A brief introduction to modified theories of gravity

(Vinc)Enzo Vitagliano
CENTRA, Lisboa

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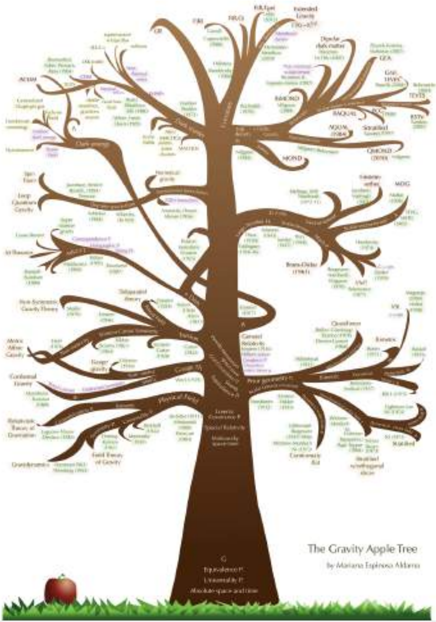
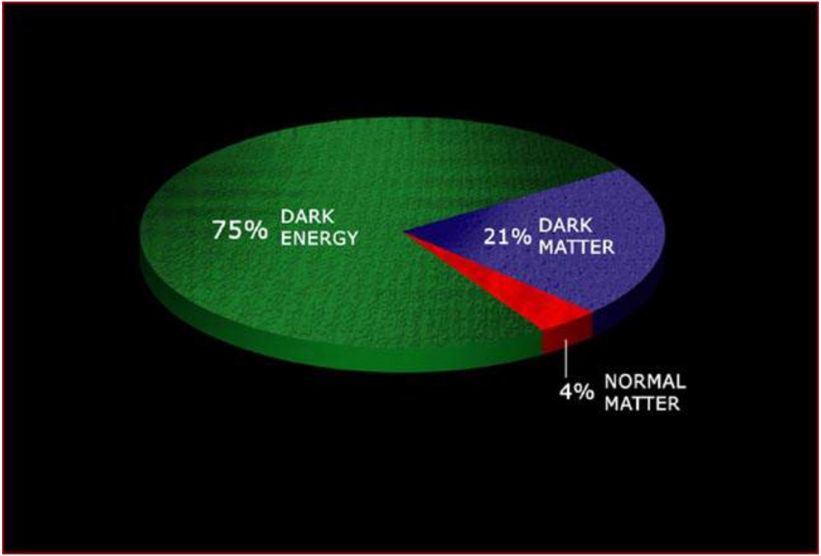
The General Theory of Relativity

- dynamics of the Universe
- behavior of Black Holes
- propagation of Gravitational Waves
- formation of Structures (I mean, really any!)
- ...


The Gravity Apple Tree
by Mariana Espinosa Aldama

Cosmological motivation



- Alternative theories of Gravity with extra fields
- Higher derivatives and non-local theories
- Higher dimensional theories

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- Higher dimensional theories

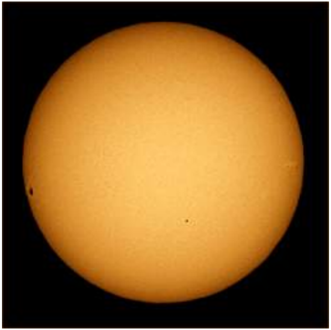
General Relativity and its foundations

Prerequisites for validity: foundational requirements and compatibility with observations

Equivalence principle(s): WEP, EEP, SEP;

Observational test of metric theories

- Solar system tests;
- Gravitational waves;
- Binary pulsars



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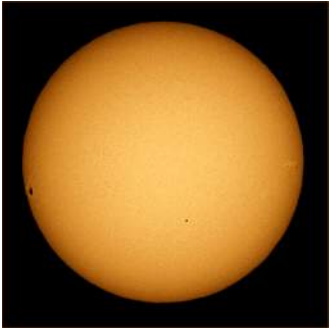
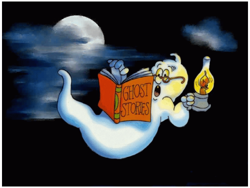




Theoretical considerations on classical and quantum fluctuations about classical solutions: ghosts and their exorcism

More theoretical requirements:

- Stable solutions
- Well motivated from Fundamental Physics
- Well-Posed Initial Value Formulation
- Strong Field Variety

Desirable property: as with most field theories, you want to derive the field equations from some action through some variational principle

Alternative formulation

In GR spacetime geometry is fully described by the metric:
it does not only define distances, which is its primary role, but also defines parallel transport.

