Stability of Hairy Black Holes with scalar or Proca hair

VI Amazonian Symposium on Physics

Belém – November 19, 2024

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Introduction and context

Context

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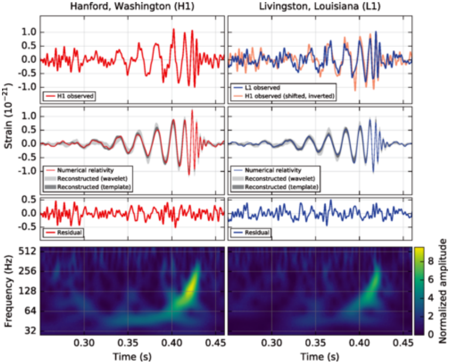
  - Detection of gravitational waves from BH binary mergers by the Ligo-Virgo-Kagra collaboration.



Credit: LIGO Scientific Collaboration and Virgo Collaboration (2016). License CC BY 3.0.

Context

- In recent years, we've witnessed incredible discoveries in astrophysics and Black Hole (BH) physics:
    - Detection of gravitational waves from BH binary mergers by the Ligo-Virgo-Kagra collaboration.
    - Direct imaging of supermassive BHs by the Event Horizon Telescope collaboration.

Credit: Event Horizon Telescope Collaboration. Source: https://www.eso.org/public/images/eso1907a/. License CC BY 4.0.

Context: some questions

- Are these objects BHs as we think (i.e. $\approx$ Kerr)? Could there be mimickers (exotic compact objects)?
- What is the nature of dark matter and can we probe it by using compact objects?
- What is the theory of (quantum) gravity?

$\Rightarrow$ It is often informative to look at simple models to understand the underlying mechanisms.
Then we can also try to assess the astrophysical relevance.

Black Holes with synchronized scalar hair

- Massive complex scalar field (SF) minimally coupled to Einstein's gravity ($c = G = 1$):

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{16\pi} - \nabla_\mu\Psi^*\nabla^\mu\Psi - \mu^2\Psi^*\Psi\right)$$

which results in the Einstein Klein-Gordon system, with stress-energy tensor

$$T_{\mu\nu} = 2\nabla_{(\mu}\Psi^*\nabla_{\nu)}\Psi - g_{\mu\nu}\left(\nabla_\mu\Psi^*\nabla^\mu\Psi + \mu^2\Psi^*\Psi\right)$$

- Solutions described in Herdeiro & Radu, PRL **112**, 221101 (2014); Herdeiro & Radu, CQG **32** 144001 (2015). Some data available at http://gravitation.web.ua.pt/

Black Holes with synchronized scalar hair

- Metric ansatz with spherical coordinates:

$$\mathrm{d}s^2 = e^{2F_1}\left(\frac{\mathrm{d}r^2}{N} + r^2\mathrm{d}\theta^2\right) + e^{2F_2}r^2\sin^2\theta\,(\mathrm{d}\varphi - W\mathrm{d}t)^2 - e^{2F_0}N\mathrm{d}t^2$$

with $N = 1 - \frac{r_H}{r}$.

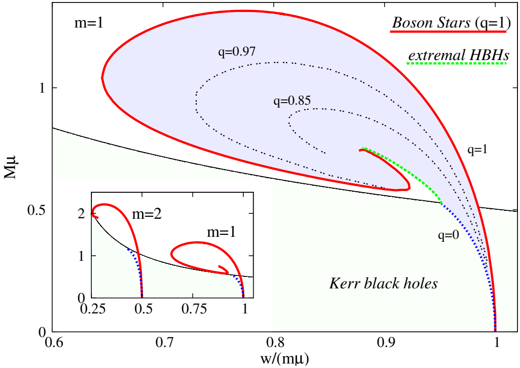
- SF ansatz:

$$\Psi = \phi(r,\theta)e^{i(m\varphi-\omega t)}$$

$\Rightarrow$ **The harmonic dependence breaks the assumptions of no-hair theorem, but still yields a stationary and axisymmetric $T_{\mu\nu}$ and spacetime!**

- The solutions are regular on and outside a horizon, located at $r = r_H$, have equatorial plane symmetry and are asymptotically flat.

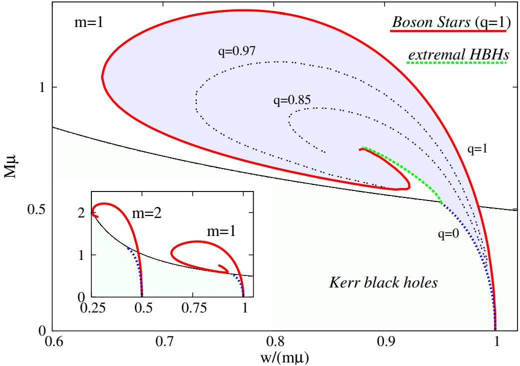
Parameter space



From Herdeiro & Radu, PRL **112**, 221101 (2014)

- Once μ is fixed, solutions are parametrized by the number of nodes n, the azimuthal number m, the frequency ω and the horizon radius r_H. We will consider only initial data with $n = 0$ and $m = 1$ here, and set $\mu = 1$.

