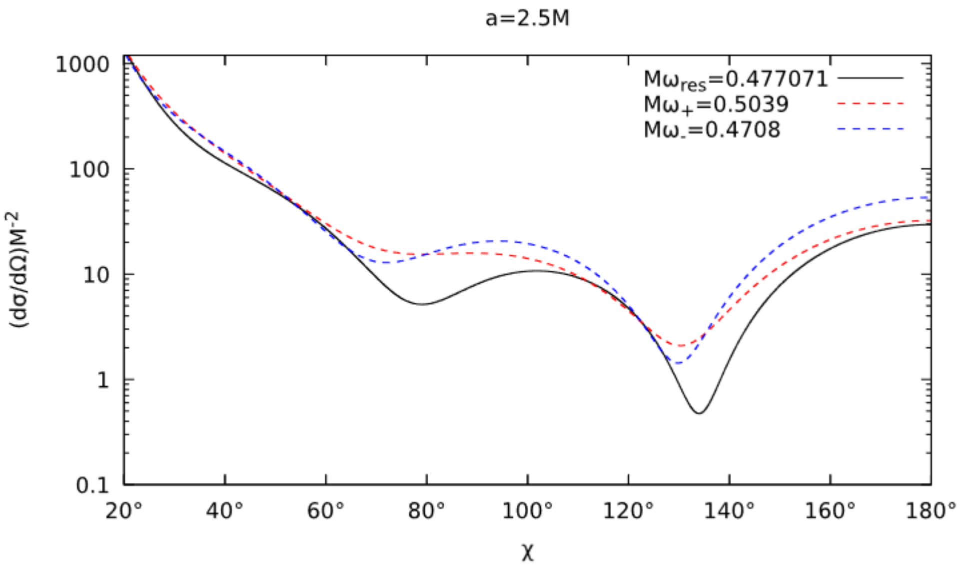
Scattering


a=2.1M
Classical
Numerical
Mω=3
(dσ/dΩ)M-2
1000
100
10
1
0.1
20°
40°
60°
80°
100°
120°
140°
160°
180°
χ


a=4M
Classical
Numerical
Mω=3
(dσ/dΩ)M-2
1000
100
10
1
0.1
20°
40°
60°
80°
100°
120°
140°
160°
180°
χ

Scattering


a=2.1M
Mωres=0.636036
Mω+=0.65
Mω-=0.61
1000
100
10
1
0.1
(dσ/dΩ)M-2
20° 40° 60° 80° 100° 120° 140° 160° 180°
χ


a=2.5M
Mωres=0.477071
Mω+=0.5039
Mω-=0.4708
1000
100
10
1
0.1
(dσ/dΩ)M-2
20° 40° 60° 80° 100° 120° 140° 160° 180°
χ

Final remarks

- The oscillatory pattern for the absorption cross section comes from contributions for different l.
- The resonance peaks in the absorption cross section are related to the quasibound states in the potential well.
- The differential scattering cross section presents an oscillatory pattern, which comes from contributions of different impact parameter.
- The differential scattering cross section is lower for resonant frequencies.

Obrigada! Thank you!