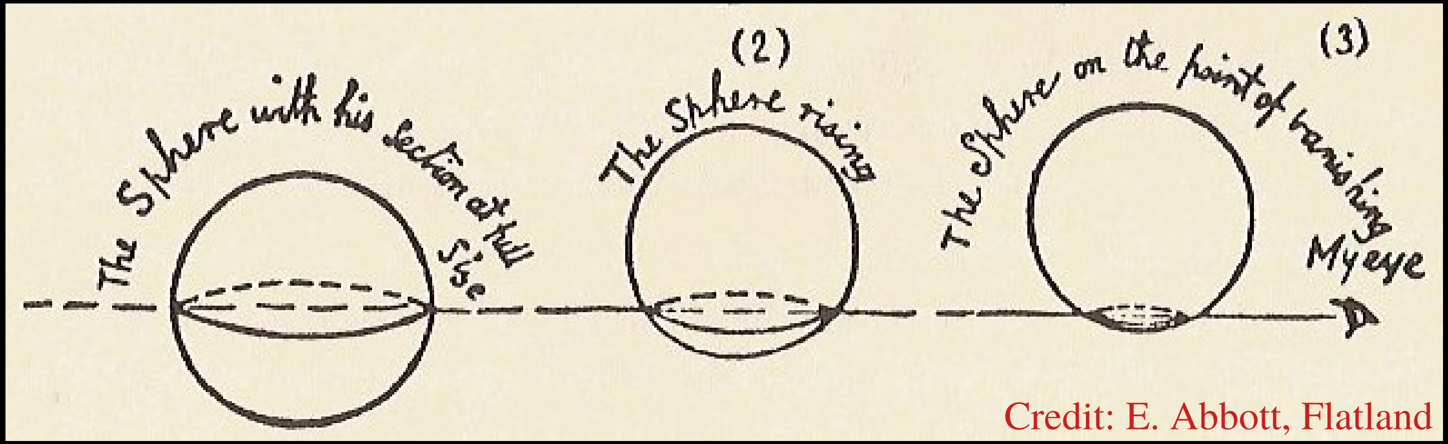
Hyper-shadows:

Imaging 5 dimensional black holes

ArXiv: 2410.05390, João P.A. Novo, Pedro V.P. Cunha, Carlos A.R. Herdeiro



Credit: E. Abbott, Flatland

Pedro Cunha

Universidade de Aveiro

universidade de aveiro

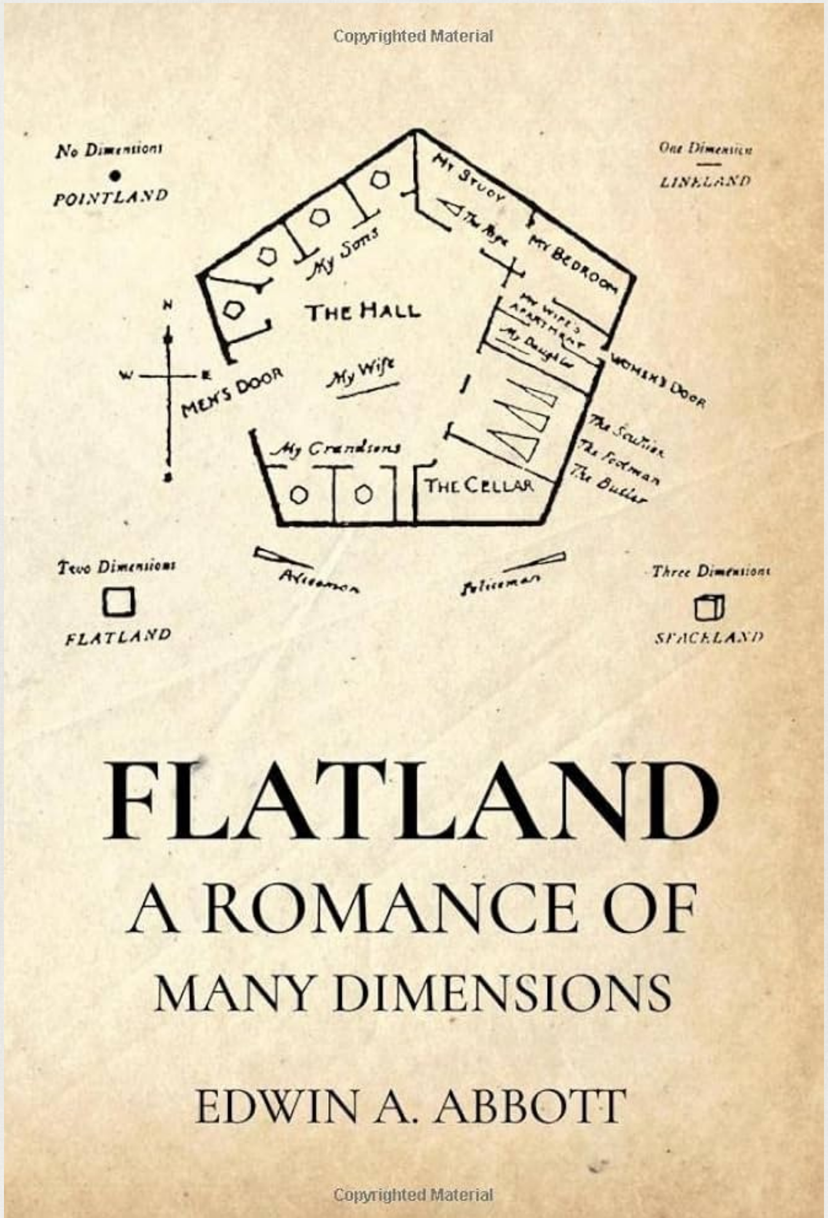
CIDMA CENTRO DE I&D EM MATEMÁTICA E APLICAÇÕES CENTER FOR R&D IN MATHEMATICS AND APPLICATIONS

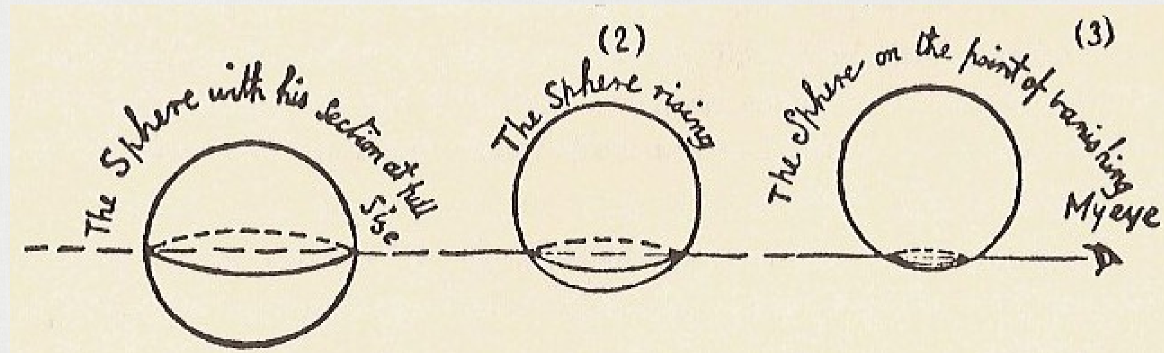
FCT Fundação para a Ciência e a Tecnologia MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

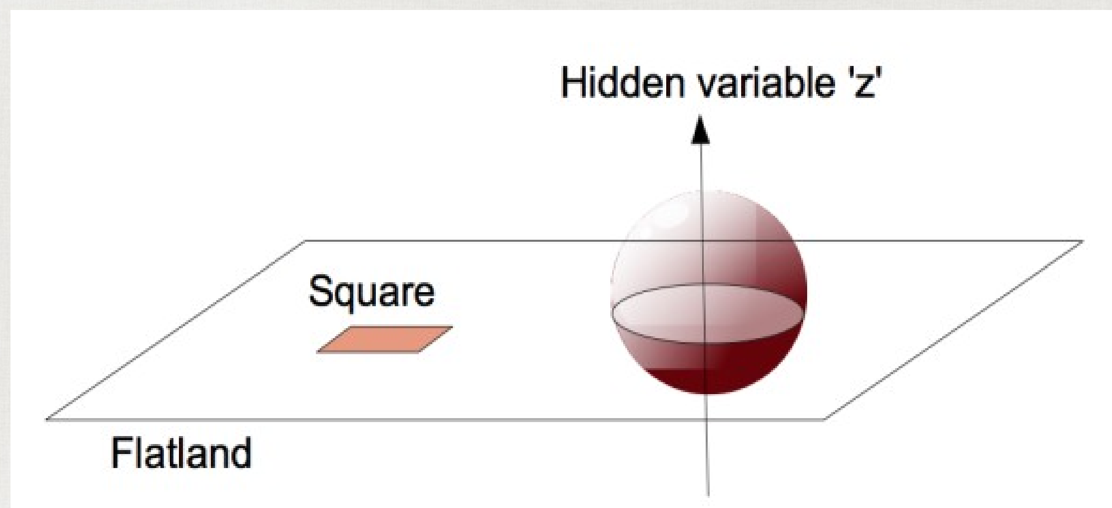
Disclaimer:

This talk has no connection whatsoever to the 'Hypershadow' character from the Sonic the Hedgehog franchise. Any resemblance is purely coincidental.

Flatland: a Romance of many dimensions

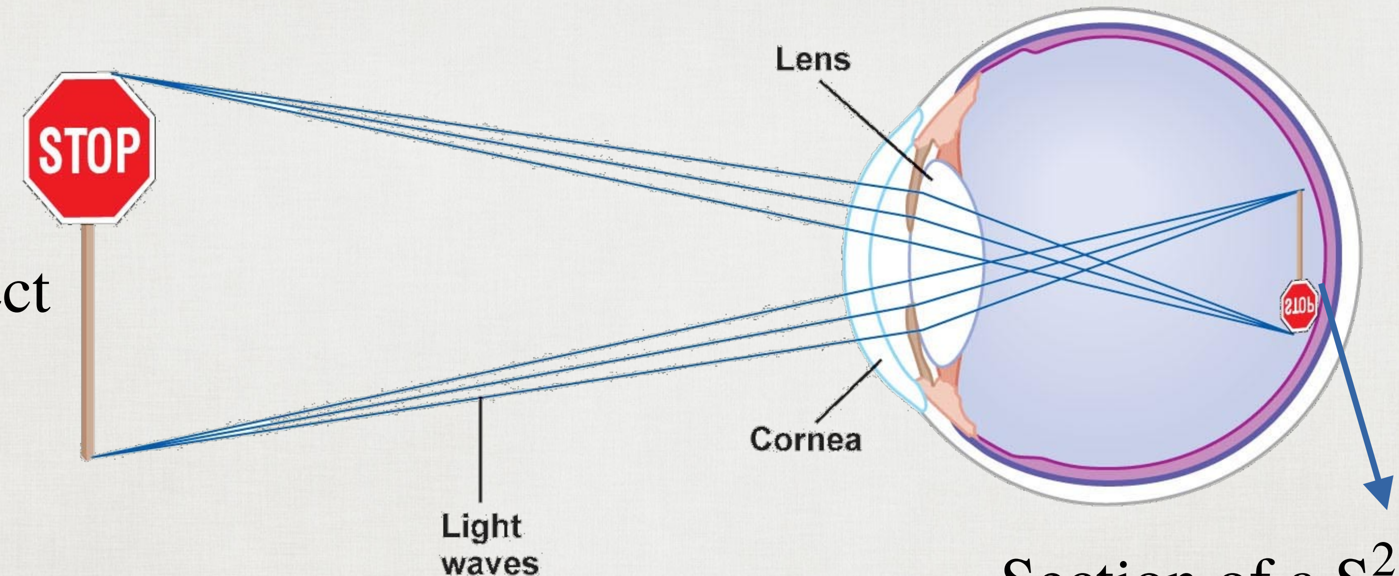




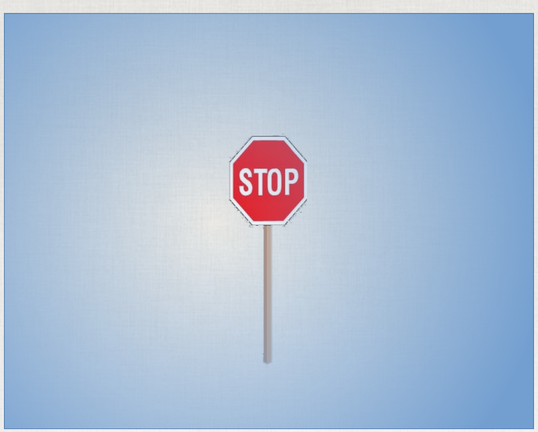


How do we “see” things in our 3D world?

3D object



Section of a S^2 sphere



Observed image is 2D

Could we “see” a 4D object by defining a <u>3D retina</u>?