However, this does not have to be a *condicio sine qua non*.
The metric and the connection can be independent quantities.

Metric variation $\Rightarrow$ Palatini approach

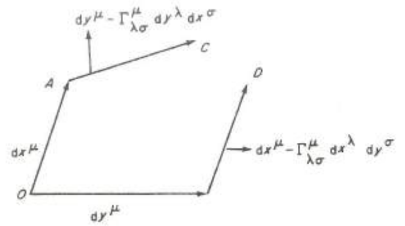
Covariant derivative as gauge derivative of a (translation) group:

Covariant derivative of a vector:	$\nabla_\mu A^\nu = \partial_\mu A^\nu + \Gamma^\nu{}_{\mu\rho} A^\rho$
Electromagnetism:	$D_\mu = \partial_\mu + ie\phi_\mu$
QCD:	$D_\mu = \partial_\mu + \frac{i}{2} g \lambda_\alpha \phi^\alpha_\mu$

"...the essential achievement of GR, namely to overcome 'rigid' space (ie the inertial frame), is only indirectly connected with the introduction of a Riemannian metric. The directly relevant conceptual element is the 'displacement field' Γ^k_{ij}, which express the infinitesimal displacement of vectors. It is this which replaces the parallelism of spatially arbitrary separated vectors fixed by the inertial frame (ie the equality of corresponding components) by an infinitesimal operation. This makes possible to construct tensors by differentiation and hence to dispense with the introduction of 'rigid' space (the inertial frame). In the face of this, it seems to be of secondary importance in some sense that some particular Γ field can be deduced from a Riemannian metric..."

A. Einstein, 4th April 1955

Geometrical interpretation



$$\text{Torsion} \Rightarrow \quad \Gamma^\lambda{}_{\mu\nu} \neq \Gamma^\lambda{}_{\nu\mu}$$
$$\Longrightarrow \mathbf{OC} - \mathbf{OD} =$$
$$= \mathbf{OA} + \mathbf{AC} - \mathbf{OB} - \mathbf{BD} =$$
$$= \left(\Gamma^\mu{}_{\lambda\sigma} - \Gamma^\mu{}_{\sigma\lambda}\right) dx^\lambda dy^\sigma \neq 0$$

$Q_{\lambda\mu\nu} \neq 0 \quad \Longrightarrow$ lengths (inner products) not preserved

$$D\left(g_{\mu\nu} u^\mu w^\nu\right) = \left(D g_{\mu\nu}\right) u^\mu w^\nu = u^\mu w^\nu \nabla_\chi g_{\mu\nu} d\xi^\chi =$$
$$= -u^\mu w^\nu Q_{\chi\mu\nu} d\xi^\chi$$

Palatini procedure

$$S = \frac{1}{16\pi G}\int \sqrt{-g}\left[g^{\mu\nu}\; R^{\Gamma}_{\mu\nu} - 2\Lambda\right] d^4x + \int \mathcal{L}_m(g_{\mu\nu},\psi) d^4x$$

$$G_{(\mu\nu)} + \Lambda g_{\mu\nu} = 8\pi G\,\tilde{T}_{\mu\nu}$$

$$P_{\alpha\mu\beta} = S_{\mu\alpha\beta} + 2g_{\mu[\alpha}S_{\beta]} + g_{\mu[\alpha}Q_{\beta]} - g_{\mu[\alpha}\bar{Q}_{\nu]\beta}{}^{\nu} = 0$$

