Black holes immersed in strong magnetic fields

Haroldo Lima Jr., Pedro V. P. Cunha, Carlos A. R. Herdeiro
and Luís C. B Crispino. [Based on arXiv:2104.09577 (2021)]




Outline

Why magnetic fields?

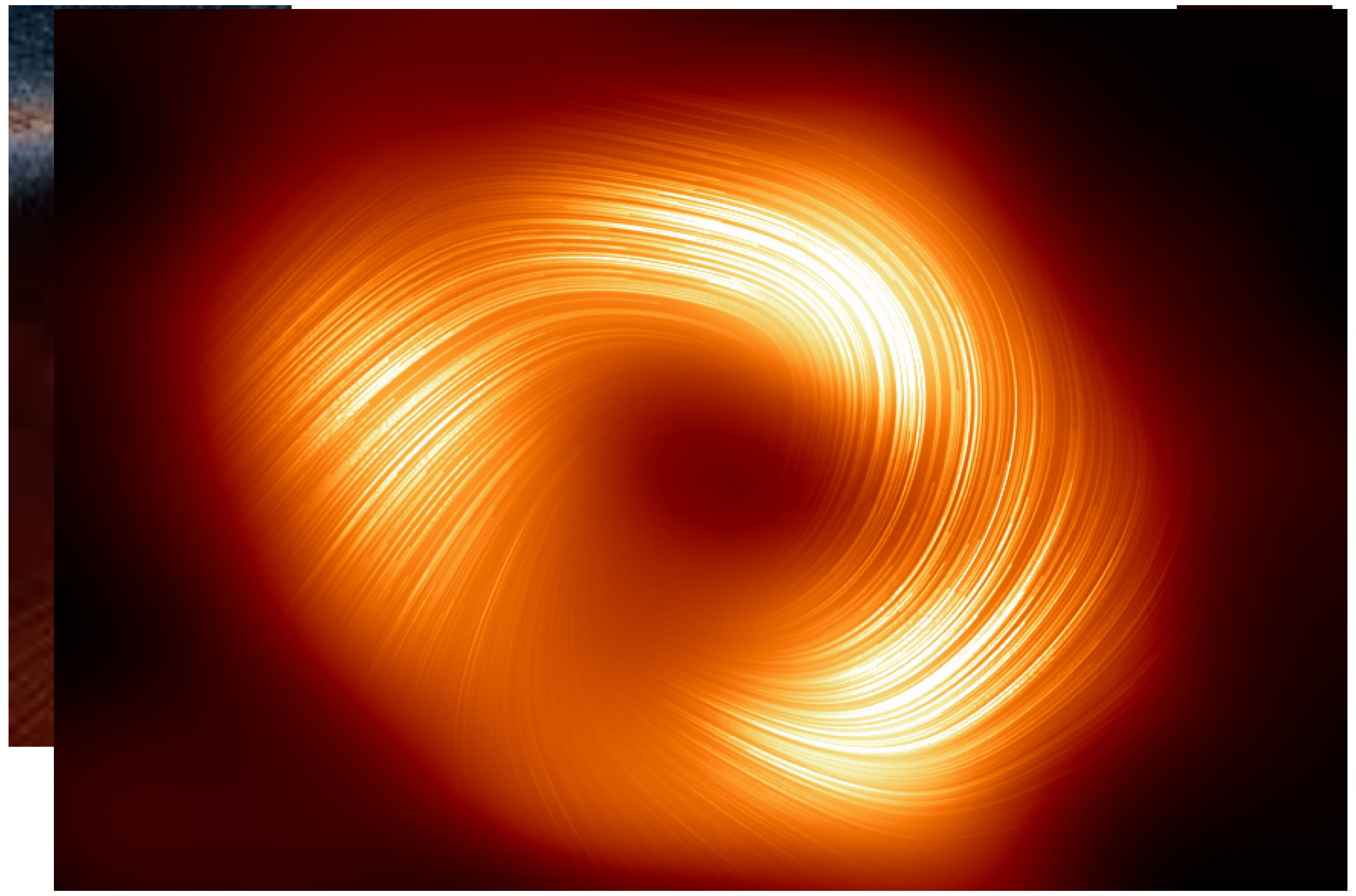
How to model magnetic fields?

What are the effects of magnetic fields?

Black holes and magnetic fields

The Kerr solution describes an (eternally) isolated black hole.

In astrophysical scenarios, additional ingredients can play a role.



- Accretion
- DM Halo
- Magnetic Fields

Magnetic fields around black holes

One possibility to model magnetic fields around BHs is to use numerical relativity.

Non-accessible. Computationally expensive.

One simple analytical approach considers the magnetic field as a test field.

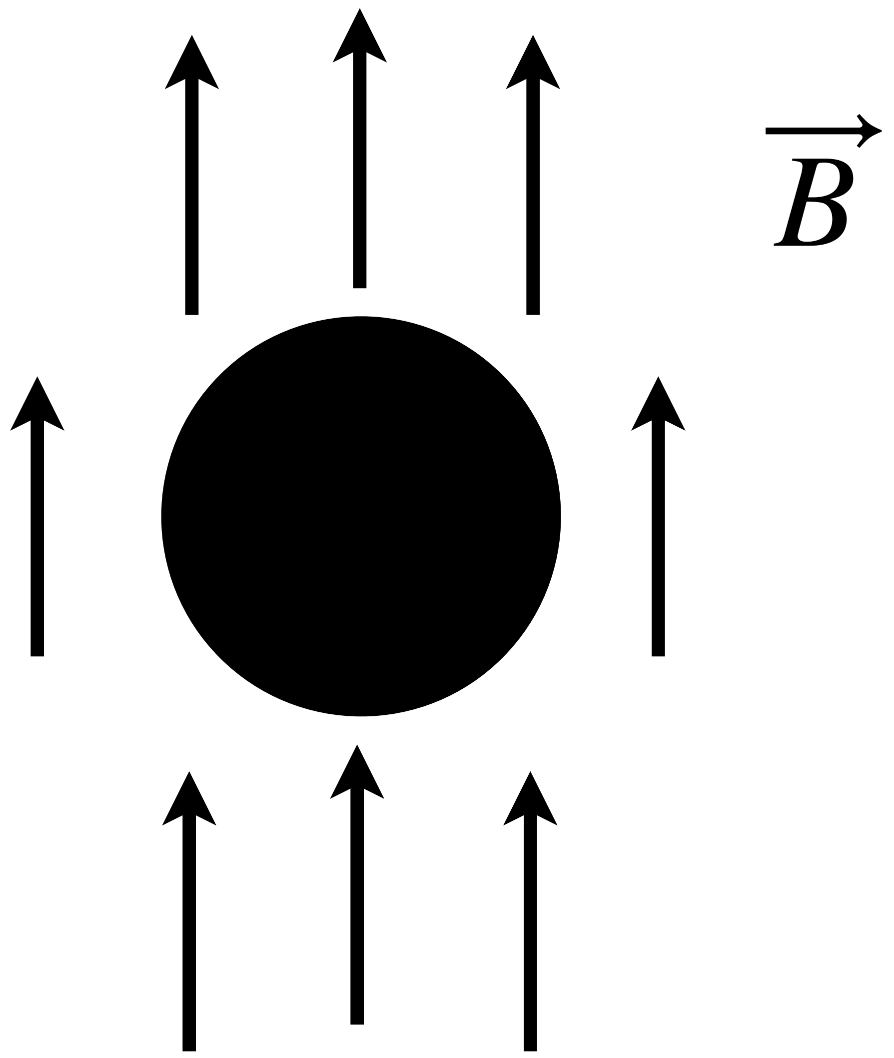
Black hole in a uniform magnetic field*

Robert M. Wald
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 22 April 1974)

Using the fact that a Killing vector in a vacuum spacetime serves as a vector potential for a Maxwell test field, we derive the solution for the electromagnetic field occurring when a stationary, axisymmetric black hole is placed in an originally uniform magnetic field aligned along the symmetry axis of the black hole. It is shown that a black hole in a magnetic field will selectively accrete charges until its charge becomes $Q=2B_0J$, where B_0 is the strength of the magnetic field and J is the angular momentum of the black hole. As a by-product of the analysis given here, we prove that the gyromagnetic ratio of a slightly charged, stationary, axisymmetric black hole (not assumed to be Kerr) must have the value $g=2$.