Retina is a 2D local patch, parameterized by the coordinates (θ, ϕ)

S^2 sphere in spherical coordinates:

$$z = r \cos\theta$$
$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$

$$r \in [0, \infty[, \ \theta \in [0, \pi], \ \phi \in [0, 2\pi],$$

Retina is a 2D local patch, parameterized by the coordinates (θ, ϕ)

S^2 sphere in spherical coordinates:

$$z = r\cos\theta$$
$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$

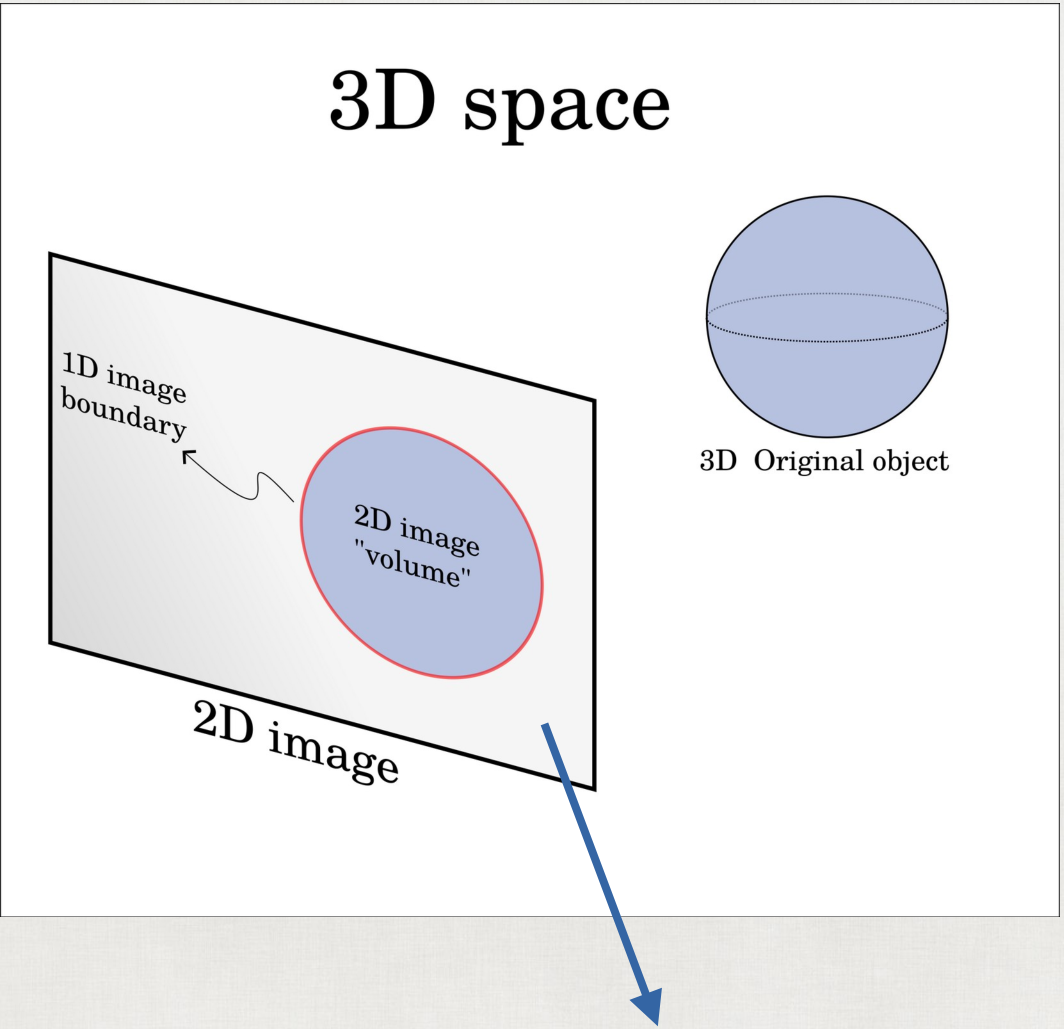
$$r \in [0, \infty[, \ \theta \in [0, \pi], \ \phi \in [0, 2\pi],$$

Retina is a 3D local patch, parameterized by the coordinates (ψ, θ, ϕ)

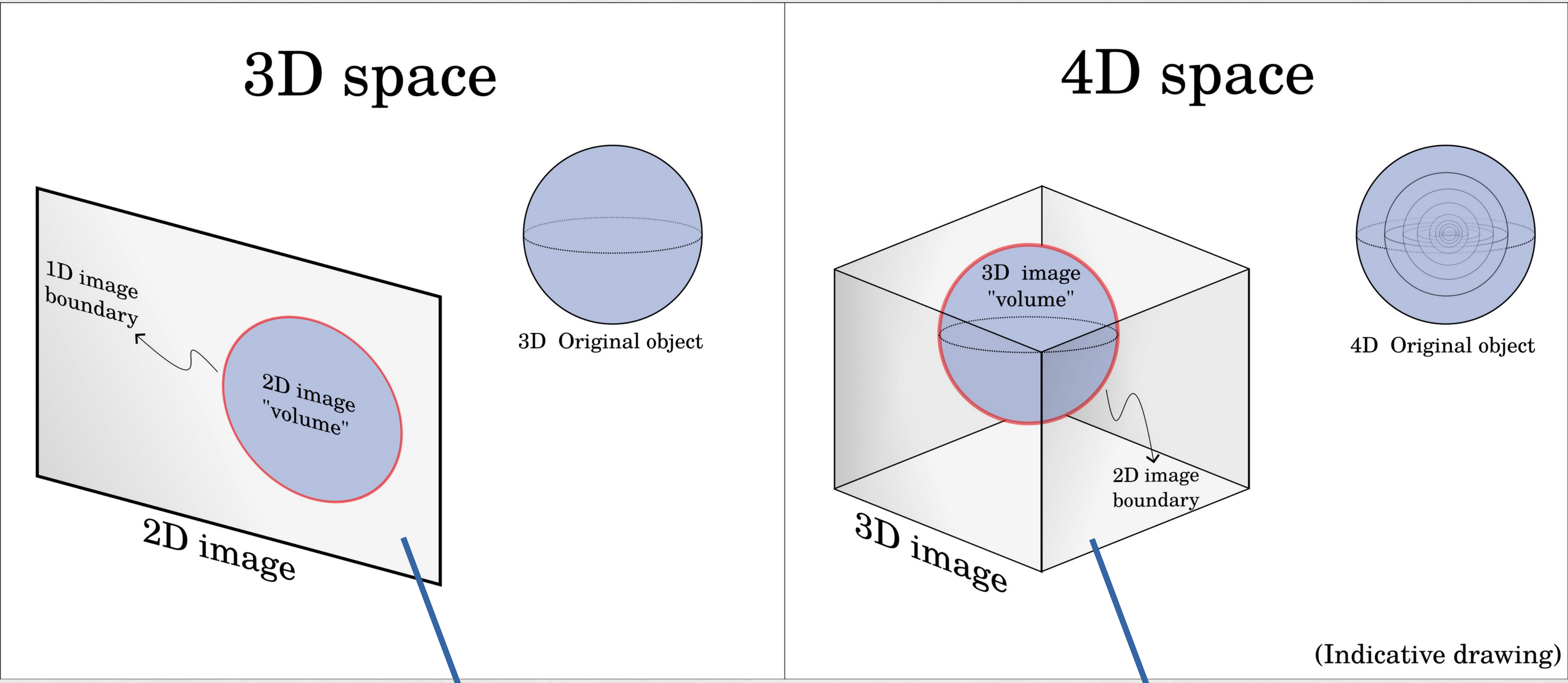
S^3 sphere in spherical coordinates:

$$w = r\cos\psi$$
$$z = r\sin\psi\cos\theta$$
$$x = r\sin\psi\sin\theta\cos\phi$$
$$y = r\sin\psi\sin\theta\sin\phi$$

$$r \in [0, \infty[, \ \psi \in [0, \pi], \ \theta \in [0, \pi], \ \phi \in [0, 2\pi],$$



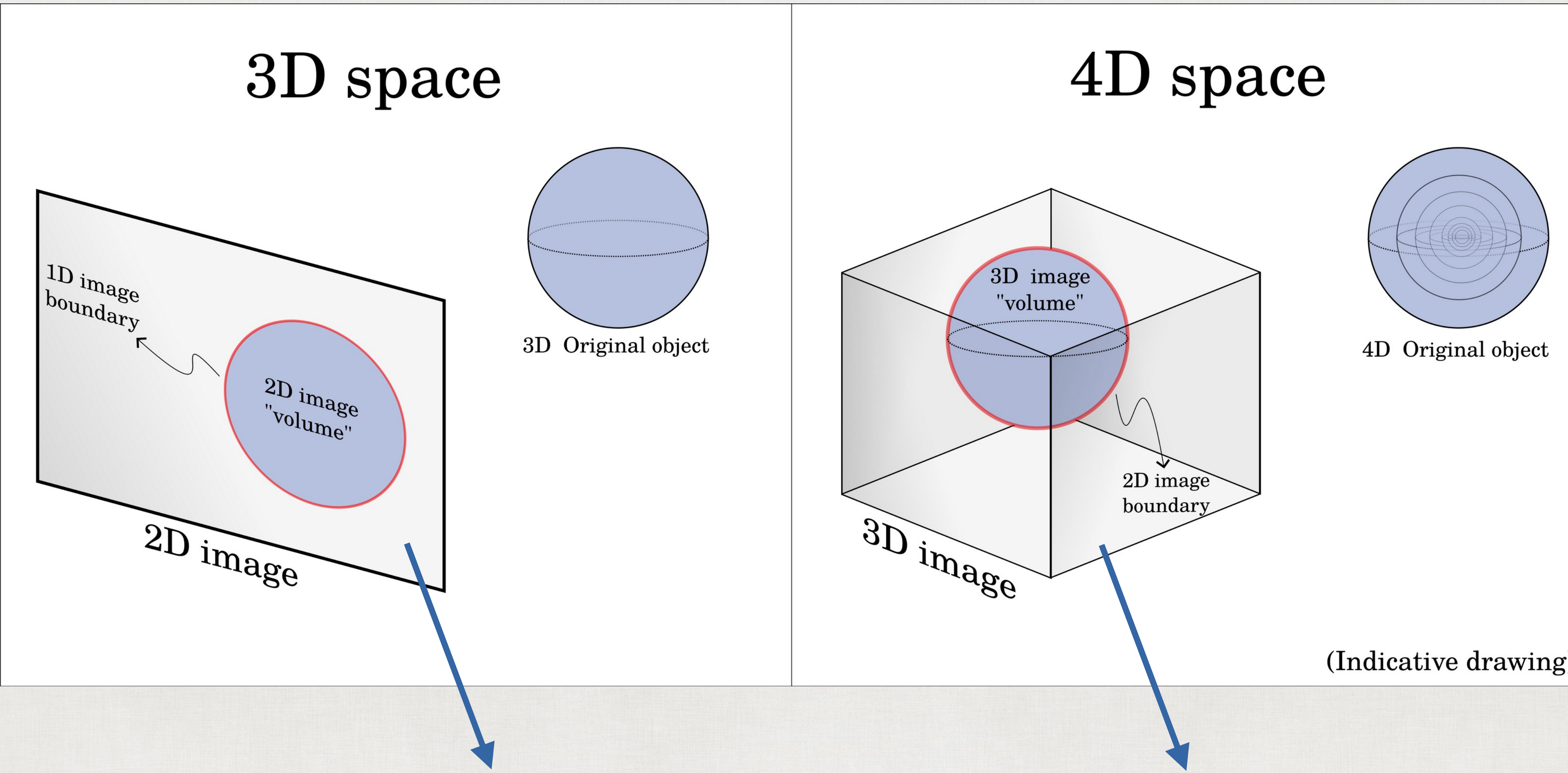
local patch of a S^2 sphere



local patch of a S^2 sphere

local patch of a S^3 sphere

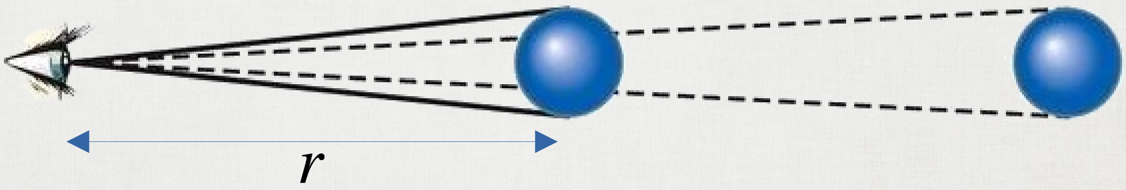
Hyper-image → 3D image of a 4D object



local patch of a S^2 sphere

local patch of a S^3 sphere

In our 3D world, the observed angular size of an object decreases as $\sim \frac{1}{r}$ with the distance to the observer r .