Parameter space



From Herdeiro & Radu, PRL **112**, 221101 (2014)

- The scalar field has to obey the synchronization condition $\omega = m\Omega_H$ where $\Omega_H = W(r = r_H)$ is the horizon angular velocity (onset of supperradiance).

Dynamical stability

- **Main question:** Are these hairy BHs stable dynamically? If not, what is the end state (Kerr, another hairy BH?)

- The answer may depend on the region of the parameter space:
  - In the region where they coexist with Kerr BHs, the BHs with scalar hair are entropically favored.
  - But spinning scalar boson stars suffer from non axisymmetric instabilities and collapse to BH – contrary to Proca stars. Sanchis-Gual *et al.*, PRL **123**, 221101 (2019).

⇒ We perform full 3D numerical evolutions to investigate the dynamical behavior!

⚠ So far, results have only been obtained for the scalar case, BHs with Proca hair are work in progress!

Numerical evolution

- To perform the numerical evolution, we use the Einstein Toolkit simulation suite (`https://einsteintoolkit.org/`), an open-source widely used set of tools for Numerical Relativity.
- It includes the evolution code (Lean), some analysis tools such as an apparent horizon finder, quasi-local measures, some diagnostics, and a Python module for post-processing.
- It relies on finite differences on a Cartesian grid, features mesh refinement and parallelization.

ET references: Brandt *et al.*, doi:10.5281/zenodo.12588764; Löffler *et al.*, CQG 29(11):115001, 2012; Bozzola, J. Open Source Softw., 6:3099, 2021; Witek *et al.*, doi:10.5281/zenodo.3565474; Cunha *et al.*, PRD 96:104040, 2017; Dreyer *et al.*, PRD 67:024018, 2003; Schnetter *et al.*, CQG 21:1465-1488, 2004; Thornburg, CQG 21:743-766, 2004;

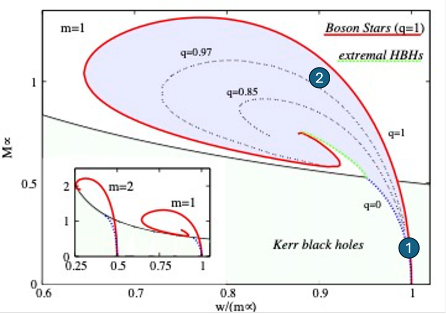
Numerical evolution

- We use the celebrated BSSN system (Baumgarte - Shapiro - Shibata - Nakamura), which relies on the 3+1 formalism (**way to cast Einstein's equations as a Cauchy problem**).

- The initial data ansatz is transformed into quasi-isotropic coordinates, which allows to extend it with a copy inside the horizon.
$\Rightarrow$ This results in a **puncture** (singularity) at the origin.

Numerical results
(Preliminary!)

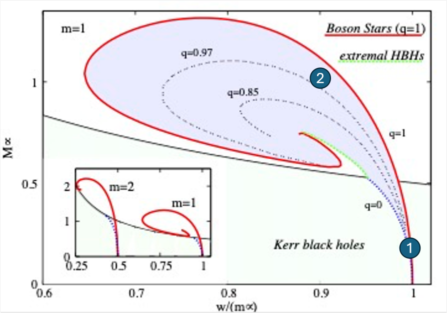
Physical systems



Adapted from Herdeiro & Radu, PRL **112**, 221101 (2014)

- Two Hairy BHs with scalar hair, $m = 1$, $\mu = 1$:
  1. In the region where they coexist with Kerr
  2. Close to the Boson Star limit (very hairy)