We want to solve Maxwell's equations

$$\nabla_\mu F^{\mu\nu} = 0, \qquad \nabla_{[\sigma} F_{\mu\nu]} = 0,$$



as a test field on Kerr background

$$ds^2 = -\left(1 - \frac{2\mu r}{\Sigma}\right) c^2 dt^2 - \frac{4\mu a c r \sin^2\theta}{\Sigma} dt d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2\mu r a^2 \sin^2\theta}{\Sigma}\right) \sin^2\theta d\phi^2$$

One simple and very useful idea to solve Maxwell's equations on a curved background is to consider the 4-potentials as Killing vectors:

$$F_{\mu\nu} = \nabla_\mu \psi_\nu - \nabla_\nu \psi_\mu = -2\nabla_\nu \psi_\mu \, .$$

Hence

$$\nabla_\nu F^{\mu\nu} = -2\nabla_\nu \nabla^\nu \psi^\mu = \cancelto{0}{-2R^\mu{}_\nu \psi^\nu} \, .$$

For a test magnetic field in Kerr spacetime:

$$\begin{aligned} ds^2 = & -(1-2mr/\Sigma)dt^2 - (4mar\sin^2\theta/\Sigma)dt\, d\varphi \\ & + \left[\frac{(r^2+a^2)^2 - \Delta a^2 \sin^2\theta}{\Sigma}\right]\sin^2\theta\, d\varphi^2 \\ & + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 \, , \end{aligned} \qquad (3.2)$$

$$\begin{aligned} F = & B_0\left[\frac{ar\sin^2\theta}{\Sigma} - \frac{ma(r^2 - a^2\cos^2\theta)(1+\cos^2\theta)}{\Sigma^2}\right]\omega^1 \wedge \omega^0 \\ & + B_0 \frac{\Delta^{1/2} r \sin\theta}{\Sigma}\omega^1 \wedge \omega^3 + B_0 \frac{\Delta^{1/2} a \sin\theta\cos\theta}{\Sigma}\omega^2 \wedge \omega^0 \\ & + \frac{B_0 \cos\theta}{\Sigma}\left[r^2 + a^2 - \frac{2mra^2(1+\cos^2\theta)}{\Sigma}\right]\omega^2 \wedge \omega^3 \, . \end{aligned}$$

(3.9)

In the presence of a uniform magnetic field, a black hole accretes charge up to a maximal value:

$$Q = 2B_0 J$$

For extremal Kerr black hole:

$$B \approx 10^{-4} - 10^{-5} G,$$

$$Q/M \approx 10^{-24}$$

Can we compute the background geometry when the magnetic field backreacts?

The symmetries of Maxwell's equations

Consider the free Maxwell's equations

$$\nabla \,.\, \overrightarrow{E} = 0, \qquad \nabla \,.\, \overrightarrow{B} = 0,$$

$$\nabla \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t}, \qquad \nabla \times \overrightarrow{B} = \frac{\partial \overrightarrow{E}}{\partial t}\,.$$

This system is invariant under the following transformations:

$$\overrightarrow{E} \to \overrightarrow{B}, \quad \overrightarrow{B} \to -\overrightarrow{E}\,.$$

$$\overrightarrow{E} \to \overrightarrow{E}\cos\theta + \overrightarrow{B}\sin\theta,$$
$$\overrightarrow{B} \to \overrightarrow{B}\cos\theta - \overrightarrow{E}\sin\theta,$$

The symmetries of Einstein-Maxwell's equations

The Ernst formalism in General Relativity is important for generating stationary and axially symmetric solutions. Consider Einstein-Maxwell's field equation:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 2\left(F_{\mu\alpha}F^{\alpha}_{\ \nu} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right), \qquad \nabla_{\mu}F^{\mu\nu} = 0, \qquad \nabla_{[\sigma}F_{\mu\nu]} = 0.$$

In the Lewis-Weyl-Papapetrou form, we have:

$$ds^2 = f\left(dt - \omega d\phi\right)^2 - f^{-1}\left[r^2 d\phi^2 + e^{2\gamma}\left(dr^2 + dz^2\right)\right],$$

$$A = A_t(r,z)dt + A_\phi(r,z)d\phi\,.$$

The field equations in these coordinates are:

$$\left(\Re(\mathscr{E}) + |\Phi|^2\right)\nabla^2\mathscr{E} = (\nabla\mathscr{E} + 2\Phi^*\nabla\Phi)\,.\,\nabla\mathscr{E},$$

$$\left(\Re(\mathscr{E}) + |\Phi|^2\right)\nabla^2\Phi = (\nabla\mathscr{E} + 2\Phi^*\nabla\Phi)\,.\,\nabla\Phi,$$

with

$$\Phi = A_t + i\tilde{A}_\phi, \qquad \mathscr{E} = f - |\Phi\Phi^*| + ih,$$

$$\nabla\tilde{A}_\phi = \frac{f}{\rho}\hat{e}_\phi \times (\nabla A_\phi + \omega\nabla A_t), \qquad \nabla h = -\frac{f^2}{\rho}\hat{e}_\phi \times \nabla\omega - 2\Im(\Phi^*\nabla\Phi),$$

The Ernst equations admits a group of symmetry. Some elements of this group are:

$$\mathcal{E}' = \frac{\mathcal{E}}{1 + i\,a\,\mathcal{E}}, \qquad \Phi' = \frac{\Phi}{1 + i\,a\,\Phi}$$

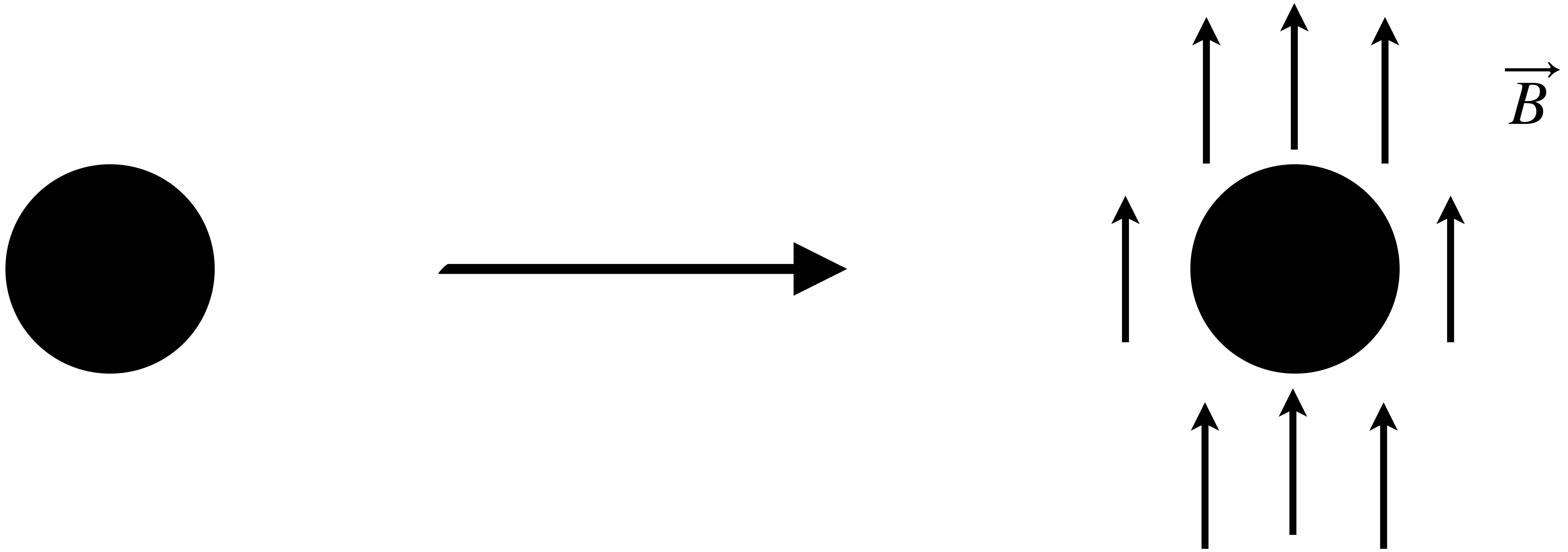
Schwarzschild $\longrightarrow$ **Taub-Nut**

$$\mathcal{E}' = \frac{\mathcal{E}}{1 + B\,\Phi - \frac{1}{4}B^2\mathcal{E}}, \qquad \Phi' = \frac{\Phi - \frac{1}{2}B\mathcal{E}}{1 + B\,\Phi - \frac{1}{4}B^2\mathcal{E}}\,.$$

Schwarzschild $\longrightarrow$ **Magnetized Schwarzschild**

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\Omega^2,$$

$$ds^2 = -\Lambda^2\left(1 - \frac{2M}{r}\right)dt^2 + \frac{\Lambda^2}{\left(1 - \frac{2M}{r}\right)}dr^2 + r^2\Lambda^2 d\theta^2 + \frac{r^2\sin^2\theta}{\Lambda^2}d\phi^2,$$