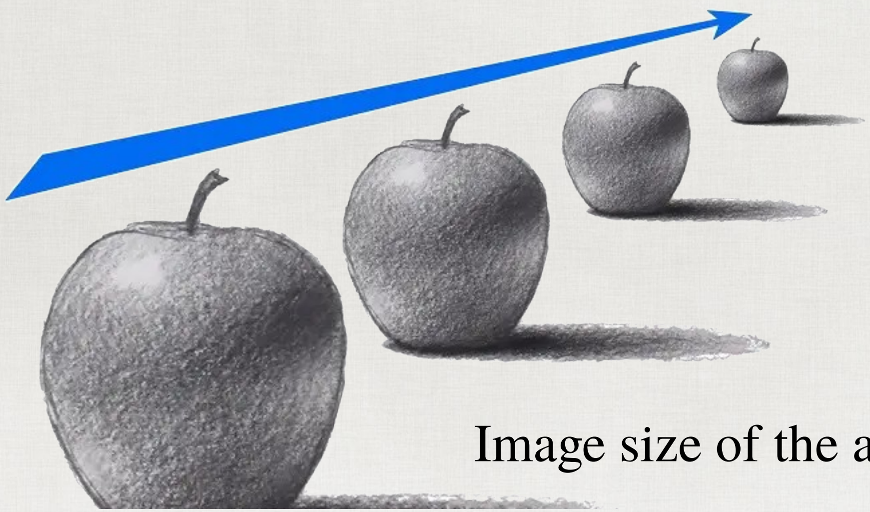
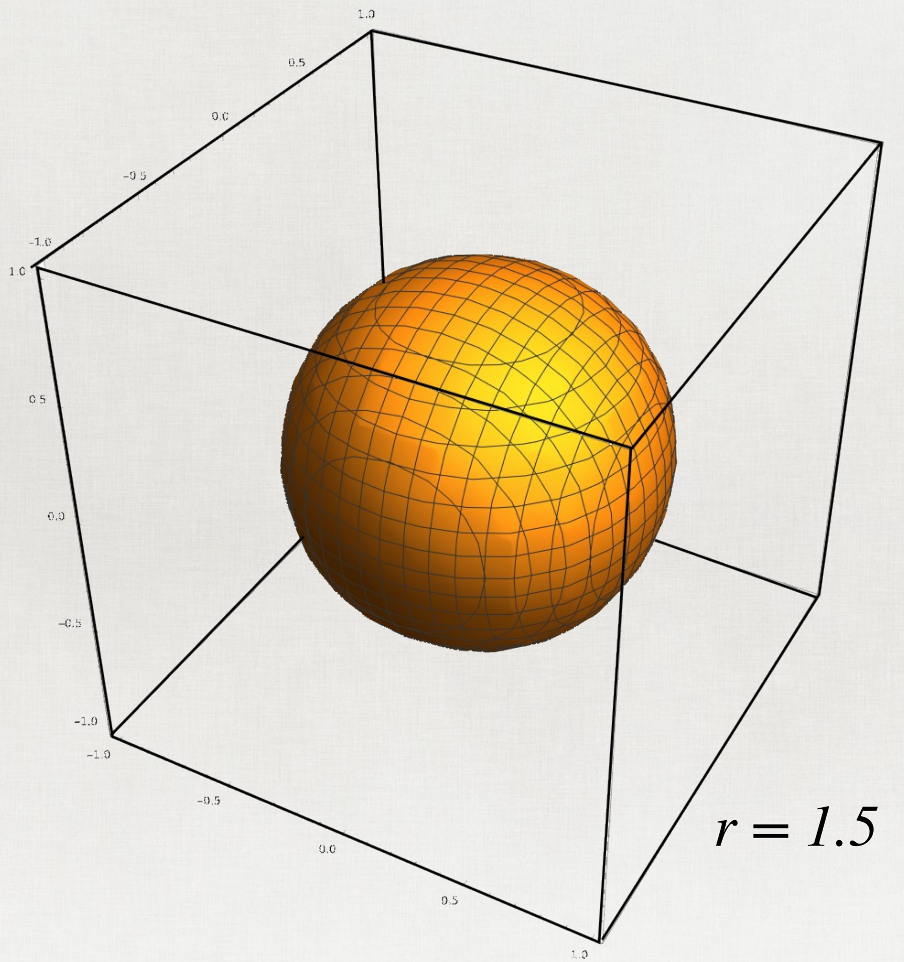
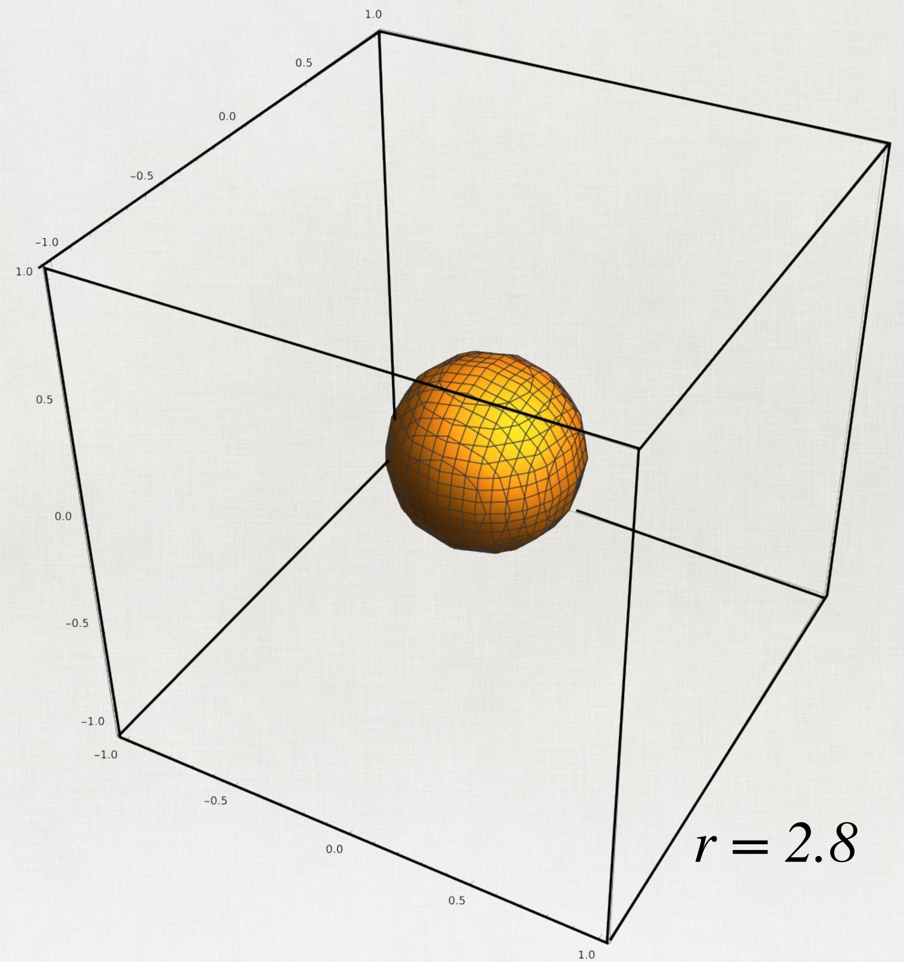


Image size of the apple decreases with distance.

In a 4D world, the observed <u>angular size</u> of an object also decreases with the <u>distance to the observer</u> r .

Example in **flat space** of 3D hyper-images of an S^3-sphere with unit radius, observed at diferent distances r:





Could we then “see” the image of higher dimensional black holes?

Why consider extra dimensions?

- Impact of Higher Dimensions:
Adding extra dimensions alters the expectations from 4D gravity, revealing a richer landscape of solutions.

Why consider extra dimensions?

- Impact of Higher Dimensions:
Adding extra dimensions alters the expectations from 4D gravity, revealing a richer landscape of solutions.

- Violation of Cosmic Censorship:
Some higher-dimensional Black hole horizons can develop instabilities that lead to naked singularities. If the Weak Cosmic Censorship holds in 4D gravity migh be very specific to d=4.
L. Lehner and F. Pretorius, *Phys. Rev. Lett. 105 (2010) 101102*

Why consider extra dimensions?

- Impact of Higher Dimensions:
Adding extra dimensions alters the expectations from 4D gravity, revealing a richer landscape of solutions.

- Violation of Cosmic Censorship:
Some higher-dimensional Black hole horizons can develop instabilities that lead to naked singularities. If the Weak Cosmic Censorship holds in 4D gravity migh be very specific to d=4.
L. Lehner and F. Pretorius, *Phys. Rev. Lett. 105 (2010) 101102*

- Topological Diversity:
In $d > 4$, event horizons can exhibit different topologies, such as spherical and toroidal.
R. Emparan and H.S. Reall, *Phys. Rev. Lett. 88 (2002) 101101*

Why consider extra dimensions?

- Impact of Higher Dimensions:
Adding extra dimensions alters the expectations from 4D gravity, revealing a richer landscape of solutions.

- Violation of Cosmic Censorship:
Some higher-dimensional Black hole horizons can develop instabilities that lead to naked singularities. If the Weak Cosmic Censorship holds in 4D gravity migh be very specific to d=4.
L. Lehner and F. Pretorius, *Phys. Rev. Lett. 105 (2010) 101102*

- Topological Diversity:
In $d > 4$, event horizons can exhibit different topologies, such as spherical and toroidal.
R. Emparan and H.S. Reall, *Phys. Rev. Lett. 88 (2002) 101101*

- Uniqueness Breakdown:
Black rings (toroidal horizons) in vacuum coexist with spherical-horizon solutions for the same mass and spin. R. Emparan and H.S. Reall, *Phys. Rev. Lett. 88 (2002) 101101*

Why consider extra dimensions?

- Impact of Higher Dimensions:
Adding extra dimensions alters the expectations from 4D gravity, revealing a richer landscape of solutions.

- Violation of Cosmic Censorship:
Some higher-dimensional Black hole horizons can develop instabilities that lead to naked singularities. If the Weak Cosmic Censorship holds in 4D gravity migh be very specific to d=4.
L. Lehner and F. Pretorius, *Phys. Rev. Lett. 105 (2010) 101102*

- Topological Diversity:
In $d > 4$, event horizons can exhibit different topologies, such as spherical and toroidal.
R. Emparan and H.S. Reall, *Phys. Rev. Lett. 88 (2002) 101101*

- Uniqueness Breakdown:
Black rings (toroidal horizons) in vacuum coexist with spherical-horizon solutions for the same mass and spin. R. Emparan and H.S. Reall, *Phys. Rev. Lett. 88 (2002) 101101*

- Examples 5D Solutions:
Examples include Tangherlini and Myers-Perry black holes, both with spherical horizons.
F.R. Tangherlini, *Nuovo Cim. 27 (1963) 636.*
R.C. Myers and M.J. Perry*, Annals Phys. 172 (1986) 304.*

The 5D Myers-Perry solution (equal spins)

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + \left(x + a^2\right)\left(\frac{\mathrm{d}x^2}{4\Delta} + \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2 + \cos^2\theta \mathrm{d}\psi^2\right)$$
$$+ \frac{\mu^2}{\rho^2}\left[\mathrm{d}t + a\left(\sin^2\theta \mathrm{d}\phi + \cos^2\theta \mathrm{d}\psi\right)\right]^2 \; .$$

with $\rho^2 = x + a^2, \quad \Delta = \left(x + a^2\right)^2 - \mu^2 x.$

The coordinate range is $r \in [0, \infty[$, $\theta \in [0, \pi/2]$, $\phi \in [0, 2\pi[$, $\psi \in [0, 2\pi[$.

Describes a topologically spherical Black Hole, of mass $M = 3\pi\mu^2/(8G)$.

It simultaneously rotates in two orthogonal planes with equal angular momenta $J = \frac{2}{3}Ma$.

5D → 4 spatial dimensions + 1 time dimension

Far field $x \gg \mu$

$$ds^2 = -dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\, d\psi^2\right)$$

Hypersurfaces with $t = const.$ and $r = const.$ are S^3-spheres in *Hopf coordinates.*
These are **not** spherical coordinates.

If also $\theta = const.$, the surface described by (ϕ, ψ) covers a 2D-torus.

The 5D Myers-Perry solution has three Killing vector fields, $\partial_t\,, \partial_\phi$ and ∂_ψ. This leads to Killing constants along geodesic motion, $p_t = -E\,, p_\phi = \Phi$ and $p_\psi = \Psi$. There exists a non-trivial constant of motion, $\mathcal{K}$, which makes the geodesic motion fully integrable.

The null geodesic equations with equal spins are:

$$\rho^2 \dot{t} = E\left(x + a^2\right) + \mu^2 \frac{\left(x + a^2\right)^2}{\Delta}\left(E + \frac{a}{x + a^2}\left(\Phi + \Psi\right)\right)\,,$$

$$\rho^2 \dot{\phi} = \frac{\Phi}{\sin^2\theta} - \mu^2 a \frac{x + a^2}{\Delta}\left(E + \frac{a}{x + a^2}\left(\Phi + \Psi\right)\right)\,,$$

$$\rho^2 \dot{\psi} = \frac{\Psi}{\cos^2\theta} - \mu^2 a \frac{x + a^2}{\Delta}\left(E + \frac{a}{x + a^2}\left(\Phi + \Psi\right)\right)\,,$$

$$\rho^4 \dot{x}^2 = 4\Delta\left(xE^2 - \mathcal{K}\right) + 4\mu^2\left(x + a^2\right)^2\left(E + \frac{a}{x + a^2}\left(\Phi + \Psi\right)\right)^2\,,$$

$$\rho^4 \dot{\theta}^2 = \mathcal{K} + E^2 a^2 - \frac{\Phi^2}{\sin^2\theta} - \frac{\Psi^2}{\cos^2\theta}\,.$$

Spherical Photon Orbits

The Kerr shadow is determined by a set of photon orbits with constant radial coordinate.

The same happens with the Myers-Perry *hyper-shadow*.

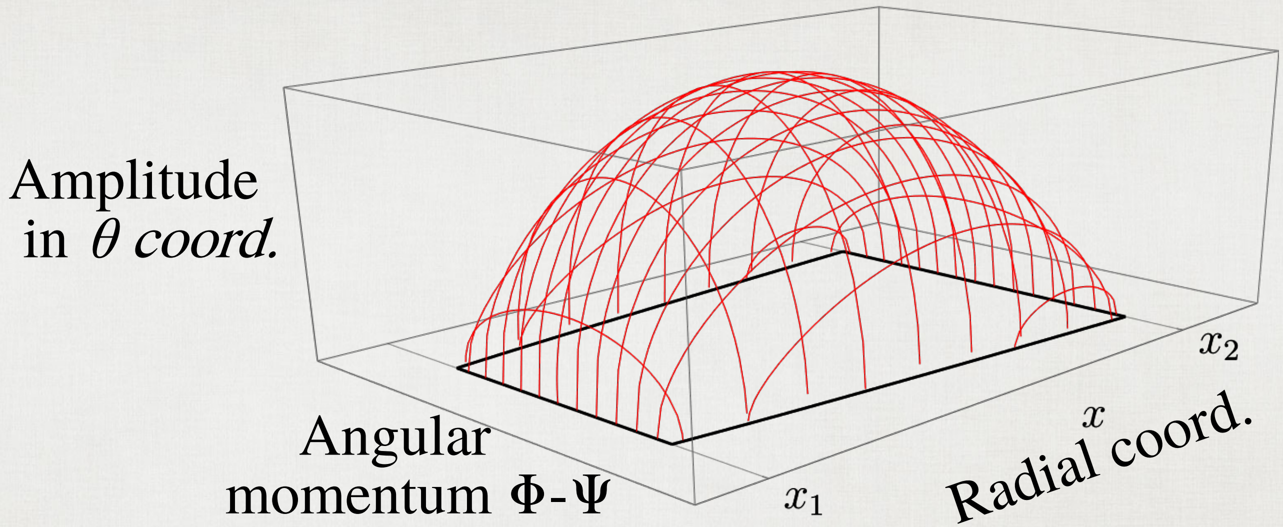
The Myers-Perry spherical photon orbits are obtained by the conditions:

$$\dot{\theta}^2 \geqslant 0$$

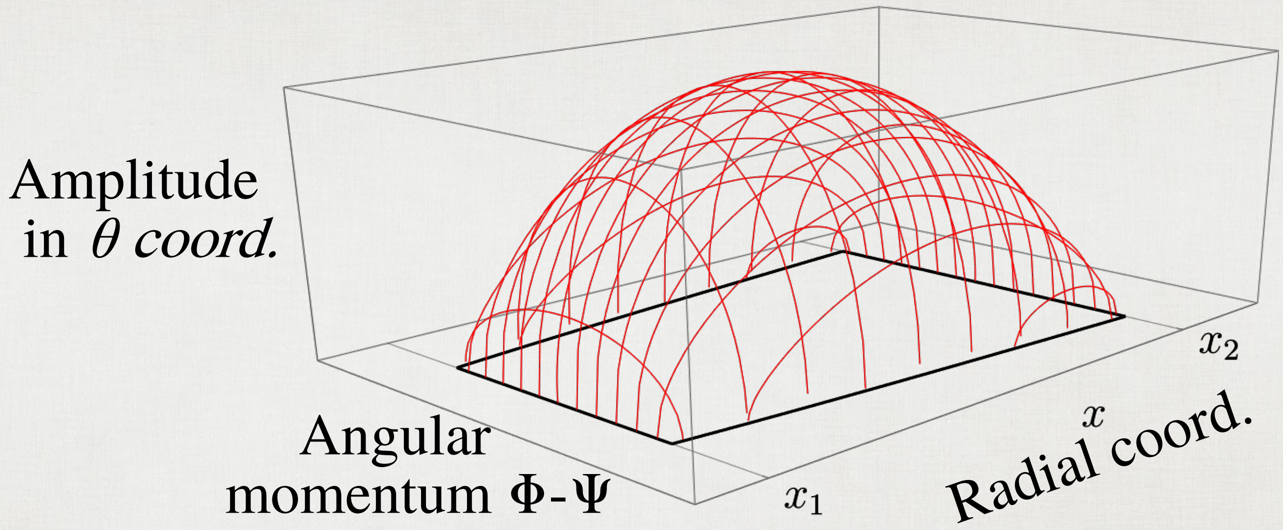
$$\chi = 0 \qquad \frac{d\chi}{dx} = 0$$