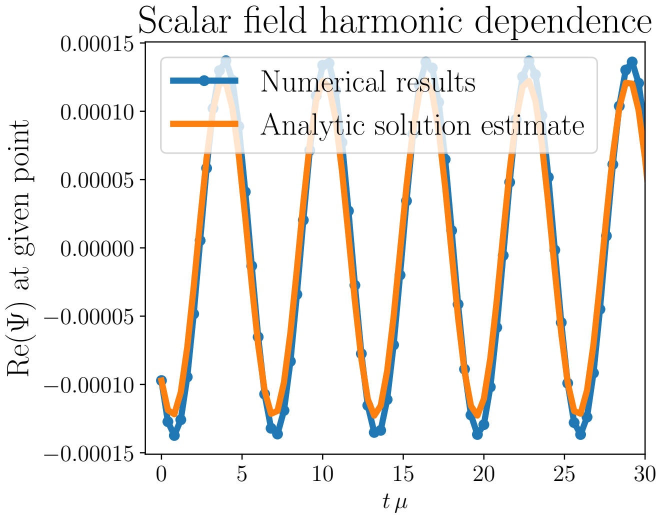
Physical systems



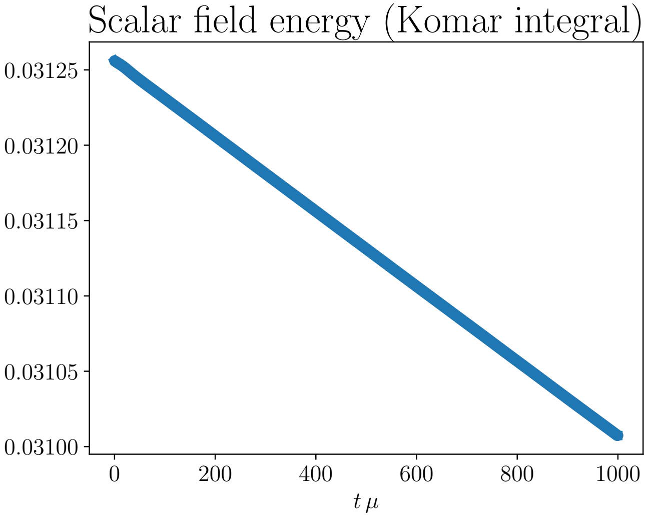
Adapted from Herdeiro & Radu, PRL **112**, 221101 (2014)

#	ω	r_H	M	J	M_H/M	J_H/J
1	0.998200	0.179026	0.127692	0.0346921	0.754980	0.0998357
2	0.90	0.2	1.01049	0.91193	0.11754	0.01217

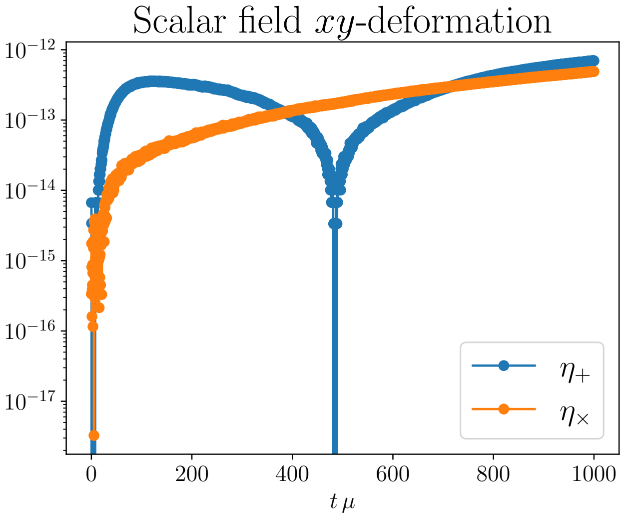
System 1: Scalar field time dependence



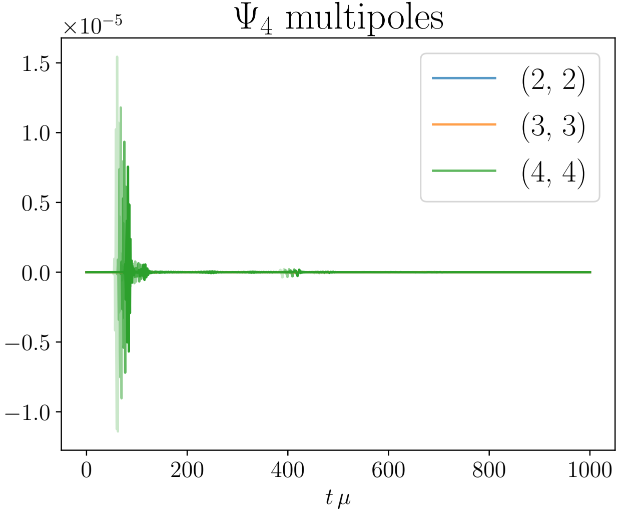
System 1: Scalar field energy



System 1: Scalar field deformation (quadrupole)



System 1: Gravitational waves



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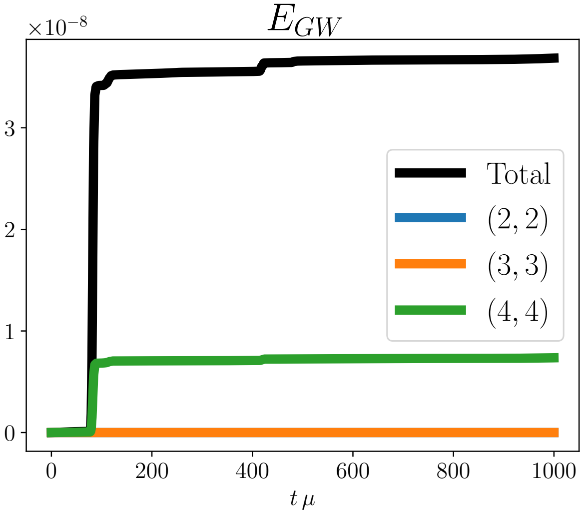