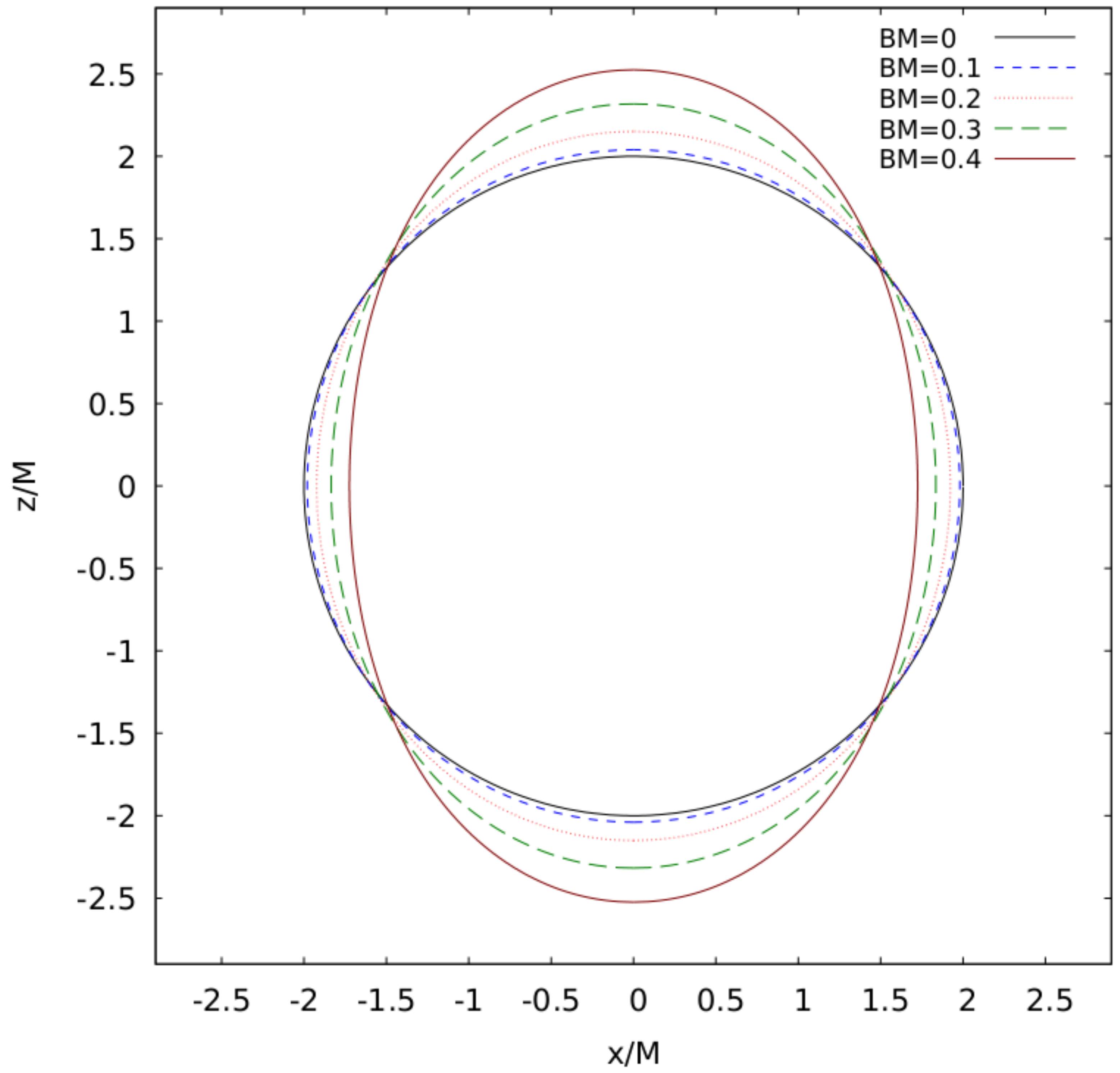
The magnetized Schwarzschild geometry

The magnetized Schwarzschild geometry is given by

$$ds^2 = -\Lambda^2\left(1-\frac{2M}{r}\right)dt^2 + \frac{\Lambda^2}{\left(1-\frac{2M}{r}\right)}dr^2 + r^2\Lambda^2 d\theta^2 + \frac{r^2\sin^2\theta}{\Lambda^2}d\phi^2,$$

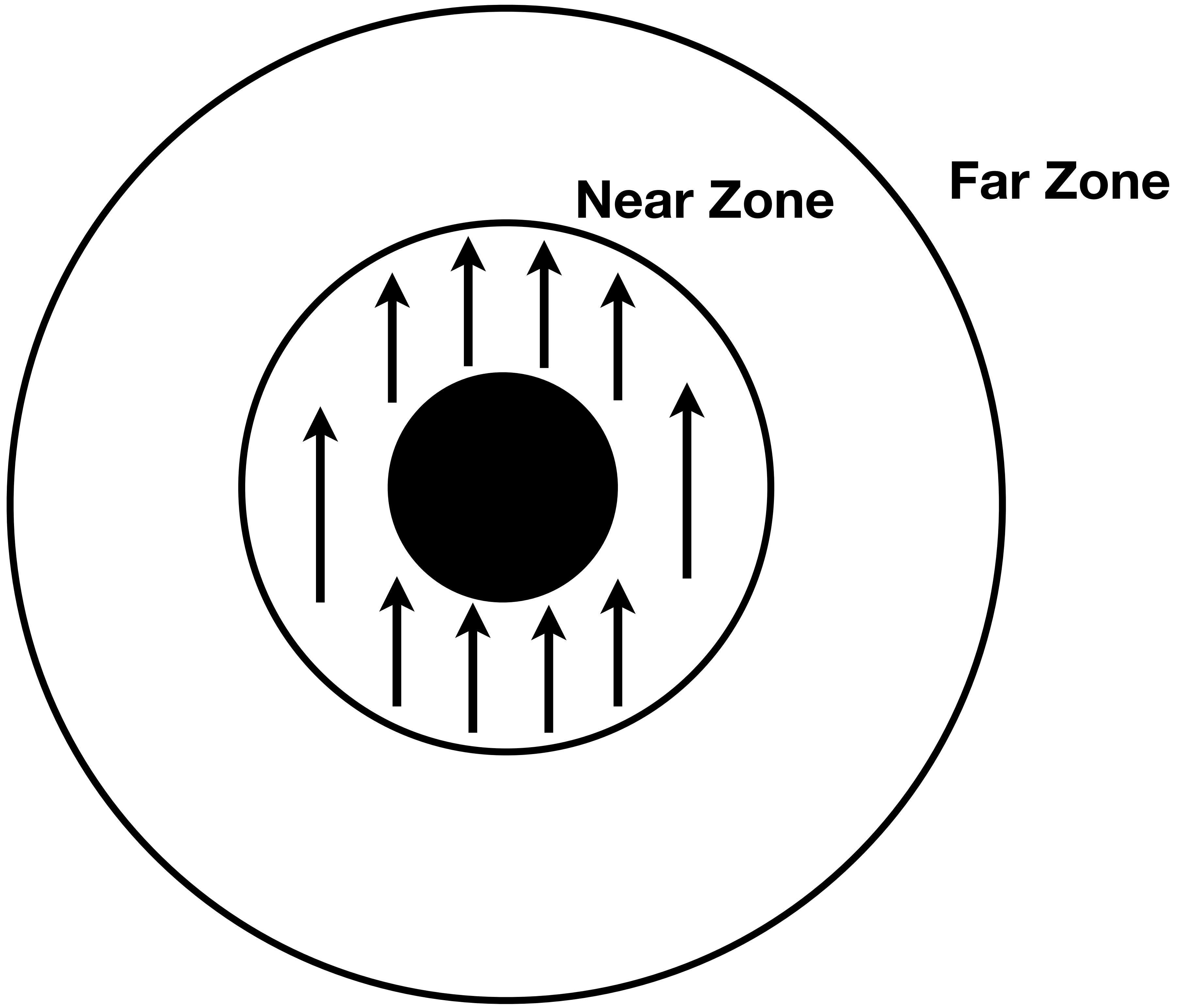
$$\Lambda = 1 + \frac{B^2r^2\sin^2\theta}{4}.$$

- There is an apparent horizon at $r = 2M$.
- There is a curvature singularity at $r = 0$.