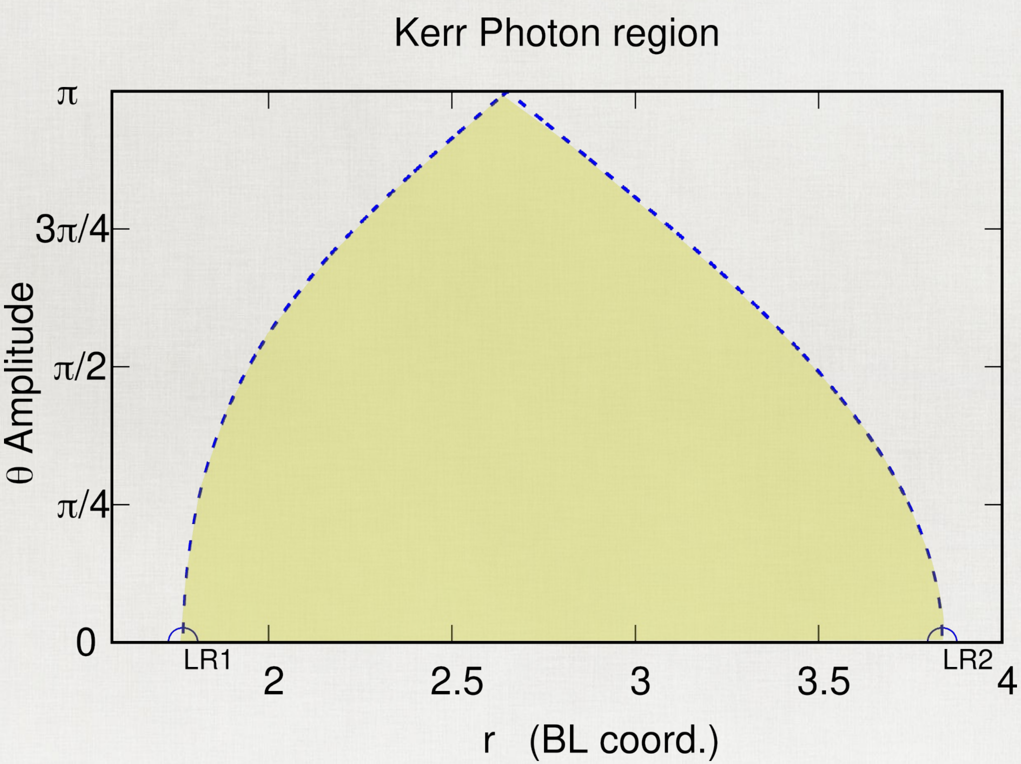
where $4\chi = \rho^4 \dot{x}^2$. It is possible to solve these equations *analytically*.

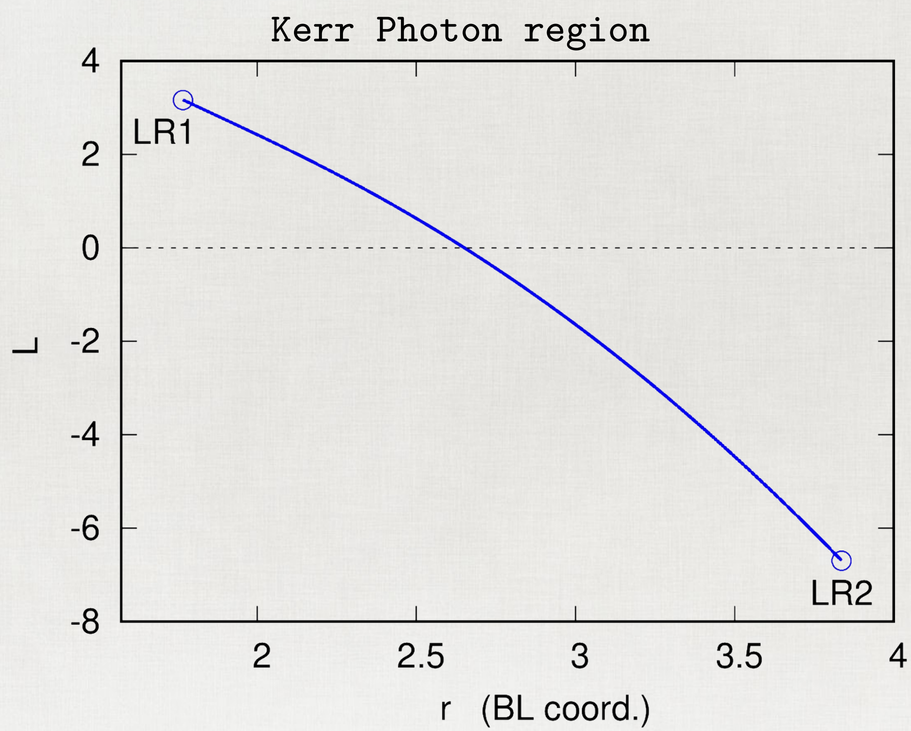
The physical domain of the Spherical Photon Orbits in the Myers-Perry spacetime:



The physical domain of the Spherical Photon Orbits in the Myers-Perry spacetime:







ZAMO observation frame

We consider a 5-dimensional generalization of a *zero angular momentum observer* (ZAMO):

The locally measured momenta of the light ray in the ZAMO frame are:

$$
\begin{aligned}
p^{(t)} &= -\hat{e}^{\mu}_{(t)} p_\mu \,, \qquad & p^{(\phi)} &= \hat{e}^{\mu}_{(\phi)} p_\mu , \\
p^{(\psi)} &= \hat{e}^{\mu}_{(\psi)} p_\mu \qquad & p^{(x)} &= \hat{e}^{\mu}_{(x)} p_\mu \\
p^{(\theta)} &= \hat{e}^{\mu}_{(\theta)} p_\mu \,. & &
\end{aligned}
$$

In this frame, the hyperimage local coordinates (X, Y, Z) can be expressed as:

$$
X = -\sqrt{x_o}\,\frac{p^{(\phi)}}{p^{(t)}} \,, \quad Y = -\sqrt{x_o}\,\frac{p^{(\psi)}}{p^{(t)}} \,, \quad Z = \sqrt{x_o}\,\frac{p^{(\theta)}}{p^{(t)}} \,.
$$

Hypershadow parametrization

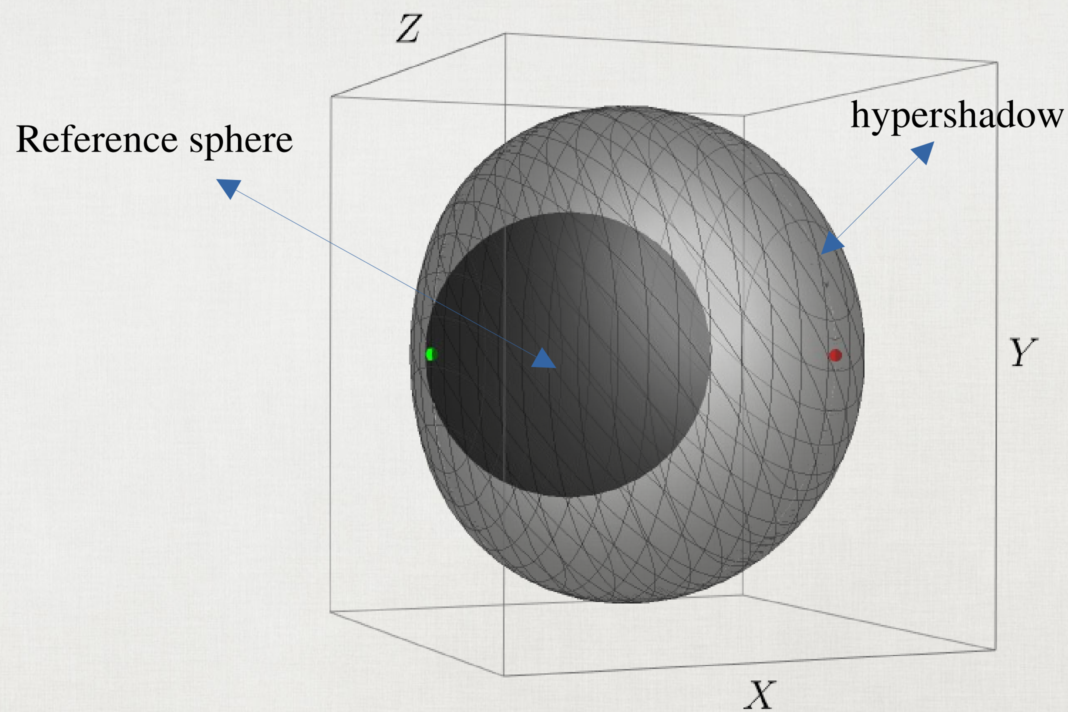
By considering an observer in the far-away limit ($x_o \to \infty$) one obtains:

$$X = -\frac{\Phi}{\sin\theta_o}\,,$$
$$Y = -\frac{\Psi}{\cos\theta_o}\,,$$
$$Z = \pm\frac{1}{\sin(2\theta_o)}\sqrt{-2(\Phi^2+\Psi^2)+2(\Psi^2-\Phi^2)\cos(2\theta)+(a^2+\mathcal{K})\sin^2(2\theta)}\,.$$

The constants of geodesic motion $\{\mathcal{K}, \Psi, \Phi\}$ are determined by the spherical photon orbits.

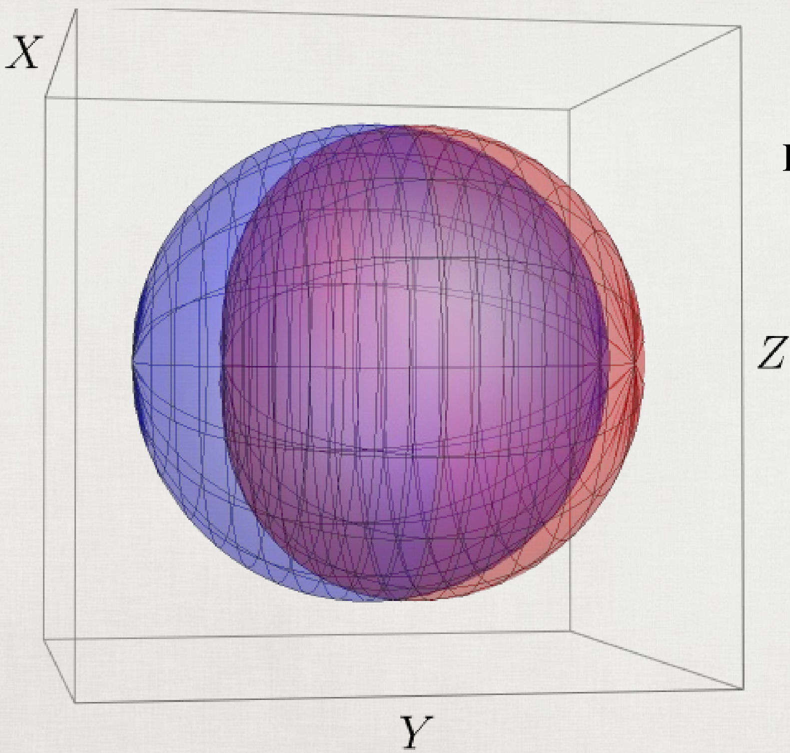
Introducing the 5D Myers-Perry Hypershadow
(with maximum spin)

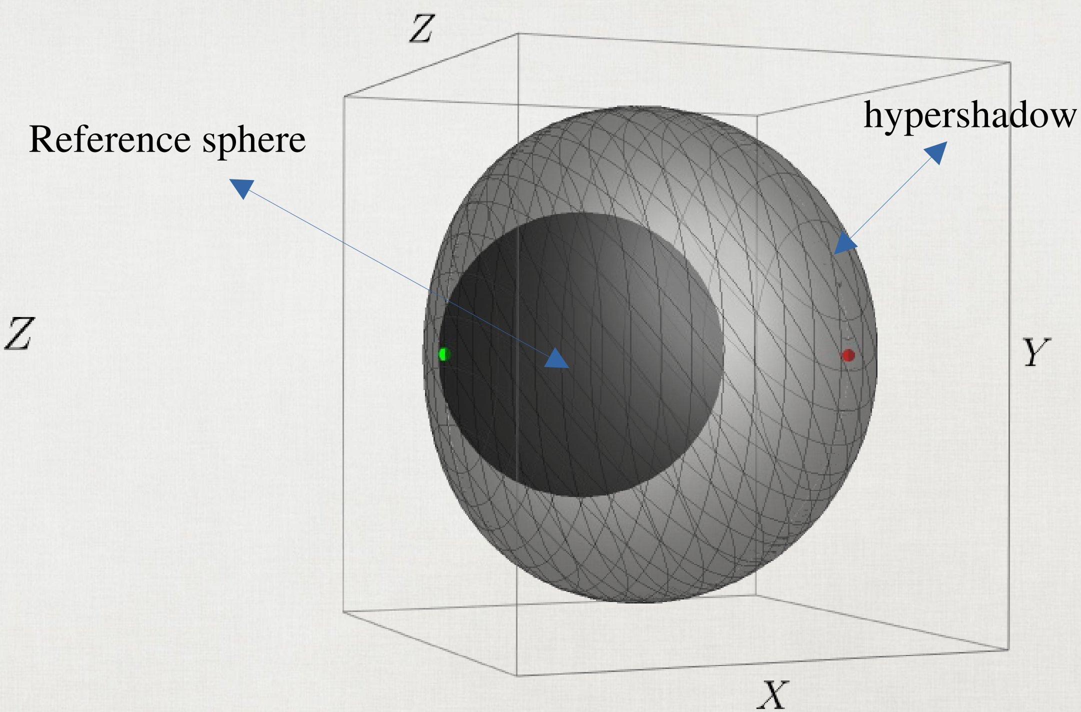
Plots of the 2D surface boundary of the Myers-Perry hypershadow



Hypershadow with spin a=0.49.
Reference sphere inside to highlight deviations from a sphere.