Further complication with MAGs

$$S = \frac{1}{16\pi G}\int \sqrt{-g}(g^{\mu\nu} R^{\Gamma}_{\mu\nu} - 2\Lambda)d^4x + \int \mathcal{L}_m(g_{\mu\nu},\Gamma^{\mu}{}_{\alpha\beta},\psi)d^4x$$

$$P_{\mu}{}^{\alpha\beta} = 8\pi G\Delta_{\mu}{}^{\alpha\beta}, \qquad T_{\mu\nu} = \tilde{T}_{\mu\nu} + \nabla_{\rho}\left[\Delta^{\rho}{}_{(\mu\nu)} - \Delta_{(\mu}{}^{\rho}{}_{\nu)} - \Delta_{(\mu\nu)}{}^{\rho}\right]$$

A useful theorem

IF metric tensor $g_{\mu\nu}$ is the only independent field

$$S = \int d^4x \mathcal{L}(g_{\mu\nu})$$

and IF the action contains up to second derivatives of $g_{\mu\nu}$, then

$$E^{\mu\nu}[\mathcal{L}] = \frac{d}{dx^\rho}\left[\frac{\partial \mathcal{L}}{\partial g_{\mu\nu,\rho}} - \frac{d}{dx^\lambda}\left(\frac{\partial \mathcal{L}}{\partial g_{\mu\nu,\rho\lambda}}\right)\right] - \frac{\partial \mathcal{L}}{\partial g_{\mu\nu}}$$

and the Euler-Lagrange equation is $E^{\mu\nu}(\mathcal{L}) = 0$.

Lovelock's Theorem

The only possible second-order Euler-Lagrange expression obtainable in a four dimensional space from a scalar density of the form $\mathcal{L} = \mathcal{L}(g_{\mu\nu})$ is

$$E^{\mu\nu} = \alpha\sqrt{-g}\left[R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right] + \lambda\sqrt{-g}g^{\mu\nu}$$

A useful theorem

So, how to overpass GR?

- Consider other fields, beyond (or rather than) the metric tensor.
- Accept higher than second derivatives of the metric in the field equations.
- Work in a space with dimensionality different from four.
- Give up on either rank (2,0) tensor field equations, symmetry of the field equations under exchange of indices, or divergence-free field equations.
- Give up with locality.

Scalar-tensor theories

In General Relativity the gravitational force is mediated by a single rank-2 tensor field. But with possible generalizations!

The simplest scenario: addition of an extra scalar field, but one might also choose to consider extra vectors, tensors, or even higher rank fields.

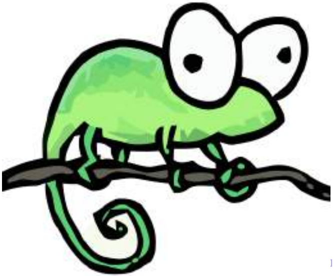
The effect of such fields needs to be suppressed at scales where GR works! (couplings very weak, or novel screening mechanisms)

Scalar-tensor theories

$$\mathcal{L} = \frac{1}{16\pi}\sqrt{-g}\left[f(\phi)R - g(\phi)\nabla_\mu\phi\nabla^\mu\phi - 2\Lambda(\phi)\right] + \mathcal{L}_m(\Psi, h(\phi)g_{\mu\nu})$$