The solution is not asymptotically flat as it approaches the Melvin universe in the far away region:

$$ds^2 \approx -\Lambda^2 dt^2 + \Lambda^2 dr^2 + r^2 \Lambda^2 d\theta^2 + \frac{r^2 \sin^2\theta}{\Lambda^2} d\phi^2,$$



Light rings, shadow and gravitational lensing

The motion of light can be described using Hamilton's equations:

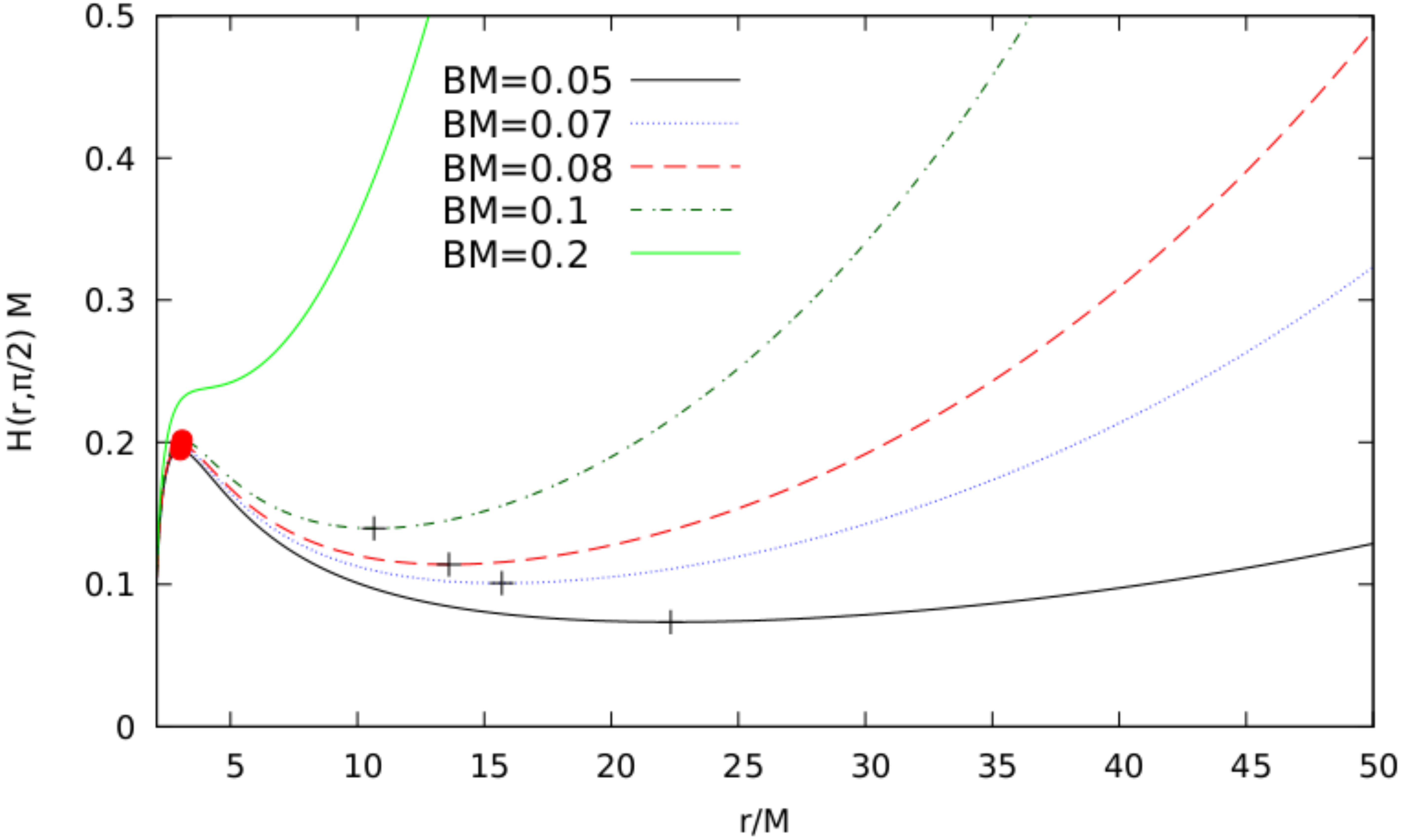
$$\dot{x}^{\mu} = \frac{\partial \mathscr{H}}{\partial p_{\mu}}, \qquad \dot{p}_{\mu} = -\frac{\partial \mathscr{H}}{\partial x^{\mu}}, \qquad \mathscr{H} = \frac{1}{2} g^{\mu\nu} p_{\mu} p_{\nu},$$

We can split the Hamiltonian in two parts:

$$\mathscr{H} = T(r,\theta) + V(r,\theta,E,L),$$

$$T(r,\theta) = g^{rr}(p_r)^2 + g^{\theta\theta}(p_{\theta})^2,$$

$$V(r,\theta,E,L) = \frac{L^2}{\Lambda^2\left(1-\frac{2M}{r}\right)}\left(H(r,\theta)+\frac{1}{\eta}\right)\left(H(r,\theta)-\frac{1}{\eta}\right)$$



$$H(r,\theta) = \frac{\Lambda^2 \left(1 - \frac{2M}{r}\right)^{\frac{1}{2}}}{r\sin\theta}$$

There exist a critical value B_c such that

- Undercritical: For $B < B_c$, there are two light rings outside the apparent horizon.

- Overcritical: For $B > B_c$, there are no light rings at all outside the apparent horizon!

The exact value of B_c is:

$$B_c M = \frac{2}{5}\sqrt{\frac{169 - 38\sqrt{19}}{15}} \approx 0.1893$$

How can a black hole have no light ring?

Stationary Black Holes and Light Rings

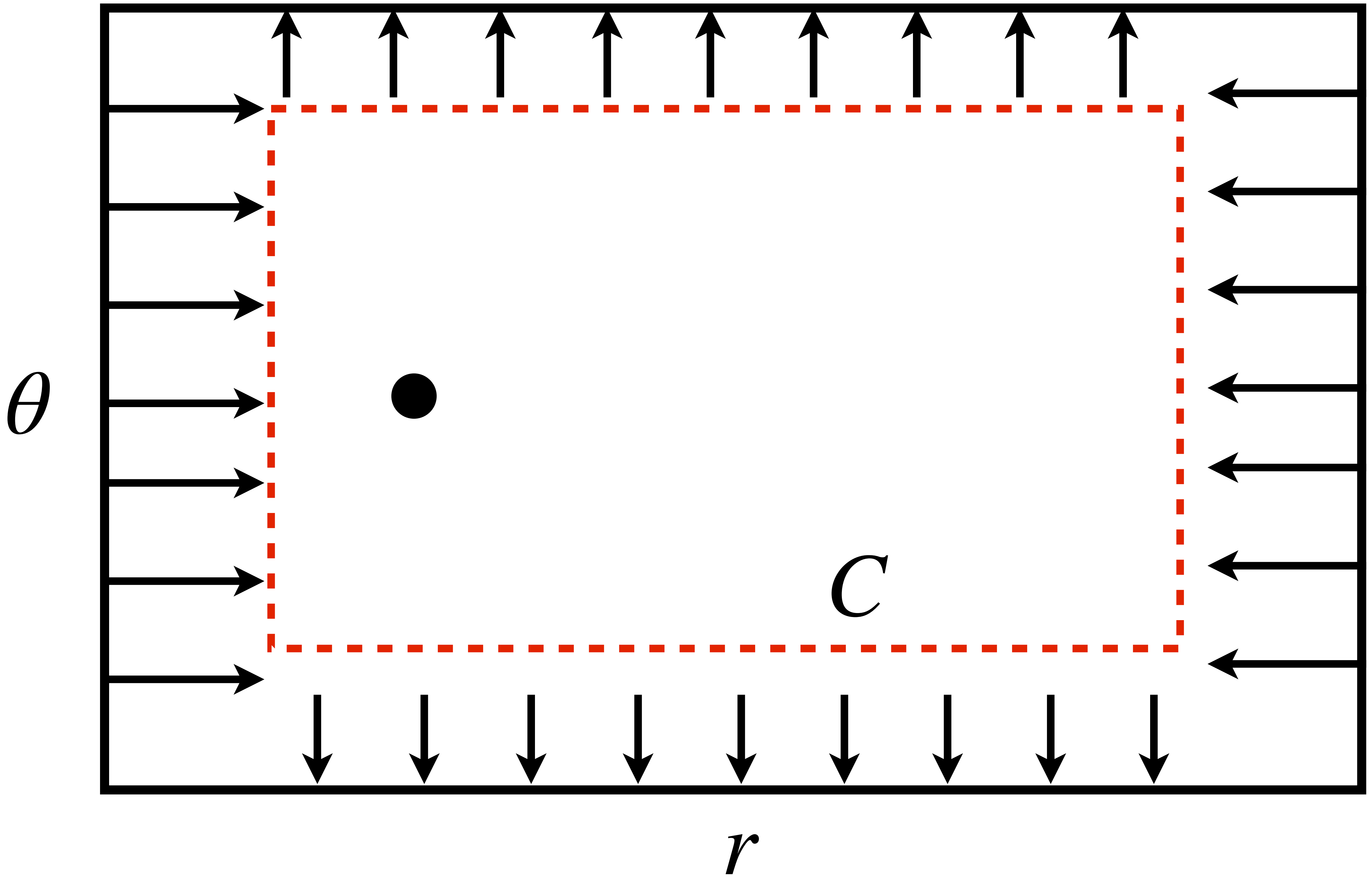
Pedro V. P. Cunha[1] and Carlos A. R. Herdeiro[2]
[1]*Max Planck Institute for Gravitational Physics—Albert Einstein Institute, Am Mühlenberg 1, Potsdam 14476, Germany*
[2]*Departamento de Matemática da Universidade de Aveiro and CIDMA, Campus de Santiago, 3810-183 Aveiro, Portugal*