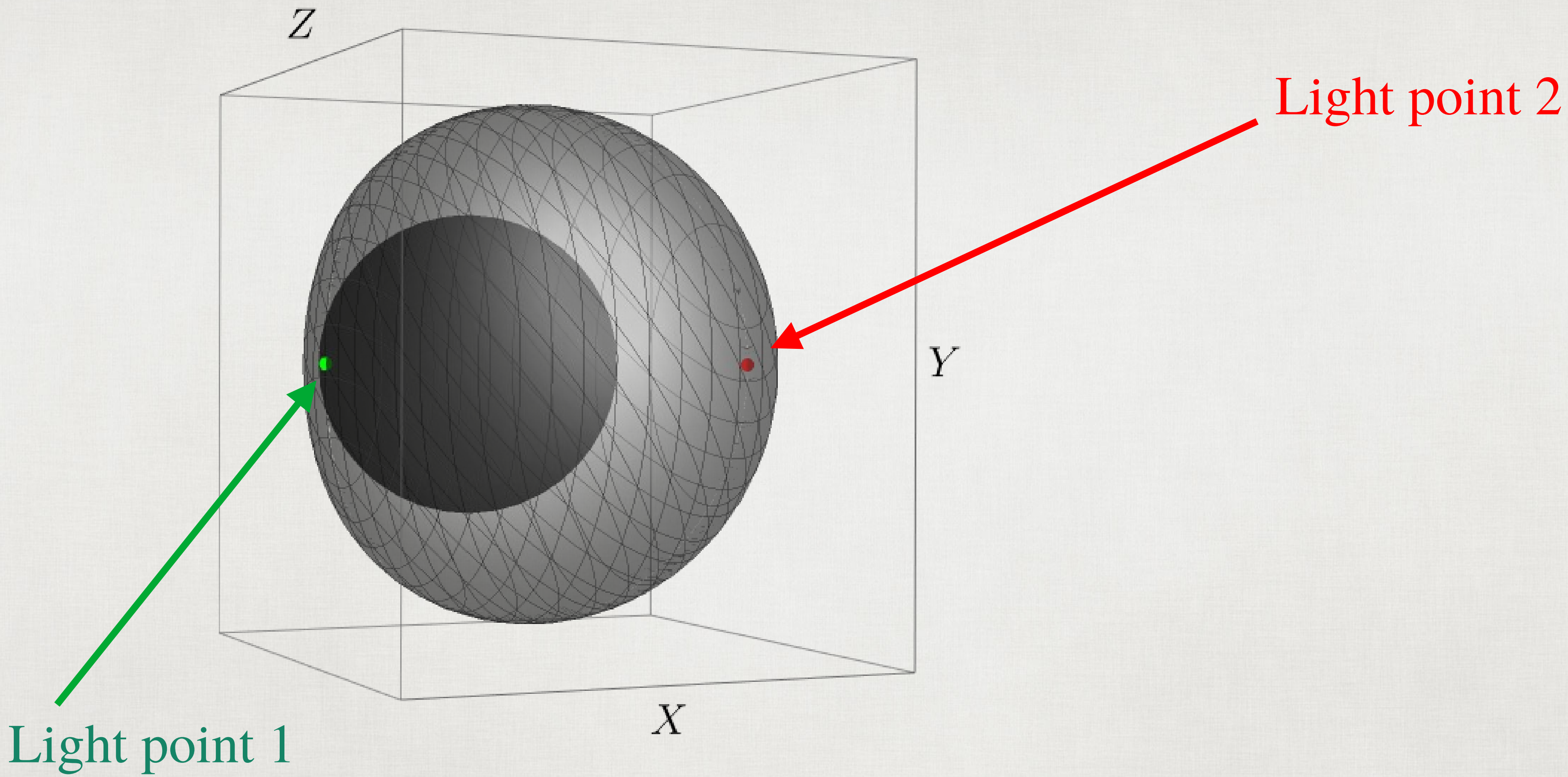
Plots of the 2D surface boundary of the Myers-Perry hypershadow

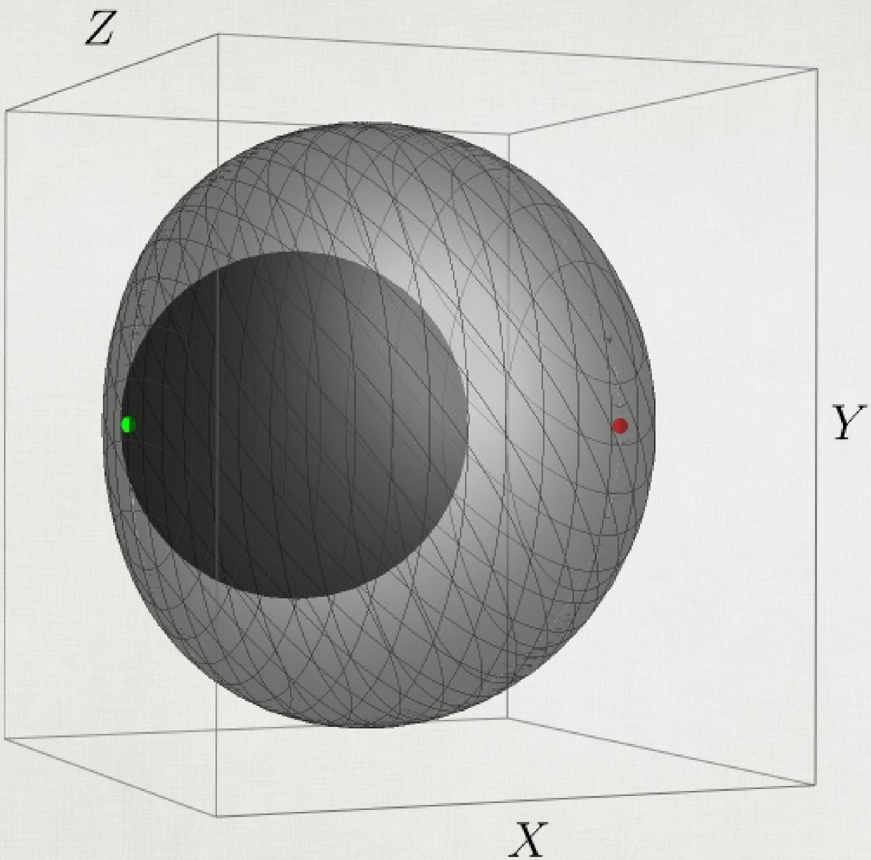


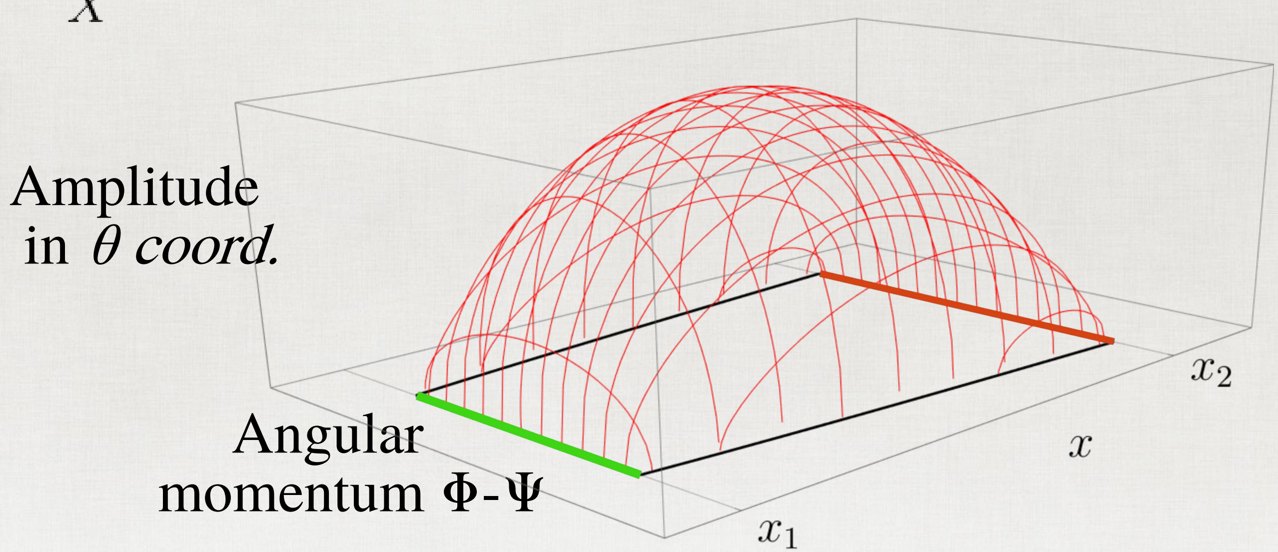


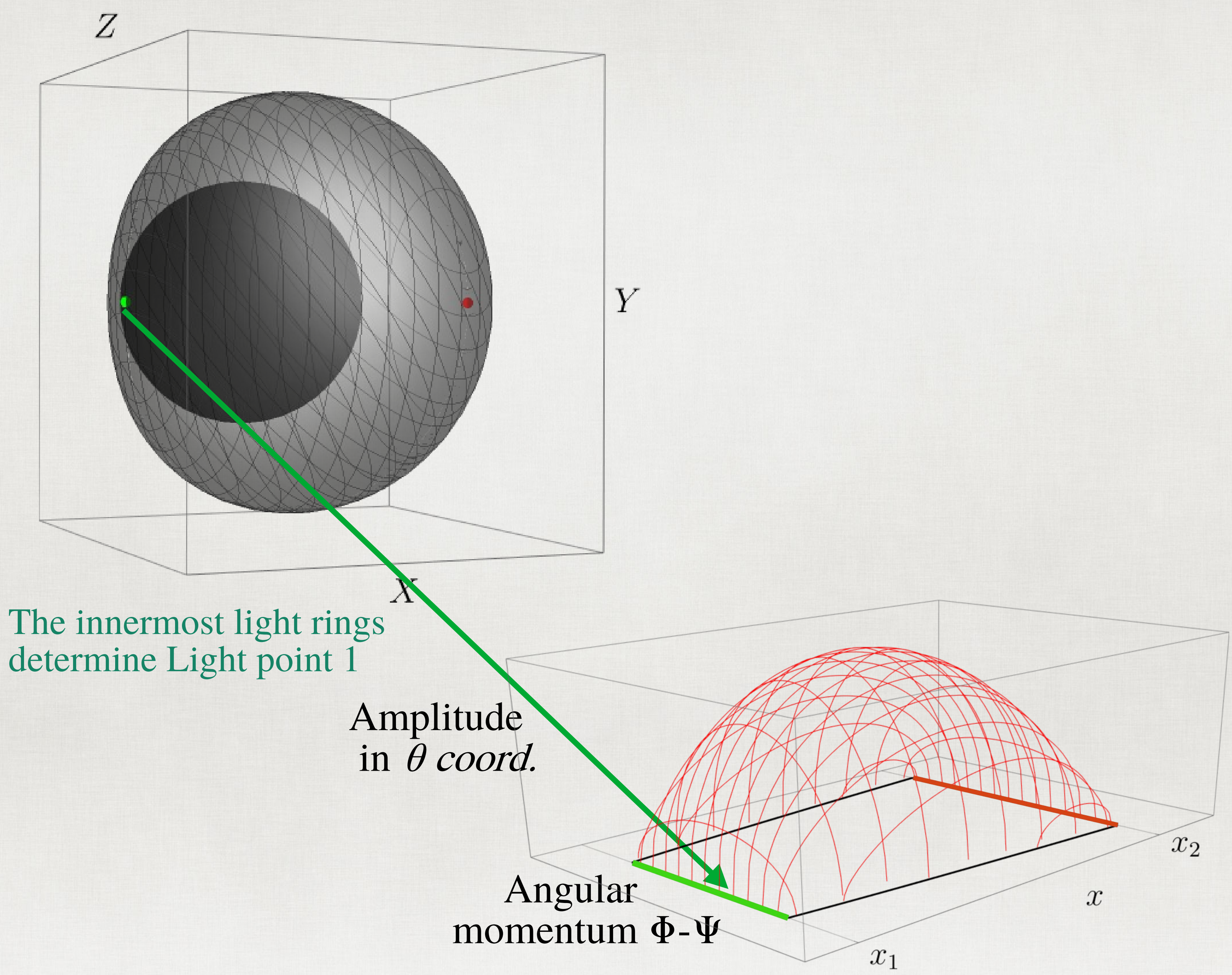
Two superimposed hypershadows with spins a=0.1 (blue), and a=0.49 (red).