Einstein frame $\Longleftrightarrow$ Jordan frame

Chameleon mechanism at works



Scalar-tensor theories

Horndeski

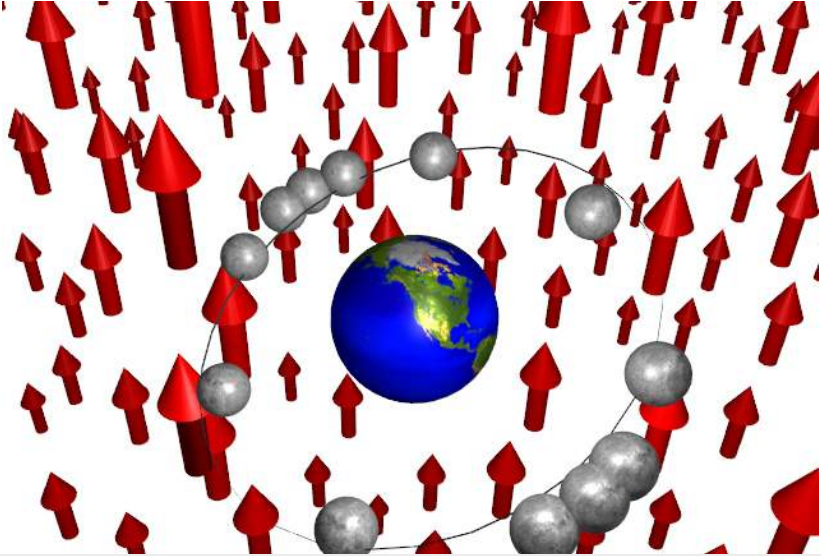
$$\begin{aligned}\mathcal{L}_H \quad &= \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\left[\kappa_1 \nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} - \frac{4}{3}\kappa_{1,X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi \right.\\ &\qquad \left. +\kappa_2\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma} - 4\kappa_{2,X}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi\right]\\ &+\delta^{\alpha\beta}_{\mu\nu}\left[(F+2W)R_{\alpha\beta}{}^{\mu\nu} - 4F_{,X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_3\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\right]\\ &-3[2(F+2W)_{,\phi} + X\kappa_3]\nabla_\mu\nabla^\mu\phi + \kappa_4(\phi, X)\end{aligned}$$

Scalar-tensor theories

Horndeski

$$
\begin{aligned}
\mathcal{L}_H \quad &= \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\Big[\kappa_1 \nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} - \frac{4}{3}\kappa_{1,X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi \\
&\qquad +\kappa_2\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma} - 4\kappa_{2,X}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi\Big] \\
&+\delta^{\alpha\beta}_{\mu\nu}\Big[(F+2W)R_{\alpha\beta}{}^{\mu\nu} - 4F_{,X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_3\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\Big] \\
&-3[2(F+2W)_{,\phi} + X\kappa_3]\nabla_\mu\nabla^\mu\phi + \kappa_4(\phi, X)
\end{aligned}
$$

Lorentz violations



Einstein-aether theories

Modifications to Newtonian gravity on galactic scales (MOND): proposal for explaining dark matter in galaxies

Vector-tensor theories

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R + \mathcal{L}(g^{\mu\nu}, A^{\nu})\right] + S_M(g^{\mu\nu}, \Psi)$$

Einstein-æther theories

$$\mathcal{L}_{EA}(g^{\mu\nu}, A^{\nu}) \equiv \frac{1}{16\pi G}[K^{\mu\nu}{}_{\alpha\beta}\nabla_{\mu}A^{\alpha}\nabla_{\nu}A^{\beta} + \lambda(A^{\nu}A_{\nu} + 1)]$$

Einstein-Cartan-Sciama-Kibble

More general framework of modified gravities: metric-affine theories.
Simplest MAG: the lesson of the ECSK theory.

$$(L_4, g) \xrightarrow{Q=0} U_4 \xrightarrow{S=0} V_4 \xrightarrow{R=0} R_4$$

Preferred curves in Riemann-Cartan U_4:
autoparallel vs extremals curves.

Field equations:
Einstein tensor$=k*$energy momentum
torsion$=k*$spin angular momentum

Invariance of a special relativistic theory of matter under Poincaré transformation inexorably leads to U_4!

Main consequences of U_4 theory

No waves of torsion outside
the spinning matter distribution!
But gravitational waves produced
by processes involving spin...

U_4 theory predicts a new, very weak, universal spin contact interaction of gravitational origin.

Critical mass density typically huge:

$$\rho = mn, \quad s = \hbar n/2 \quad \to \overline{\rho} = \frac{m^2}{k\hbar^2}$$

To be considered in high density regimes...

Particle pair creation when the mass density reaches the critical density $\overline{\rho}$.

Coupling torsion with matter fields

Scalar field: no spin $\Rightarrow$ no coupling to torsion

Maxwell and non-Abelian gauge fields:
minimally coupling to torsion $\Rightarrow$ gauge symmetry breaking.

Proca field: problem of gauge non-invariance bypassed.