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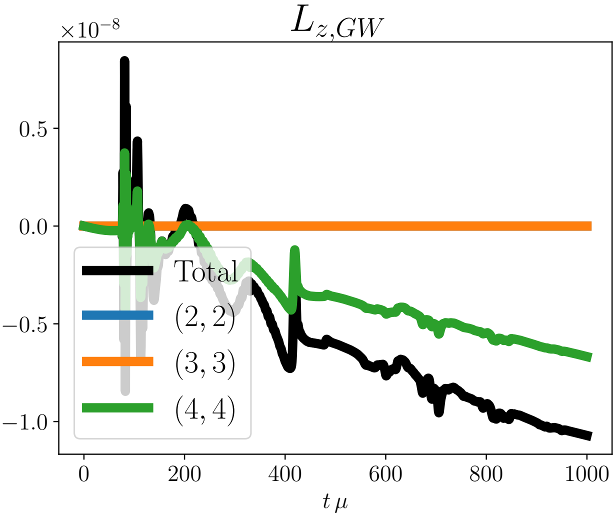
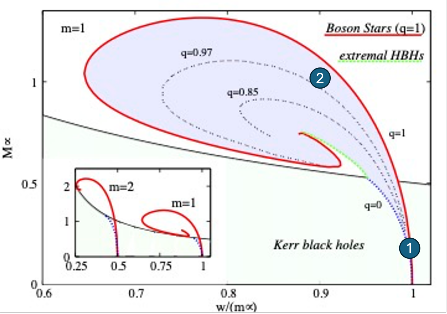


System 1

- This Hairy BH is stable on long time scales, in scales of $1/\mu$, M, $2\pi/\omega$.
- Other indicators are also stable: horizon mass, spin, deformation; SF angular momentum.
- Similar results for another Hairy BH in the same region of the parameter space with $M_H/M \approx 0.5$.

System 2: (Very) Hairy BH



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#	ω	M	J	M_H/M	J_H/J
1	0.998200	0.127692	0.0346921	0.754980	0.0998357
2	0.90	1.01049	0.91193	0.11754	0.01217

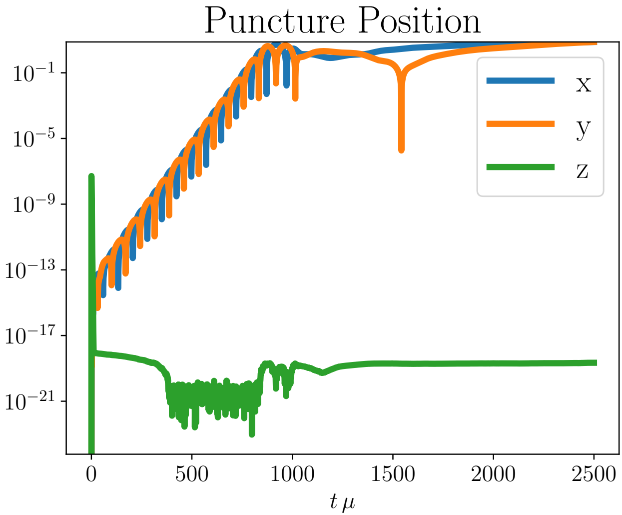
System 2: Main result

- **Driving phenomenon:** The BH starts to move, in a planar outspiral movement, in the direction of the angular momentum.

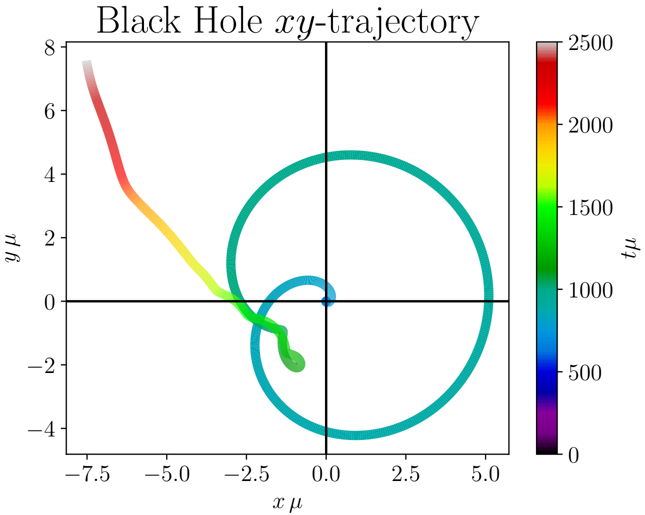
- When it gets to regions of higher SF density, **it perturbs its structure and accretes substantially**.