Hypershadow with spin a=0.49. Reference sphere inside to highlight deviations from a sphere.

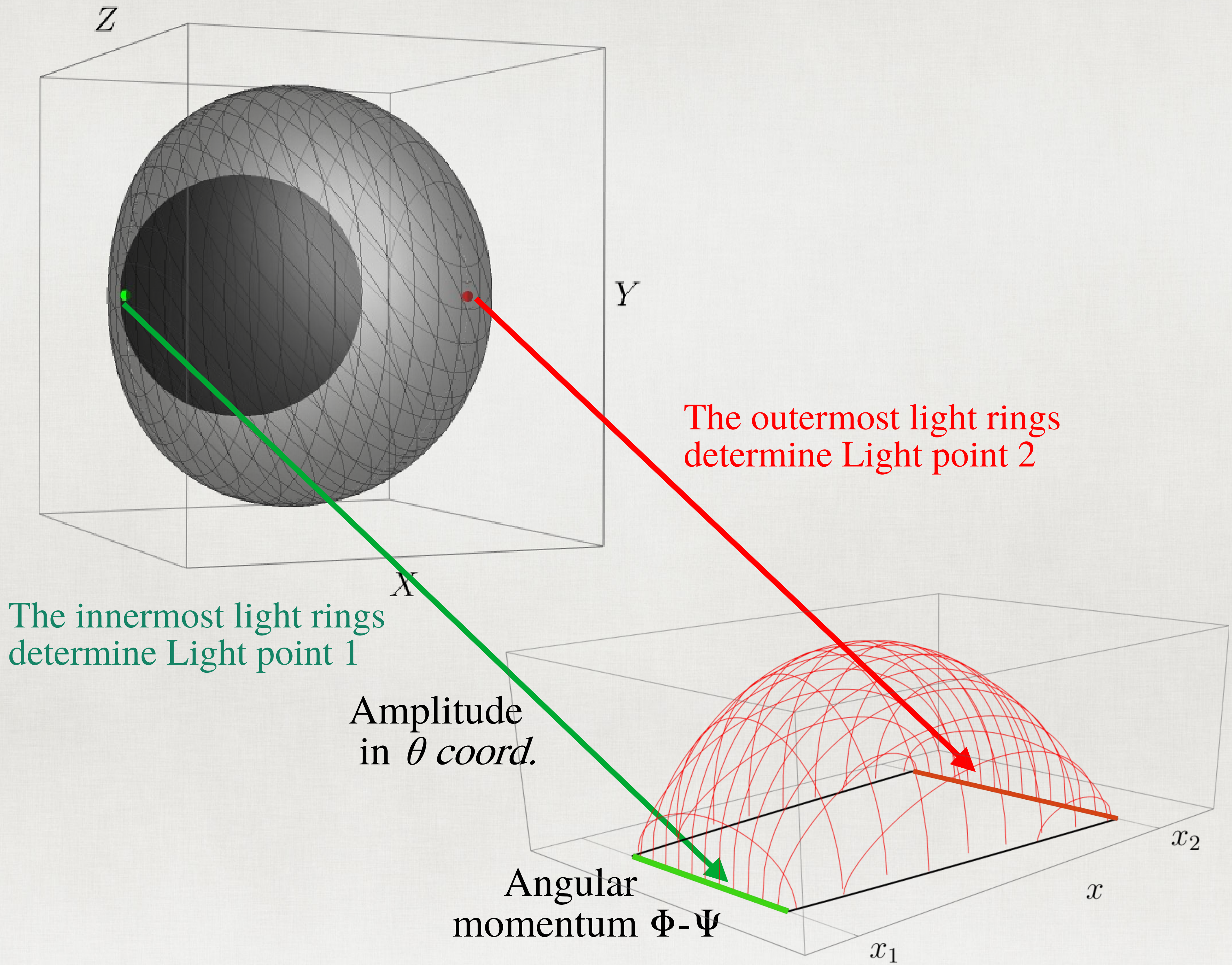

Z
Y
X
Light point 2
Light point 1


Z
Y
X

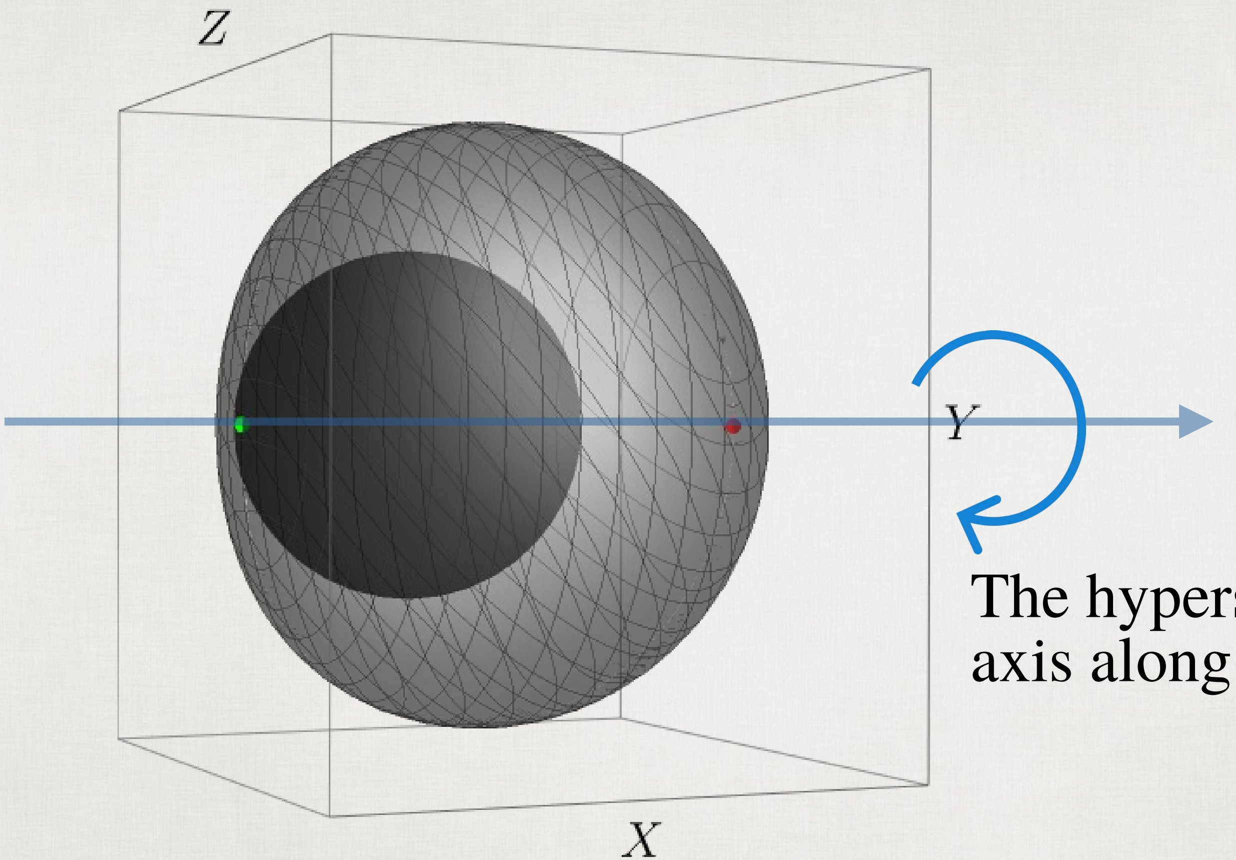

Amplitude
in θ *coord.*
Angular
momentum Φ-Ψ
x_2
x
x_1



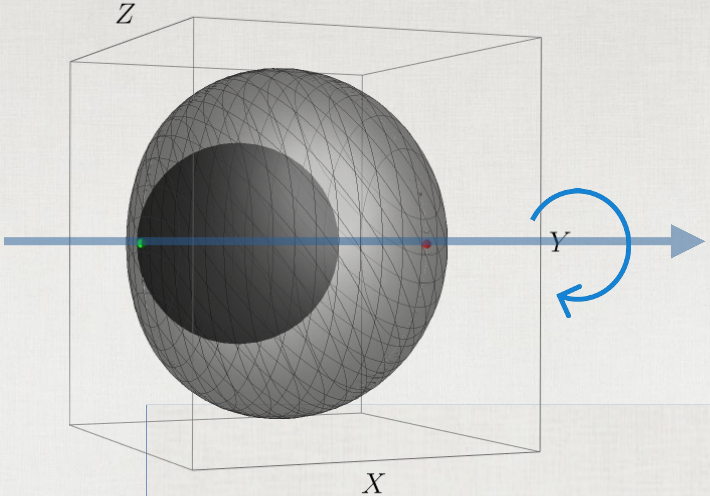
Z
Y
X
The innermost light rings
determine Light point 1
Amplitude
in θ coord.
Angular
momentum Φ-Ψ
x_2
x
x_1



Z
Y
X
The outermost light rings
determine Light point 2
The innermost light rings
determine Light point 1
Amplitude
in θ coord.
Angular
momentum Φ-Ψ
x_2
x
x_1



The hypershadow has a symmetry axis along the light points!



Simple Hypershadow parametrization

These symmetries can be explored to convey a much simpler parameterization of the hypershadow:

$$X = -\alpha \sin\theta_o - \cos\upsilon \cos\theta_o \sqrt{Q-\alpha^2}\,,$$
$$Y = -\alpha \cos\theta_o + \cos\upsilon \sin\theta_o \sqrt{Q-\alpha^2}\,,$$
$$Z = \sin\upsilon \sqrt{Q-\alpha^2}\,,$$

where θ_o is the observation angle, and the parametrization coordinates have the range $x \in [x_1, x_2]$ and $\upsilon \in [0, 2\pi]$. In addition:

$$Q(x) = x + a^2 + \frac{\left(x+a^2\right)^2 - x}{\left(1-\sqrt{2}\sqrt{x+a^2}\right)^2}\,,$$
$$\alpha(x) = -\,a - \frac{1}{a}\left[x + \frac{\left(a^2+x\right)^2 - x}{1-\sqrt{2}\sqrt{a^2+x}}\right]\,.$$

The Road ahead

- Hypershadows of black rings:
What is the hyperimage of other black hole solutions with toroidal topology? Is it a 3D torus?

- Continuous Symmetry:
It is not clear if the existence of a symmetry axis for the hypershadow is a consequence of equal spins or connected with deeper symmetry properties of the Myers-Perry spacetime.

Bonus topic:
Imaging the gravitational collapse of a pressureless star into a Black Hole

Metric for the collapsing dust matter [Oppenheimer + Snyder (1939)] :

$$ds^2 = -d\tau^2 + e^{\lambda} dR^2 + r^2 d\Omega^2$$

Metric for the collapsing dust matter [Oppenheimer + Snyder (1939)] :

$$ds^2 = -d\tau^2 + e^{\lambda} dR^2 + r^2 d\Omega^2$$

proper time
(dust frame)

Metric for the collapsing dust matter [Oppenheimer + Snyder (1939)] :

$$ds^2 = -d\tau^2 + e^\lambda dR^2 + r^2 d\Omega^2$$

proper time
(dust frame)

Comoving dust radius
(dust frame)

Metric for the collapsing dust matter [Oppenheimer + Snyder (1939)] :

$$ds^2 = -d\tau^2 + e^\lambda dR^2 + r^2 d\Omega^2$$

proper time
(dust frame)

Comoving dust radius
(dust frame)

Areal radius

Metric for the collapsing dust matter [Oppenheimer + Snyder (1939)] :

$$ds^2 = -d\tau^2 + e^{\lambda} dR^2 + r^2 d\Omega^2$$

proper time
(dust frame)

Comoving dust radius
(dust frame)

Areal radius

$$r := r(R, \tau)$$

$$\lambda := \lambda(R, \tau)$$

Every dust particle worldline has
$\{R, \theta, \varphi\} = \text{const.}$

Possible to find analytical solution with the dust **starting from rest** at τ=0 .

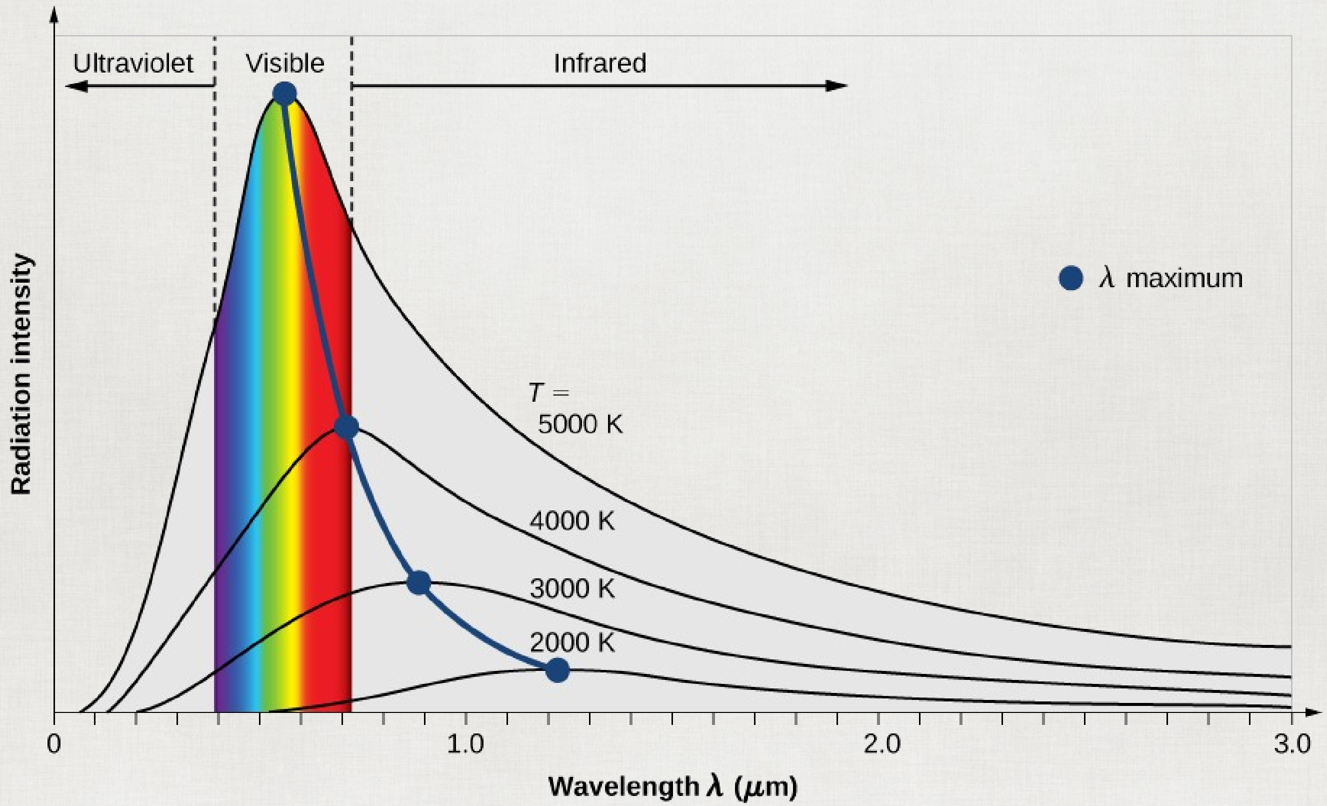
$R_o \longrightarrow$ co-moving radius of the star