(Received 19 March 2020; accepted 15 April 2020; published 8 May 2020)

The ringdown and shadow of the astrophysically significant Kerr black hole (BH) are both intimately connected to a special set of bound null orbits known as light rings (LRs). Does it hold that a *generic* equilibrium BH *must* possess such orbits? In this Letter we prove the following theorem. A stationary, axisymmetric, asymptotically flat black hole spacetime in $1+3$ dimensions, with a nonextremal, topologically spherical, Killing horizon admits, at least, one standard LR outside the horizon for each rotation sense. The proof relies on a topological argument and assumes C^2 smoothness and circularity, but makes no use of the field equations. The argument is also adapted to recover a previous theorem establishing that a horizonless ultracompact object must admit an even number of nondegenerate LRs, one of which is stable.

Topological charge of asymptotically flat black holes

Asymptotically flat case



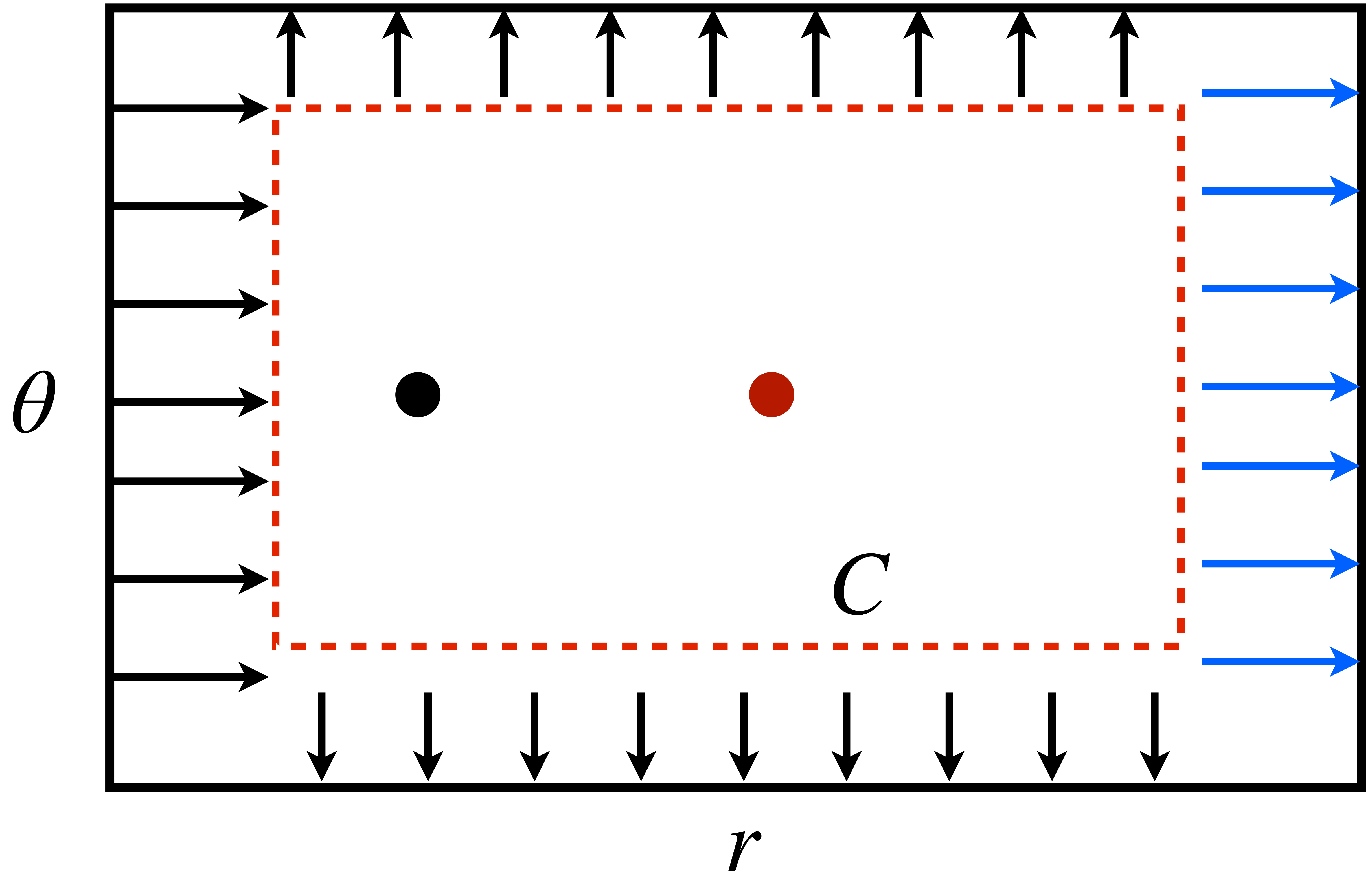
$$\vec{v} = \left(\frac{\partial_r H}{\sqrt{g_{rr}}}, \frac{\partial_\theta H}{\sqrt{g_{\theta\theta}}} \right),$$

$$w = \frac{1}{2\pi} \oint_C d\Omega \, .$$

$$w = -1$$

Topological charge of asymptotically Melvin black holes

Asymptotically flat case



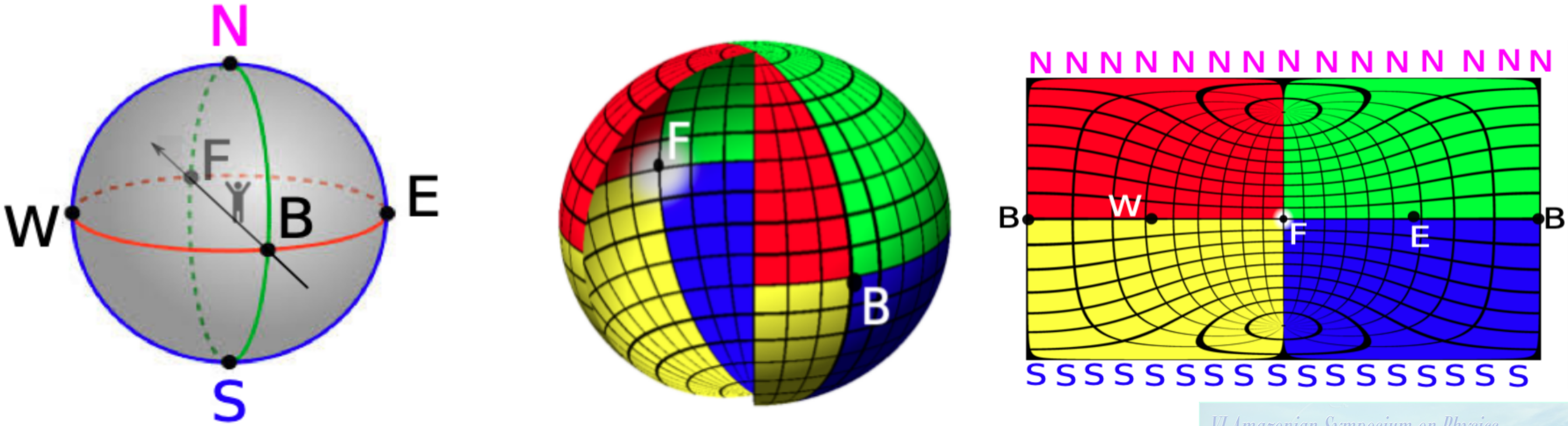
$$\vec{v} = \left(\frac{\partial_r H}{\sqrt{g_{rr}}}, \frac{\partial_\theta H}{\sqrt{g_{\theta\theta}}} \right),$$