- (Similar behavior has been observed for another physical configuration in this region of the parameter space.)

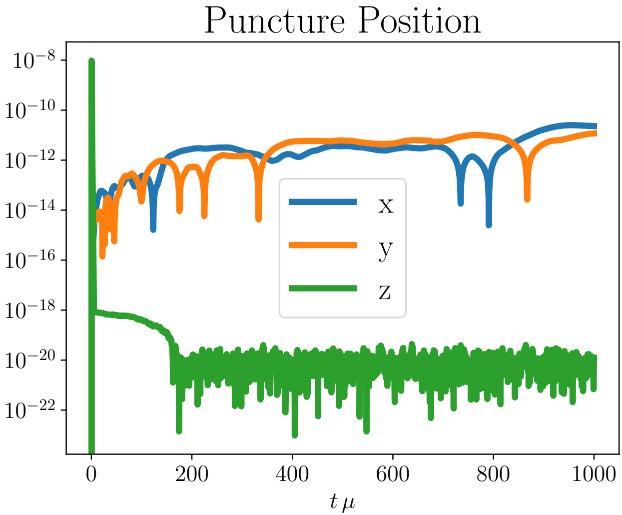
System 2: Puncture position



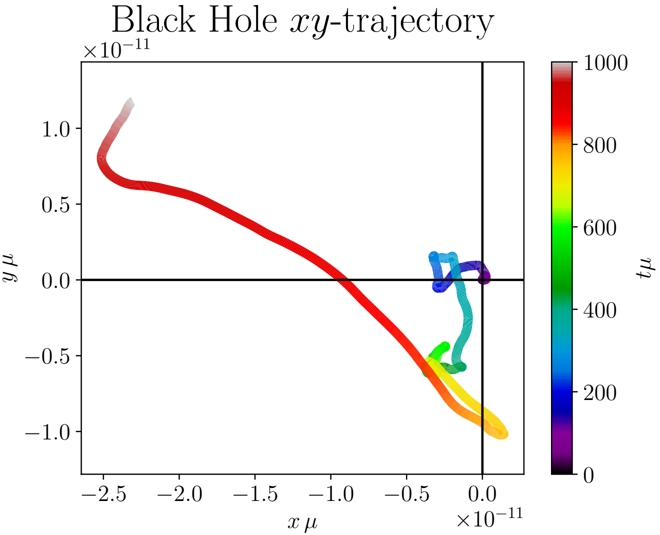
System 2: Puncture position



System 1 revisited: Puncture position



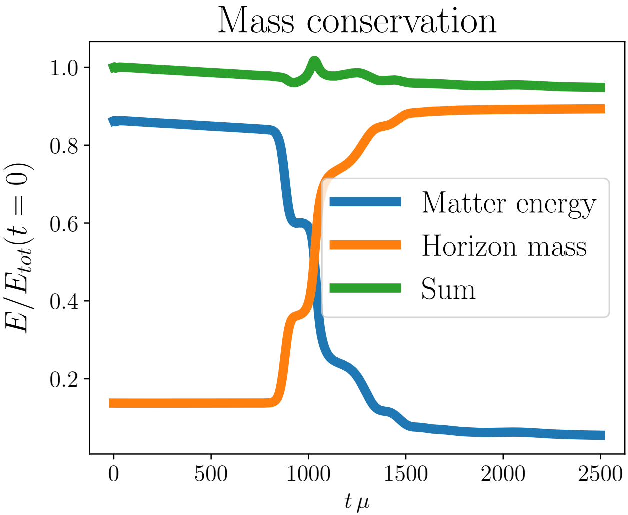
System 1 revisited: Puncture position



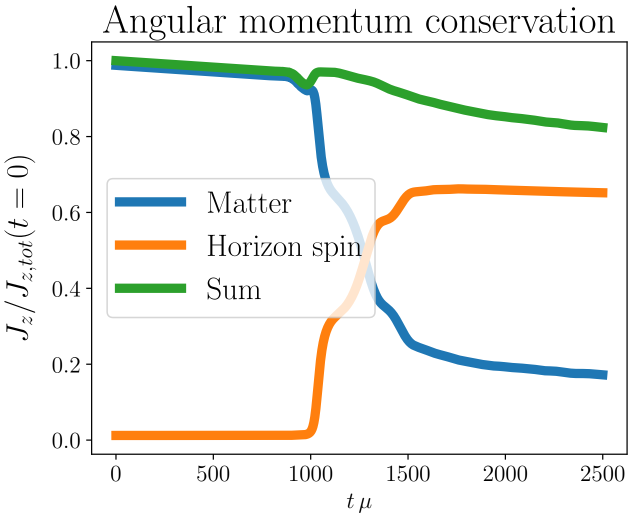
System 2: Remarks

- The Einstein Toolkit is able to track the puncture and move the mesh refinement levels accordingly (required to avoid crash!).
- Tweaking the numerics (position of the boundary, refinement levels, resolution, ...) generates a different numerical noise, and hence a different trajectory, but its properties are maintained and the overall behavior is unchanged.
- Relaxing the **equatorial symmetry** causes the BH to quickly move to a different (small) z coordinate, which then remains stable.