$M \longrightarrow$ mass of the star

Black body radiation

The star is assumed to radiate Black body radiation at constant temperature T.
The spectral radiance for the wavelength λ being:

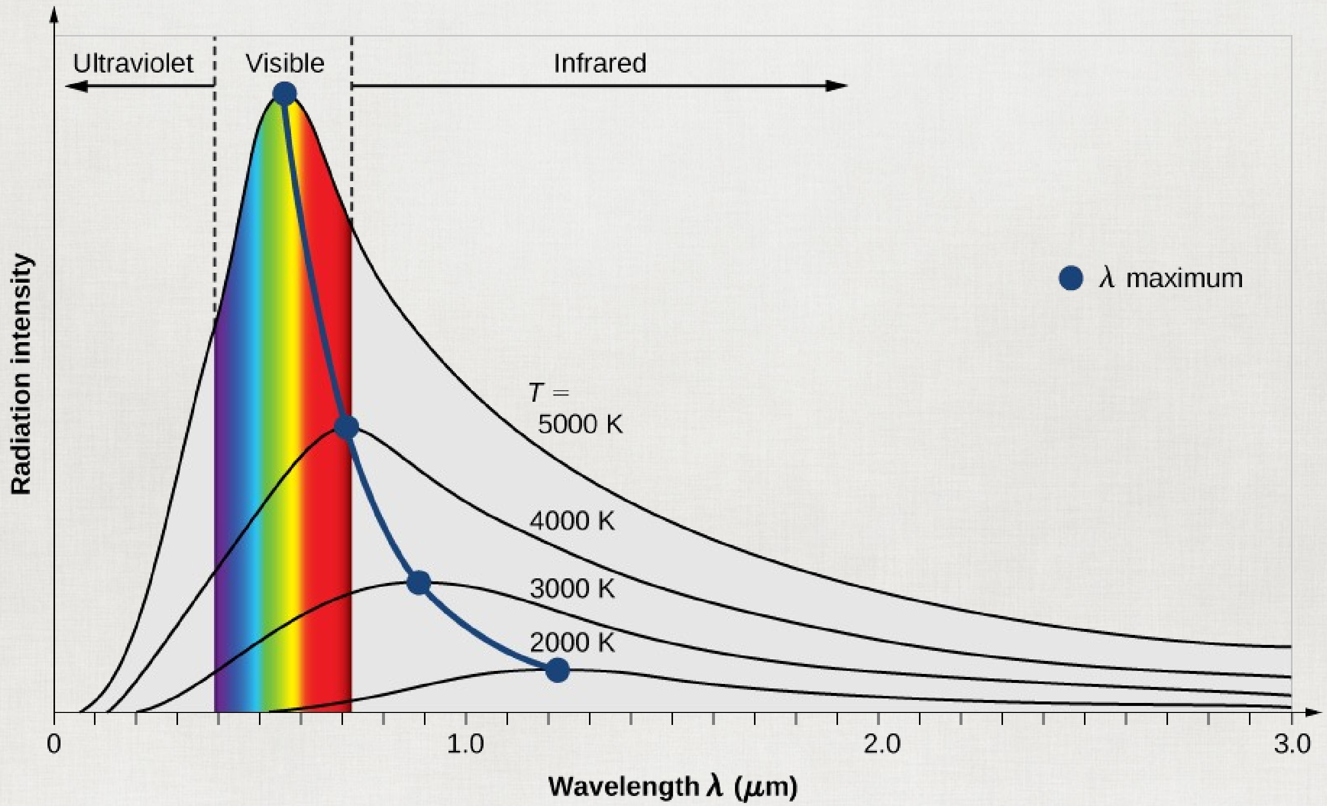
$$B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\Big(hc/(\lambda k_B T)\Big) - 1}$$



Black body radiation

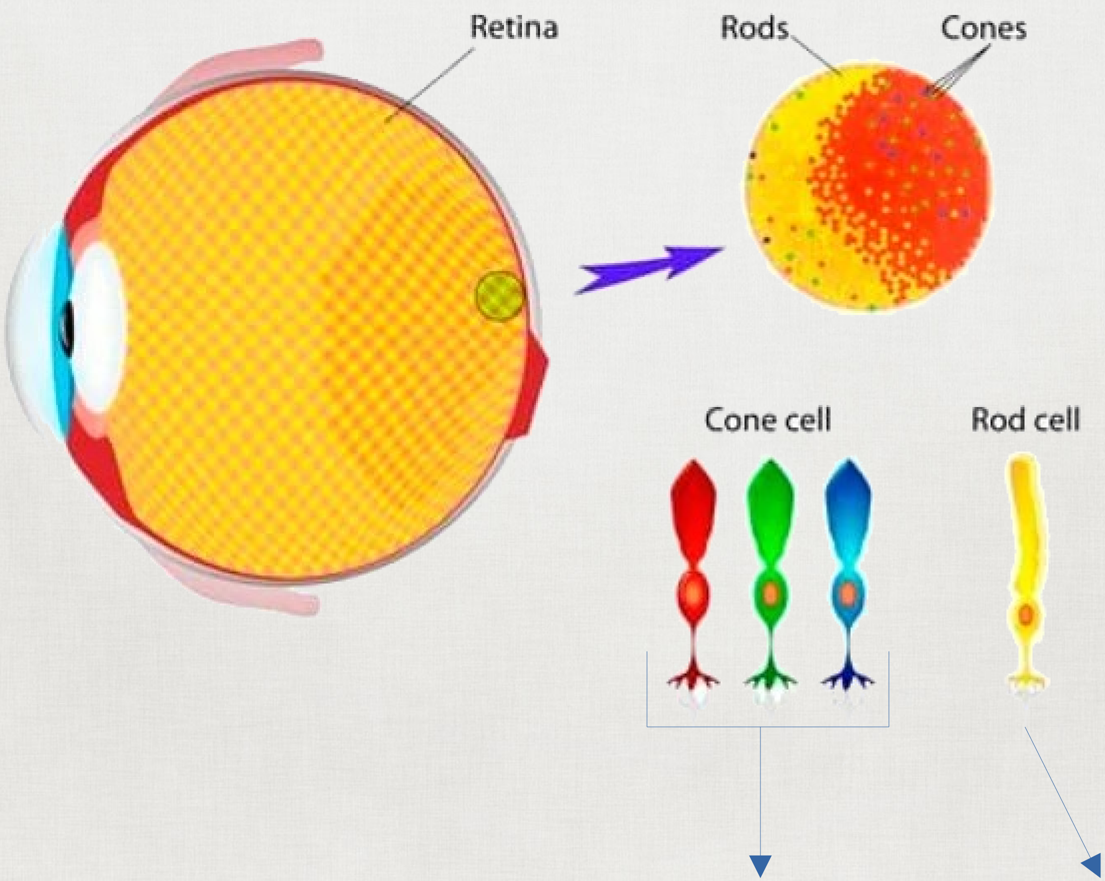
The star is assumed to radiate Black body radiation at constant temperature T. The spectral radiance for the wavelength λ being:

$$B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(hc/(\lambda k_B T)\right) - 1}$$



How to we obtain the perceived color of the star given some spectra?

Brief introduction to Colorimetry

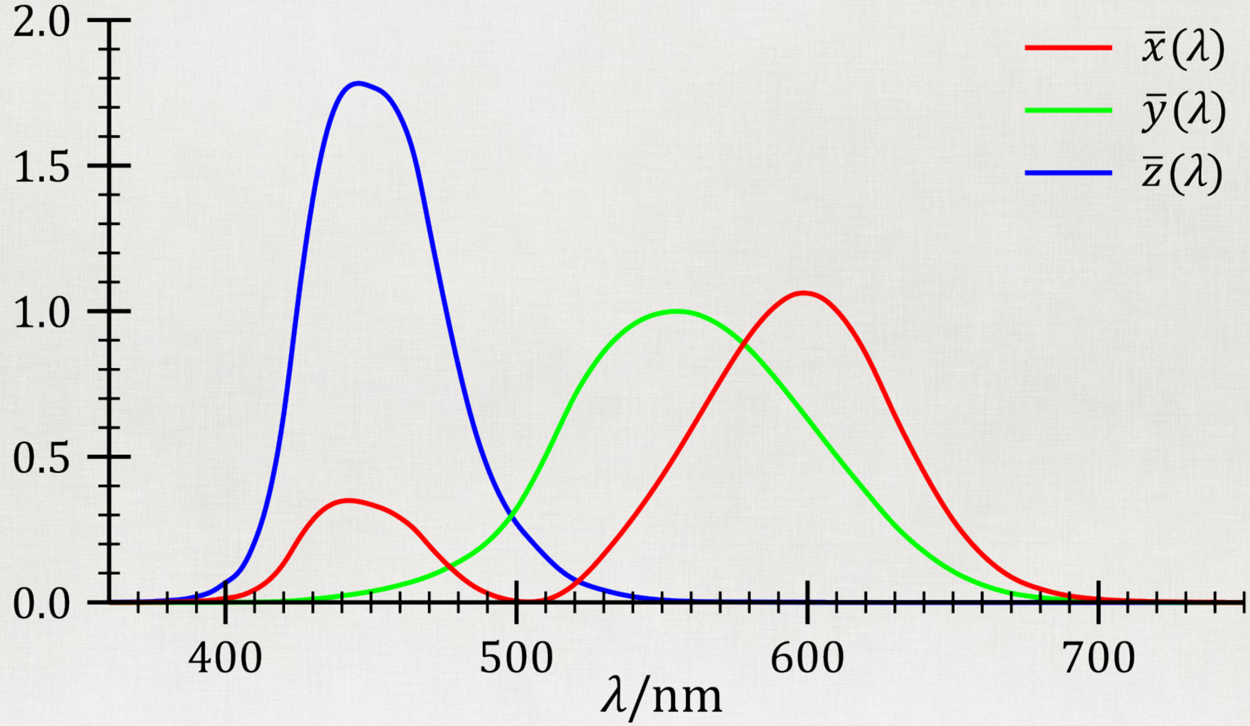


For color vision

For vision at very low light intensity

The human eye has three types of color sensors that respond to different ranges of wavelengths.

In 1931 the *International Commission on Illumination* (CIE) published the CIE 1931 color spaces which define the relationship between the **visible spectrum** and the visual sensation of specific colors by human color vision.

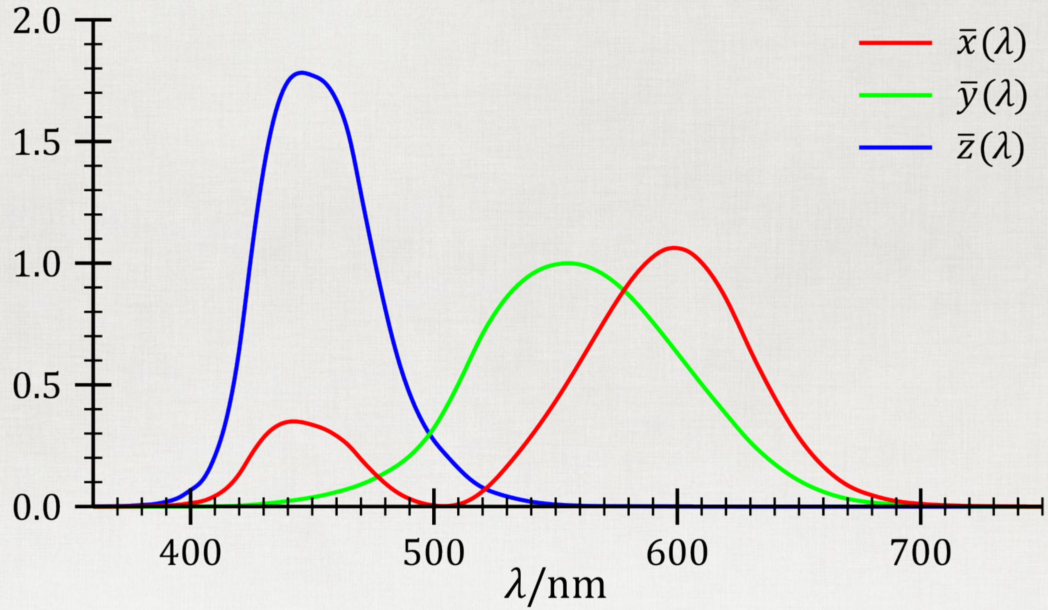


Tristimulus values XYZ

The stimulus of the light cones by standard observer is given by:

$$X = \int_\lambda B_\lambda(\lambda)\ \bar{x}(\lambda)\, d\lambda$$
$$Y = \int_\lambda B_\lambda(\lambda)\ \bar{y}(\lambda)\, d\lambda$$
$$Z = \int_\lambda B_\lambda(\lambda)\ \bar{z}(\lambda)\, d\lambda$$

These values (X, Y, Z) are approximately the Red-Green-Blue perceived colors, although there is a more complicated mapping between the two.