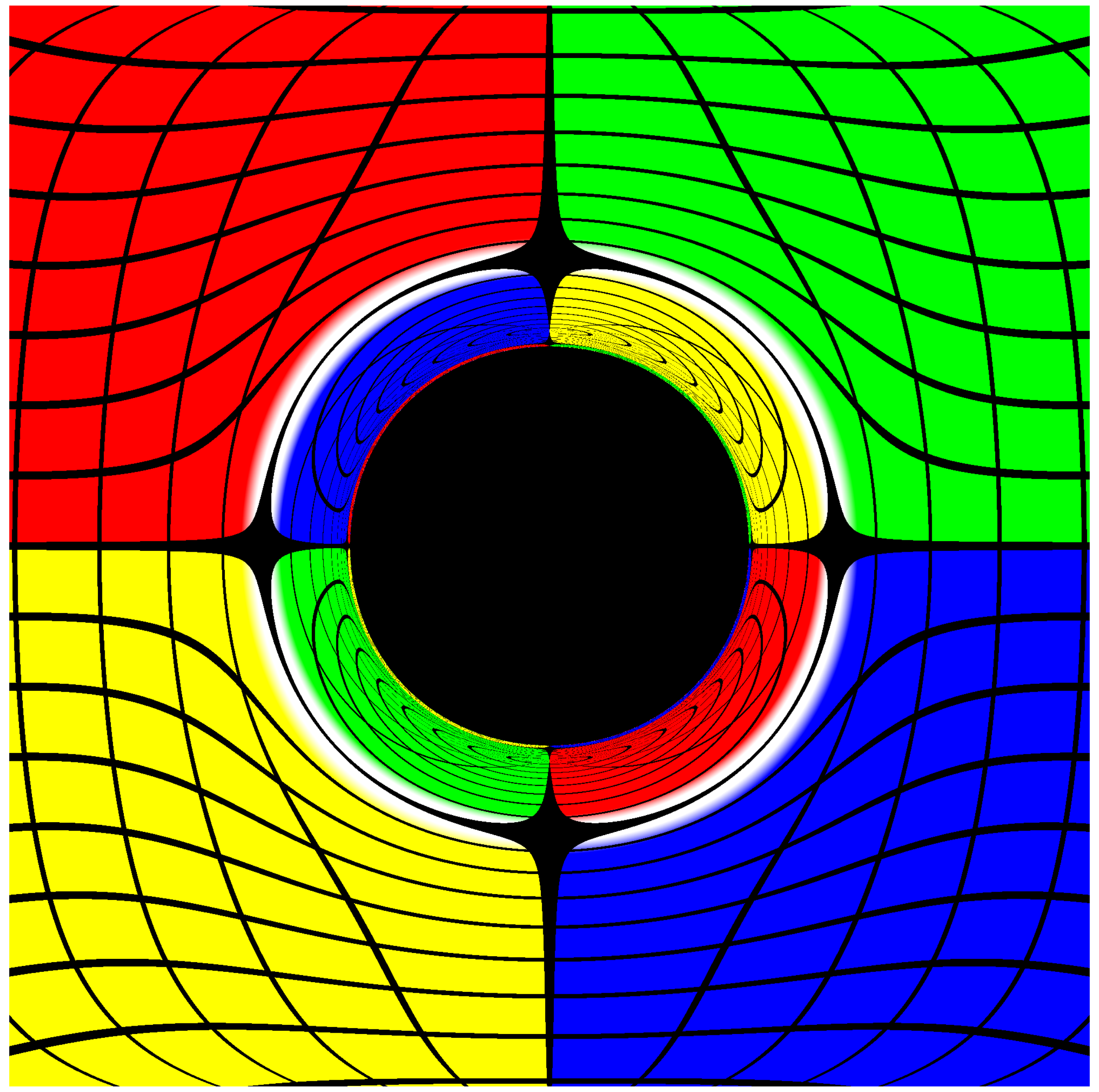
$$w = \frac{1}{2\pi} \oint_C d\Omega \,.$$