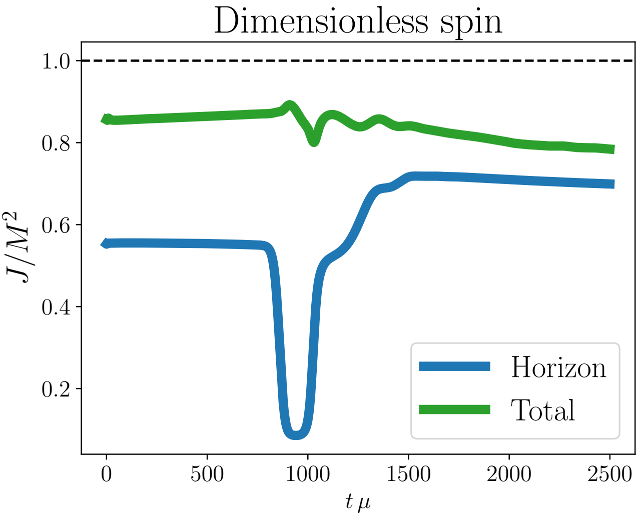
System 2: Mass exchange



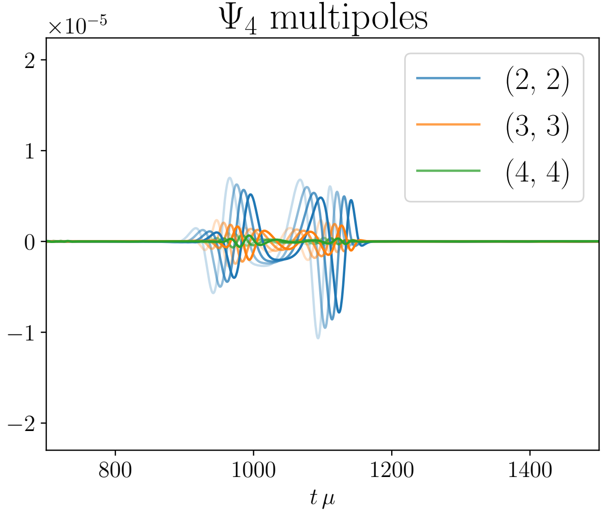
System 2: Angular momentum exchange



System 2: Dimensionless spin



System 2: Gravitational waves



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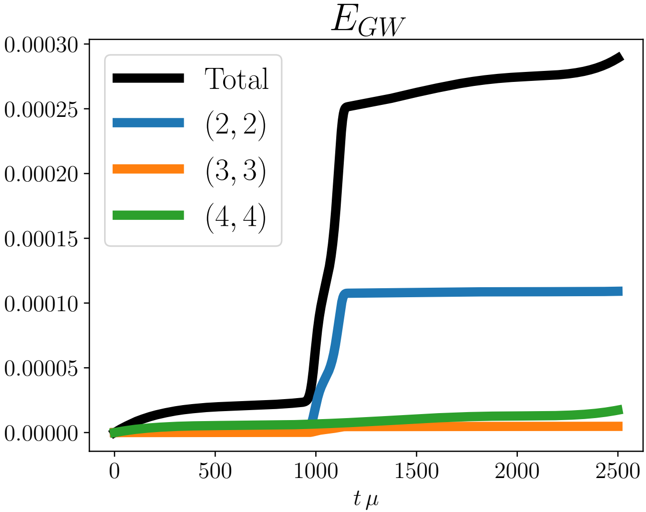

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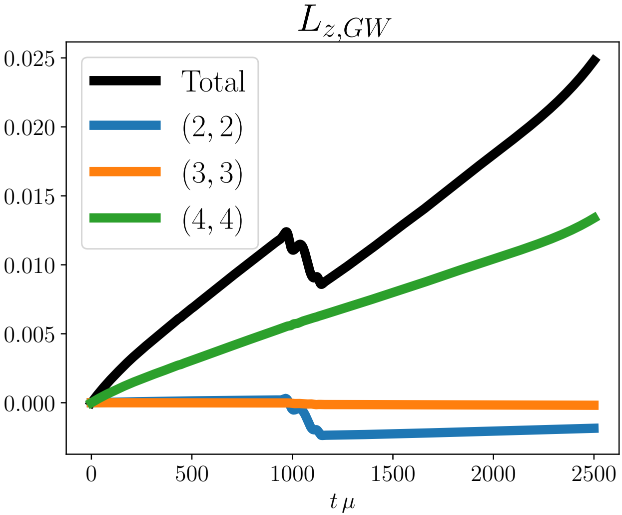

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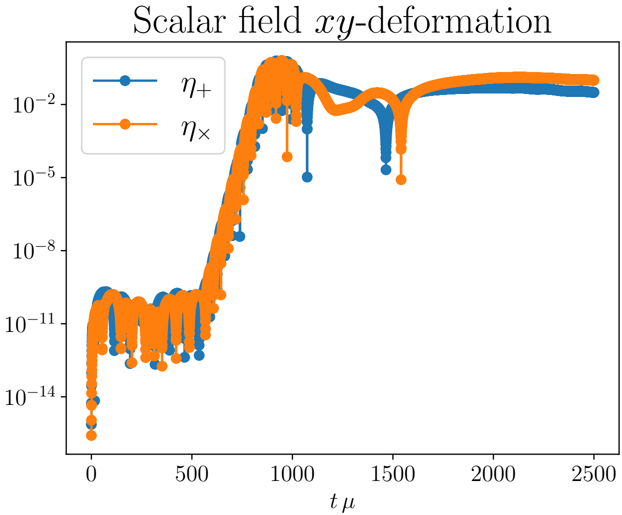
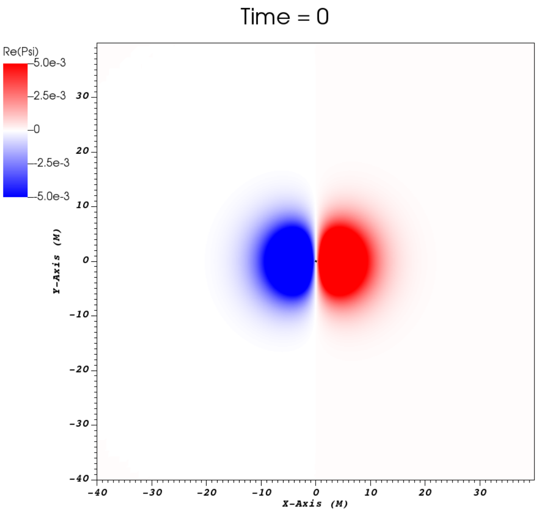
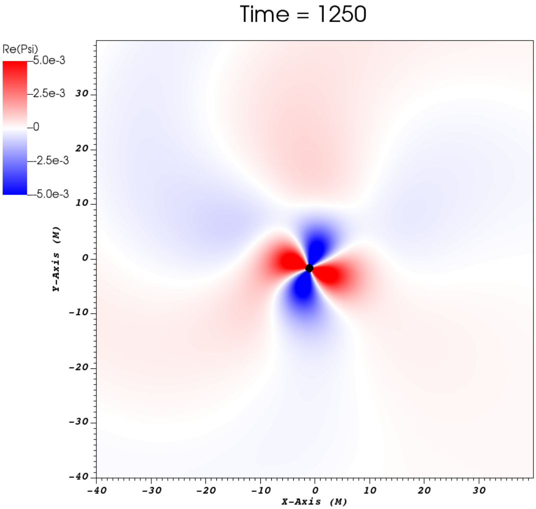


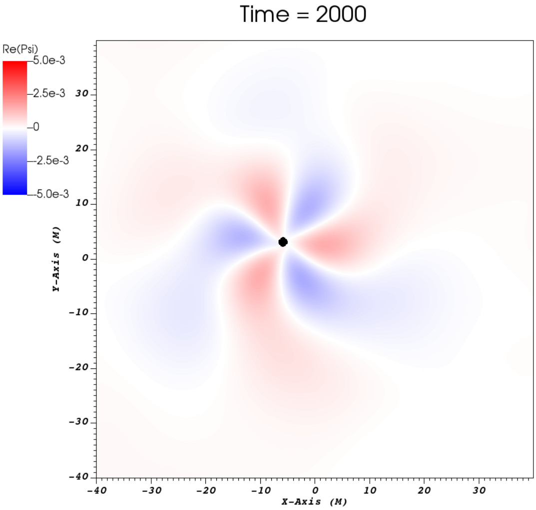
System 2: Scalar field configuration



System 2: Scalar field configuration



System 2: Scalar field configuration



System 2: Movies

Interpretation

- For a scalar boson star, the origin is an unstable point
  $\Rightarrow$ The BH, whose mass is small, is like a test-particle sitting at the origin. The numerical noise and discretization are enough to trigger the instability, resulting in the BH outspiral.
- On the other hand, it's stable against z perturbation.
- From a few preliminary tests, adding a non-axisymmetric scalar field or metric perturbation does not trigger this instability faster.
- At this stage, outgoing matter flux is unclear.