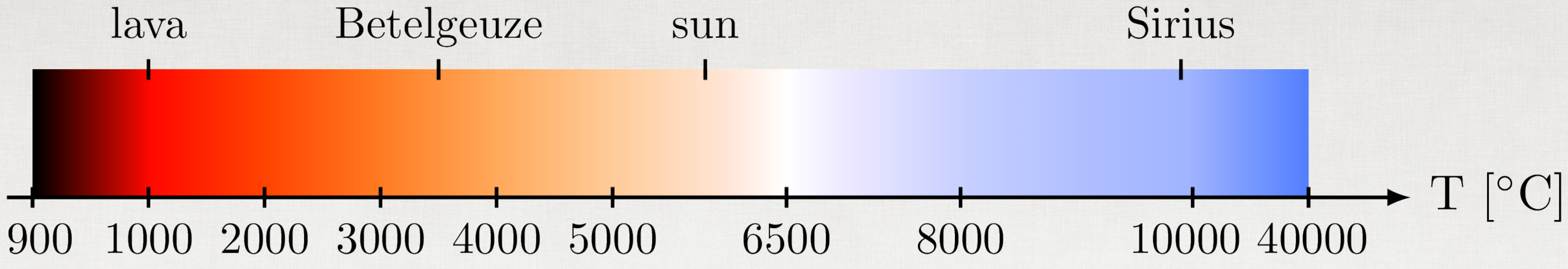
Tristimulus values XYZ

The stimulus of the light cones by standard observer is given by:

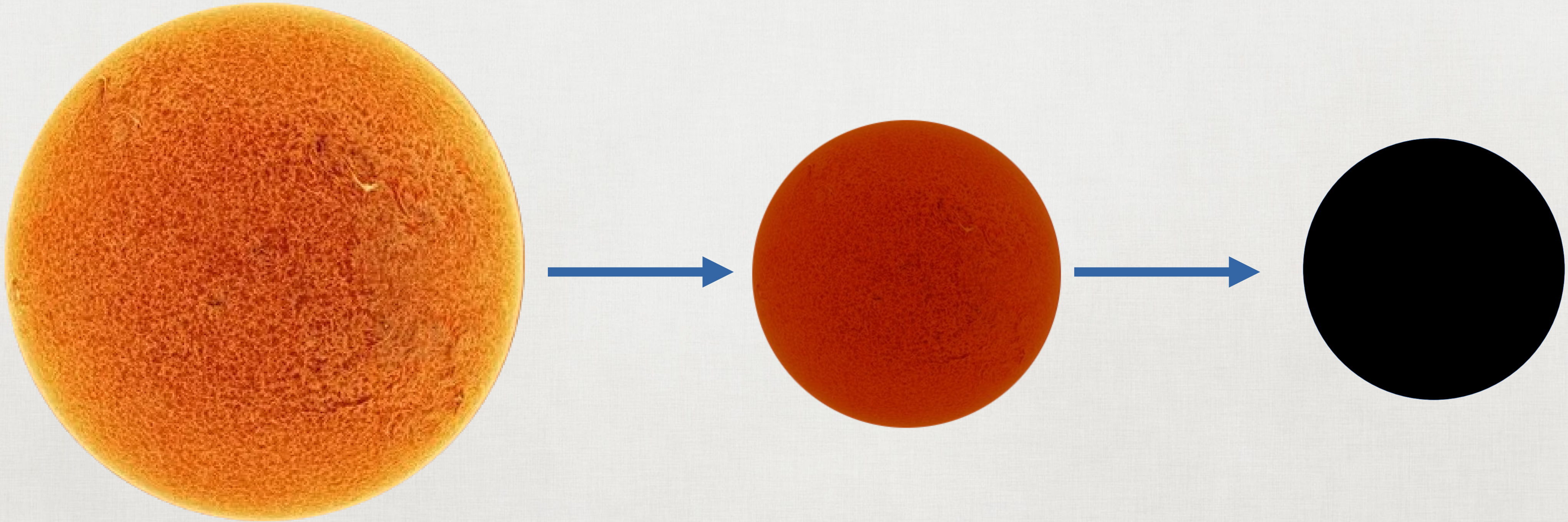
$$X = \int_\lambda B_\lambda(\lambda)\ \bar{x}(\lambda)\, d\lambda$$
$$Y = \int_\lambda B_\lambda(\lambda)\ \bar{y}(\lambda)\, d\lambda$$
$$Z = \int_\lambda B_\lambda(\lambda)\ \bar{z}(\lambda)\, d\lambda$$

These values (X, Y, Z) are approximately the Red-Green-Blue perceived colors, although there is a more complicated mapping between the two.

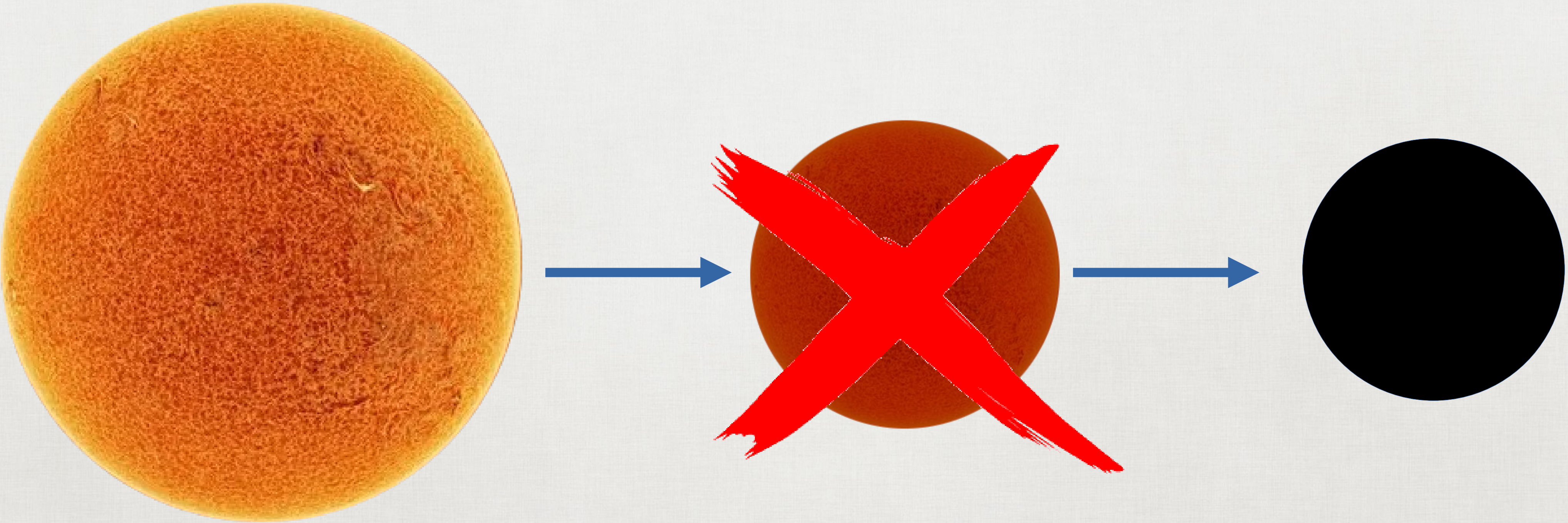
We can recover the perceived color of a black body at different temperatures



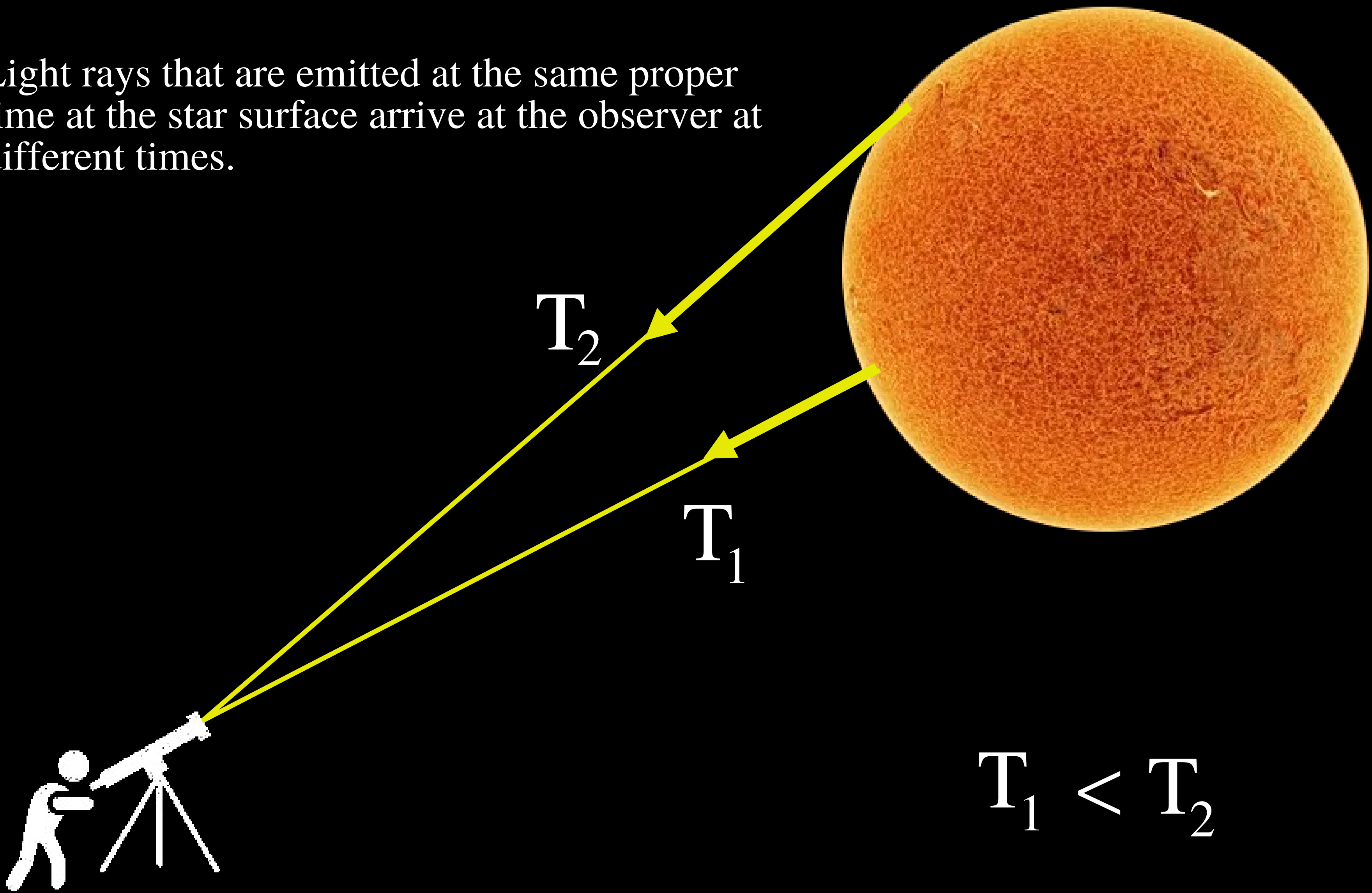
Expectation for collapse imaging?



Expectation for collapse imaging?

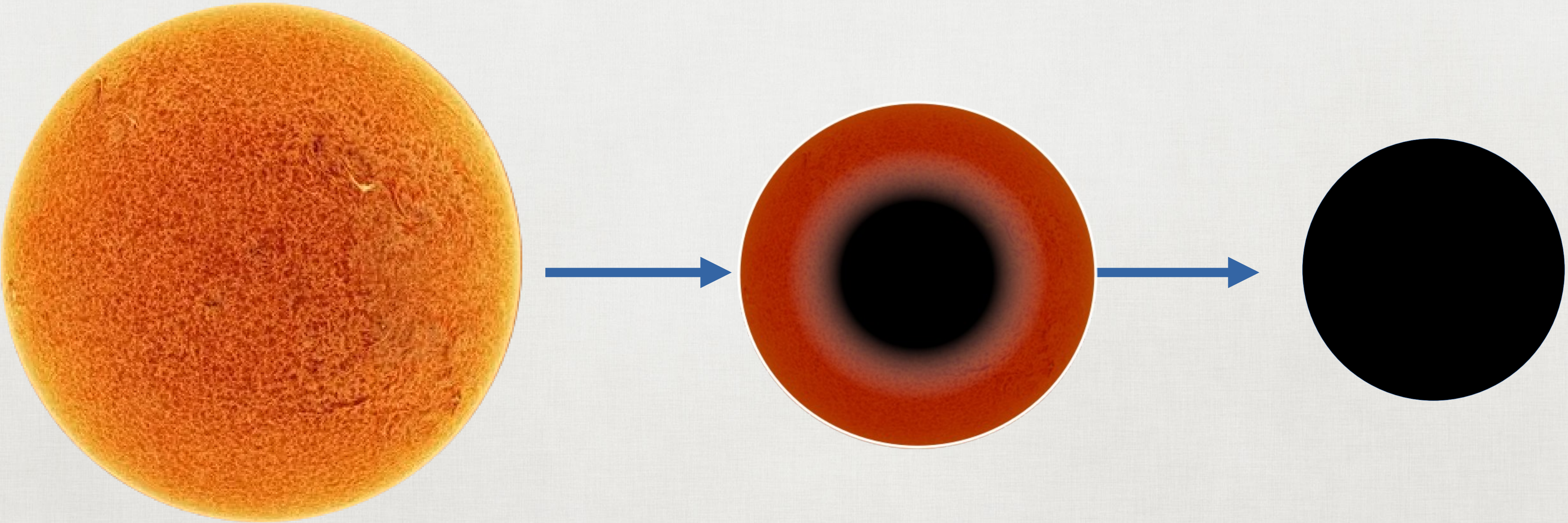


Light rays that are emitted at the same proper time at the star surface arrive at the observer at different times.



$T_1 < T_2$

Expectation for collapse imaging



Video with the collapse of a dust star, starting from rest, with an initial radius of 8M, and surface temperature of 5000 ºC: