Three non-existence results are established for self-gravitating solitons in Einstein-Maxwell-scalar models, wherein the scalar field is, generically, non-minimally coupled to the Maxwell field via a scalar function $f(\Phi)$. Firstly, a trivial Maxwell field is considered, which yields a consistent truncation of the full model. In this case, using a scaling (Derrick-type) argument, it is established that no stationary and axisymmetric self-gravitating scalar solitons exist, unless the scalar potential energy is somewhere negative in spacetime. This generalises previous results for the static and strictly stationary cases. Thus, rotation alone cannot support self-gravitating scalar solitons in this class of models. Secondly, constant sign couplings are considered. Generalising a previous argument by Heusler for electro-vacuum, it is established that no static self-gravitating electromagnetic-scalar solitons exist. Thus, a varying (but constant sign) electric permittivity alone cannot support self-gravitating scalar solitons. Finally, the second result is generalised for strictly stationary, but not necessarily static, spacetimes, using a Lichnerowicz-type argument, generalising previous results in models where the scalar and Maxwell fields are not directly coupled. The scope of validity of each of these results points out the possible paths to circumvent them, in order to obtain self-gravitating solitons in Einstein-Maxwell-scalar models.