Conclusion and prospects

Summary

- **Main result:** We notice two distinct stability behavior for BHs with **scalar** hair:
  1. Stable in the region where Kerr BH exist
  2. Unstable close to the Boson Star limit (very hairy BH), resulting in the accretion of the scalar field.

- In the latter case, the precise end state (migration to Kerr or other hairy BH) can be hard to tell numerically, as it involves long time scales.
  Superradiance could also kick in on long time scales.

Current work: BH with Proca hair

- Spinning Proca stars do not exhibit the same instabilities as spinning scalar boson stars.
  $\Rightarrow$ **How does that transfer to hairy BHs?**

- Furthermore, Proca stars / synchronized Proca hair around BHs have spherical density repartition (vs. toroidal for scalar), in which case the origin should be a stable equilibrium point.
  But the BH is directly in a higher density region and could start accreting.

- **Numerical implementation is in progress!**

Future work

Beyond the comparison between BHs with scalar and Proca hairs, here are interesting follow-up investigations:

- Perturb stable configurations.
- Explore parameter space (e.g. $J/M^2 > 1$), look for a transition between regions with different behaviors.
- Explore more exotic configurations, other m, excited states...

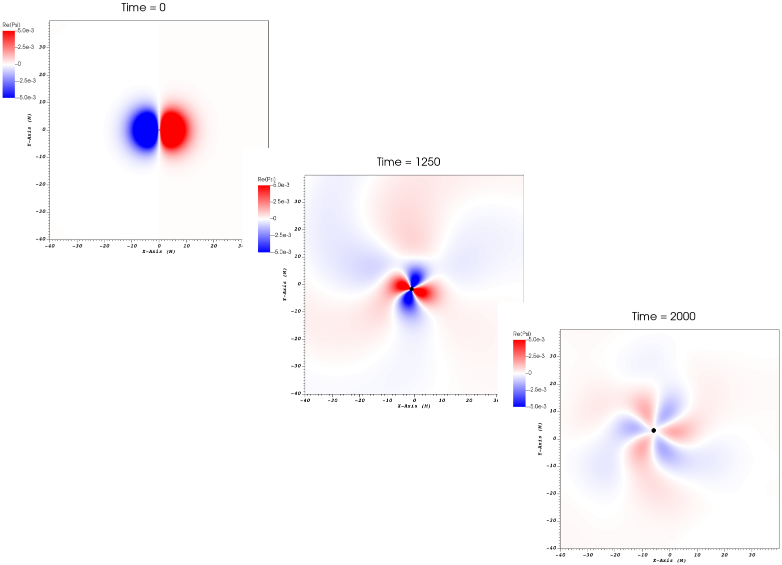
Acknowledgments

- This work was supported by CIDMA and is funded by the Fundação para a Ciência e a Tecnologia, I.P. (FCT, Funder ID = 50110000187) under Grants https://doi.org/10.54499/UIDB/04106/2020 and https://doi.org/10.54499/UIDP/04106/2020.

- The author thankfully acknowledges RES resources provided by BSC in MareNostrum to FI-2024-2-0012, FI-2024-3-0007, by BSC in MareNostrum to POR021PROD through FCT.

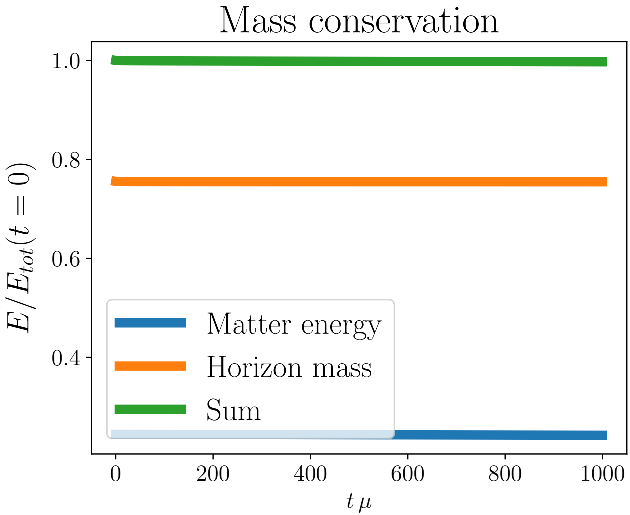
- This work was granted access to the HPC resources of MesoPSL financed by the Region Ile de France and the project Equip@Meso (reference ANR-10-EQPX-29-01) of the programme Investissements d'Avenir supervised by the Agence Nationale pour la Recherche.

Thank you!


Time = 0
Time = 1250
Time = 2000
Re(Psi)
5.0e-3
2.5e-3
0
-2.5e-3
-5.0e-3
X-Axis (M)
Y-Axis (M)

Supplementary material

System 1: Mass conservation



System 1: Angular momentum conservation