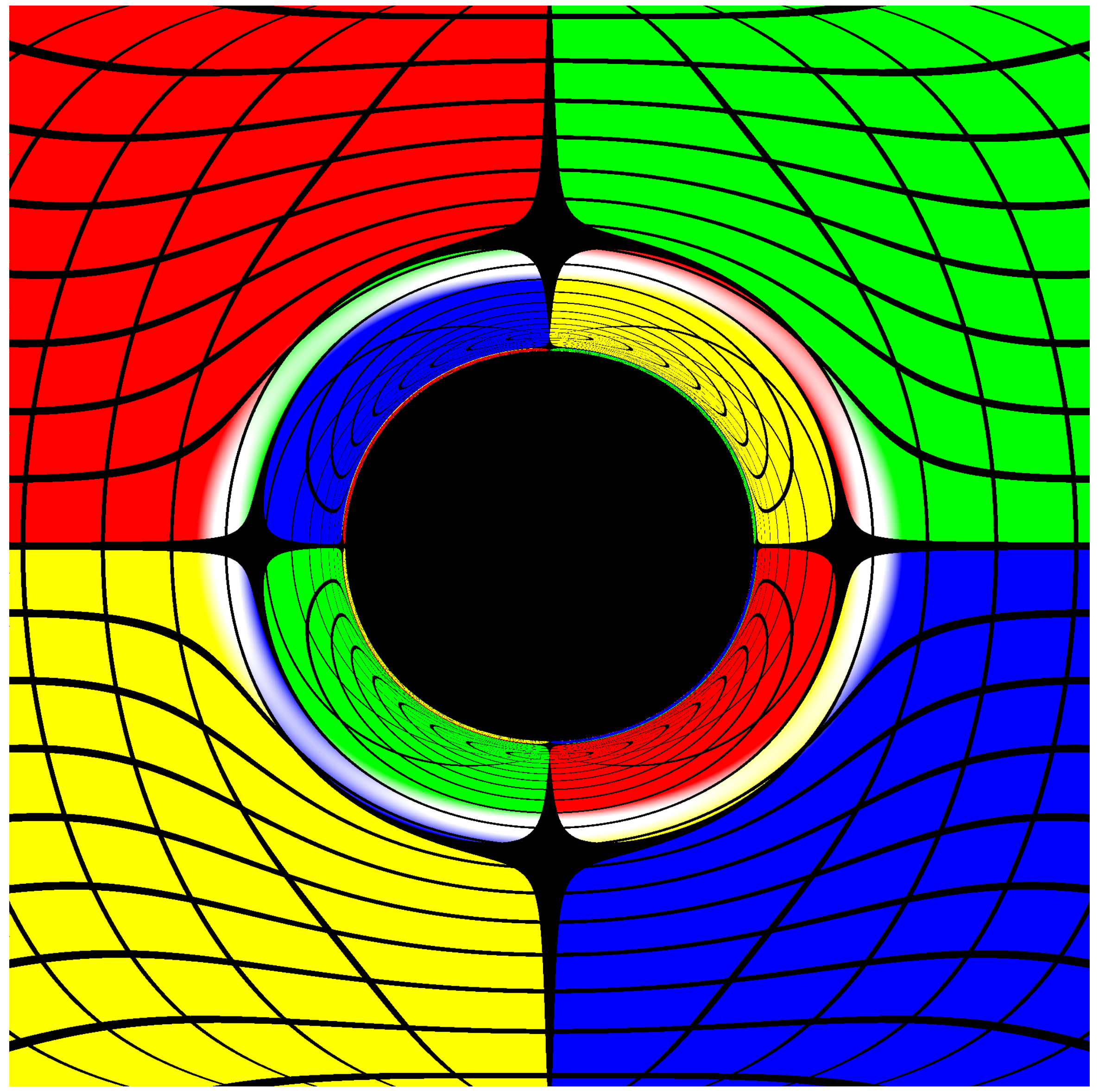
$$w = 0$$

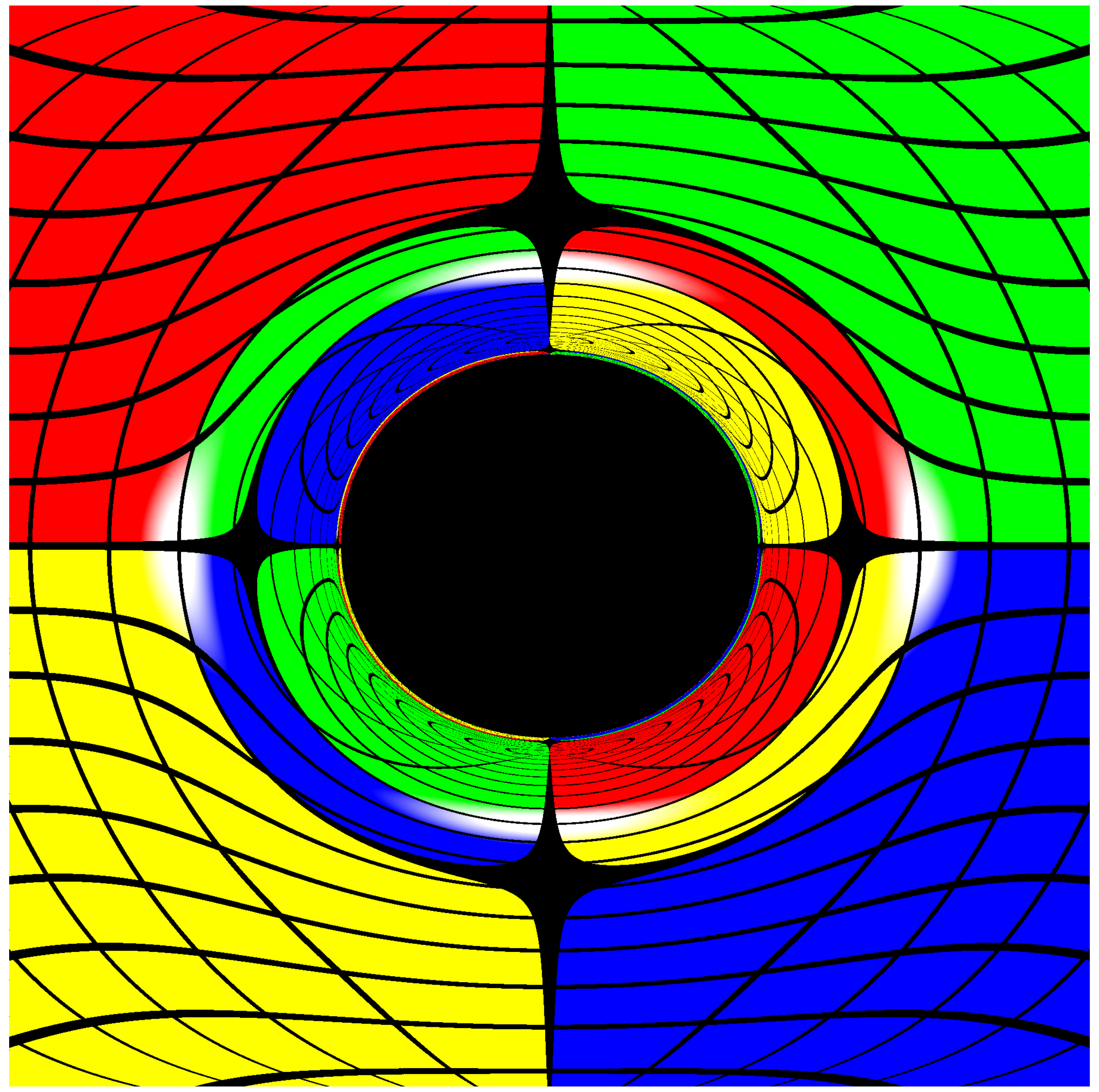
How does a black hole without LR look like?

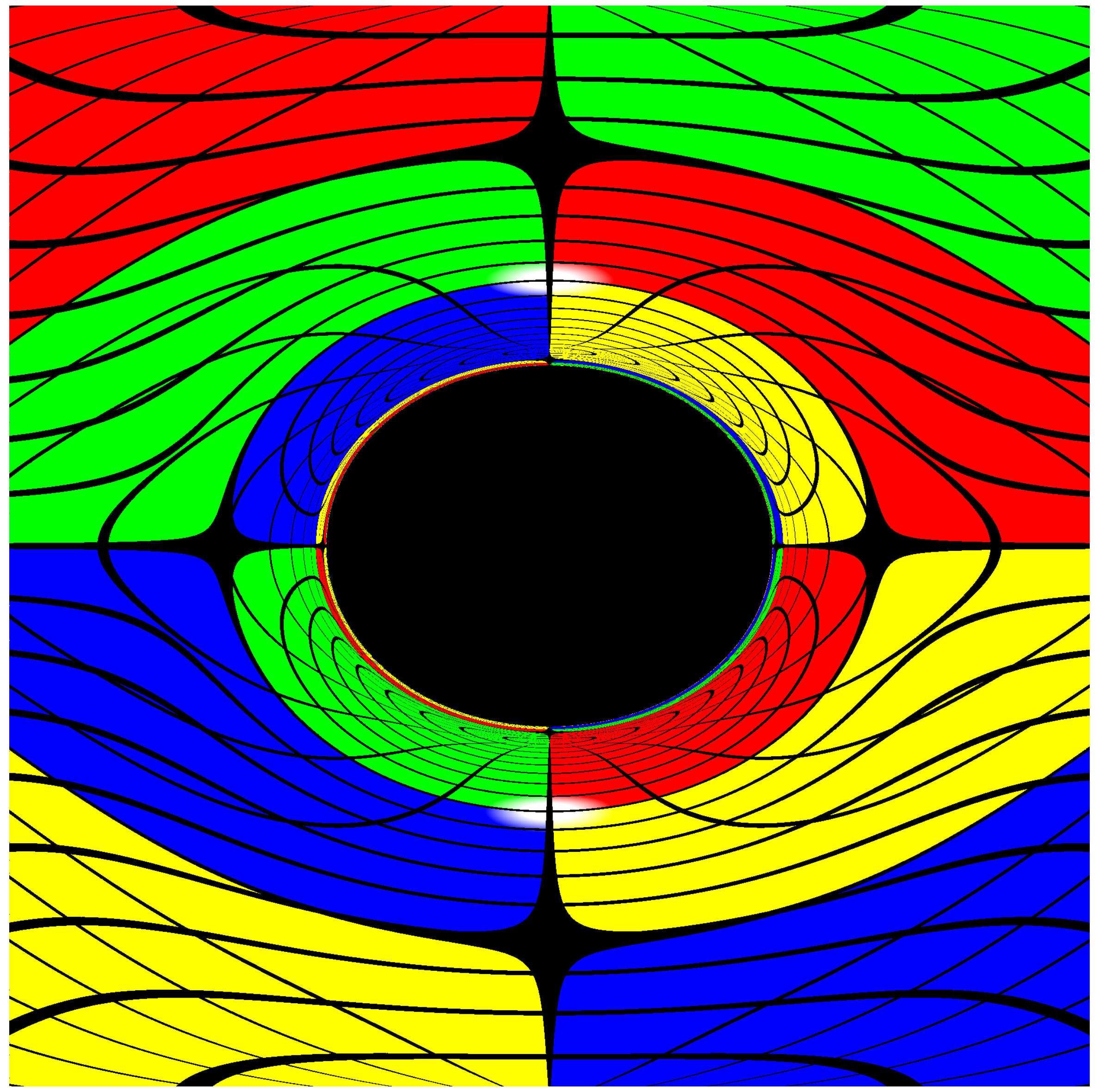
In order to simulate the shadow and gravitational lensing, we apply backwards ray-tracing techniques:



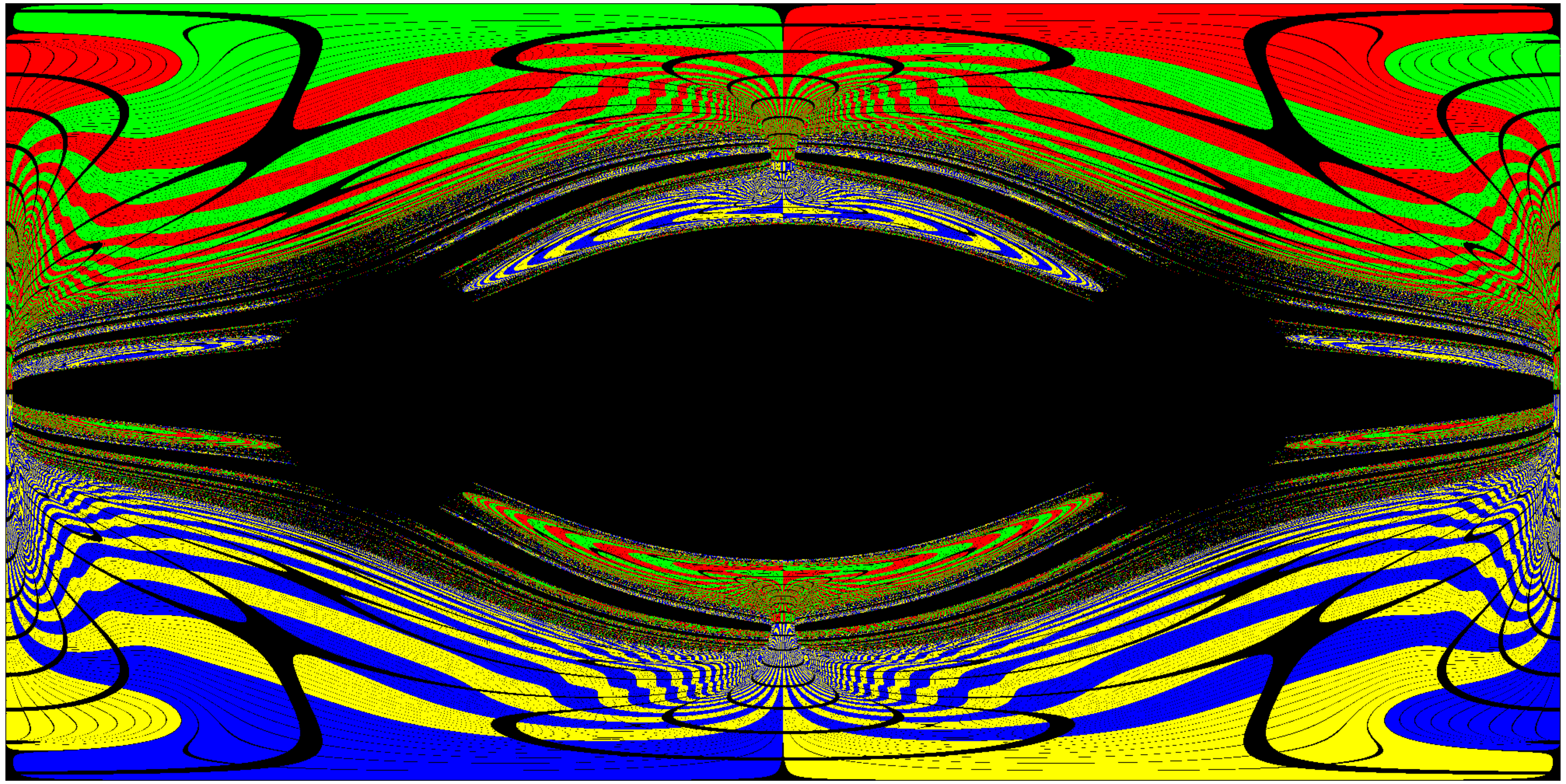

BM = 0

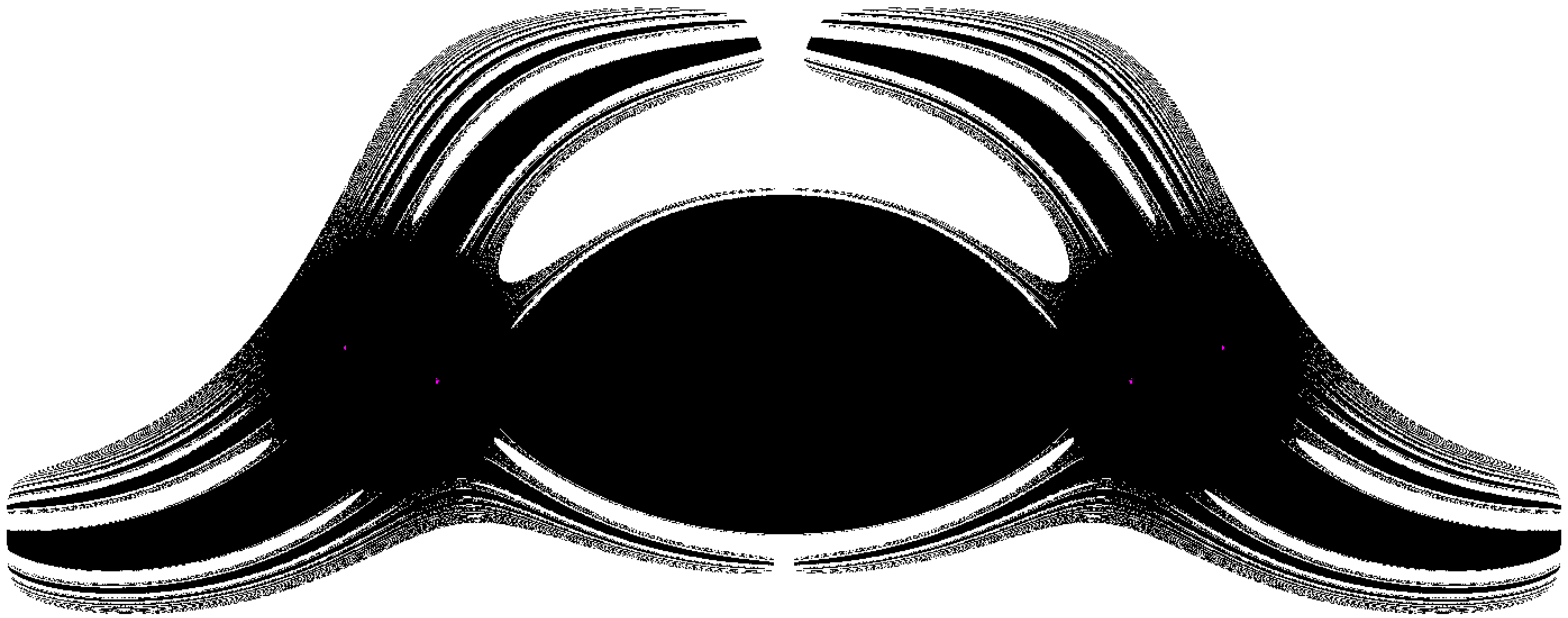

BM = 0.3


BM = 0.4

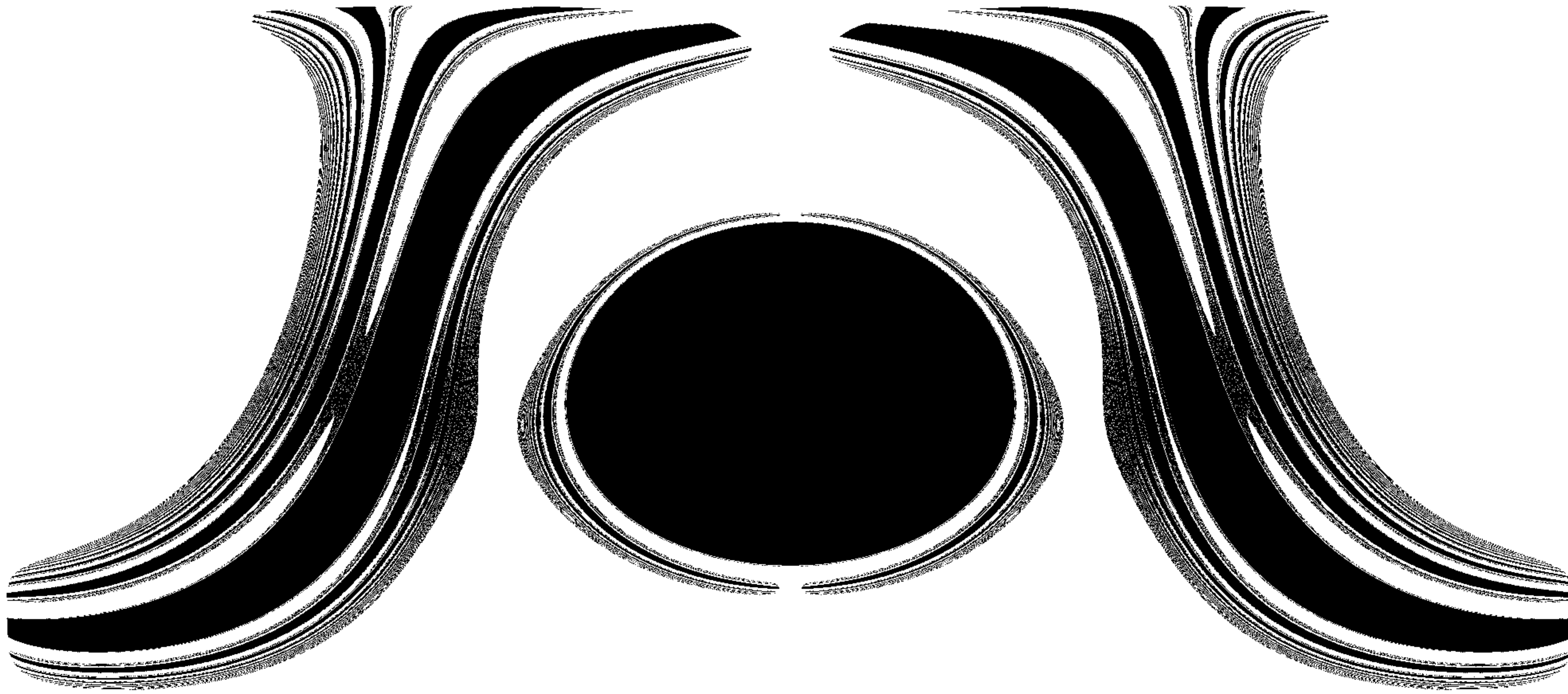

BM = 0.6

For an overcritical case:





Observer at 45º



Observer at 30º

Final Remarks

Magnetic fields can play an important role around black holes in astrophysical realistic environments.

We can explore the symmetries of Einstein-Maxwell's equations to obtain novel solutions in General Relativity.

Strong external magnetic fields can give rise to intriguing phenomena such as black holes without any light ring.

Acknowledgments

UFMA
Universidade Federal do Maranhão
A VIDA É COMBATE

UNIVERSIDADE FEDERAL DO PARÁ

universidade de aveiro

Fapespa

CNPq