Dirac field:

$$\begin{aligned}\mathcal{L}_D[\Gamma] &= (\hbar c/2)[(\overline{\nabla_\alpha\psi})\gamma^\alpha\psi - \overline{\psi}\gamma^\alpha\nabla_\alpha\psi - 2(mc/\hbar)\overline{\psi}\psi] \\ &= \mathcal{L}_D[\{\}] + \text{Spin}\otimes\text{Torsion}\end{aligned}$$

$$\gamma^\alpha\nabla_\alpha\psi + \frac{3}{8}l_P^2(\overline{\psi}\gamma_5\gamma^\alpha\psi)\gamma_5\gamma_\alpha\psi + (mc/\hbar)\psi = 0$$

Neutrinos: Dirac with no spin contact term.

Die hard: f(R)

Both standard metric and Palatini variations of EH action lead to equivalent systems of field equations.

For more general actions, this is not true anymore!

$$S = \frac{1}{l_p^2} \int dx^4 \sqrt{-g} f(\mathcal{R}) + S_M(\psi, g_{\mu\nu})$$

- metric field equations

$$f'(R) R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} + \left(g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu\right) f'(R) = \kappa T_{\mu\nu}$$

- Palatini field equations

$$f'(\mathcal{R}) \mathcal{R}_{(\mu\nu)} - \frac{1}{2} f(\mathcal{R}) g_{\mu\nu} = \kappa T_{\mu\nu}$$

$$\nabla_\lambda \left(\sqrt{-g} f'(\mathcal{R}) g^{\mu\nu}\right) = 0$$

Both are metric theories in the sense that
i) gravity is associated to a symmetric tensor (the metric),
ii) the response of matter and fields to gravity is described by
$\nabla_\mu T^{\mu\nu} = 0$.
In any case, the two theories are not equivalent.

The *a priori* independent connection in Palatini $f(R)$ gravity does not carry any dynamics: it is an auxiliary field that can algebraically eliminated.

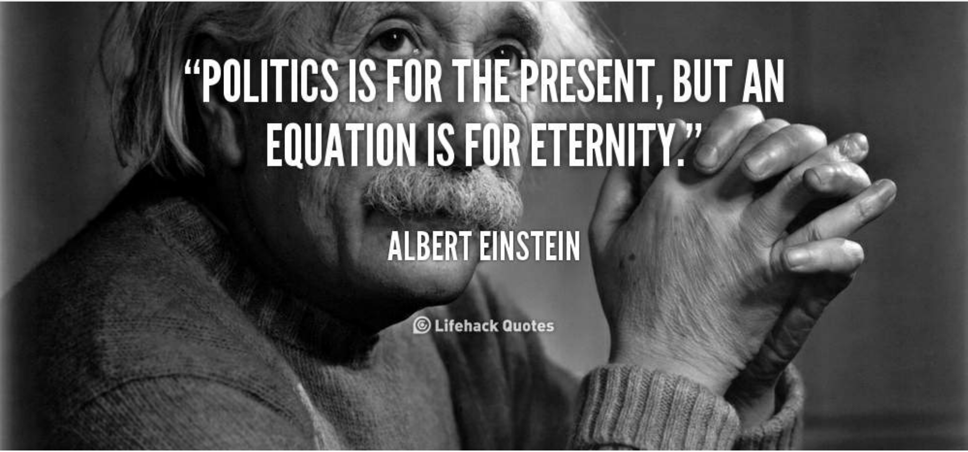
Palatini $f(R)$ dynamically equivalent to Brans-Dicke theory with $\omega_0 = -3/2$ (a particular class in which the scalar field doesn't add any new dynamics).

Eliminating the (symmetric and not) connection in Palatini

$$\Gamma^{\lambda}{}_{\mu\nu} = \left\{{}^{\lambda}{}_{\mu\nu}\right\} + \frac{1}{2f'}\left[2\partial_{(\mu} f' \delta^{\lambda}_{\nu)} - g^{\lambda\sigma} g_{\mu\nu} \partial_\sigma f'\right]$$

$$f'(\mathcal{R})\mathcal{R} - 2f(\mathcal{R}) = \kappa T$$

The algebraic equation can be generically solved to give $\mathcal{R}(T)$. Some exceptions: $f \propto \mathcal{R}^2$, no root of the equation...
Cook everything into the expression of Γ...


"POLITICS IS FOR THE PRESENT, BUT AN
EQUATION IS FOR ETERNITY."
ALBERT EINSTEIN
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Thank you!!!