Black holes in higher dimensional spacetimes

Jorge V. Rocha (Centra-IST, U. Lisboa)

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Part II

Exact solutions in higher D:

Linear instabilities

Exact solutions in higher D: Stability of Tangherlini

[Kodama, Ishibashi (2003)]

- Linear (modal) gravitational perturbations can be decomposed into

  scalar + vector + tensor

- Using a gauge-invariant formalism Kodama and Ishibashi showed that *"the master equation for each type of perturbation has no normalisable negative modes that would correspond to unstable solutions".*

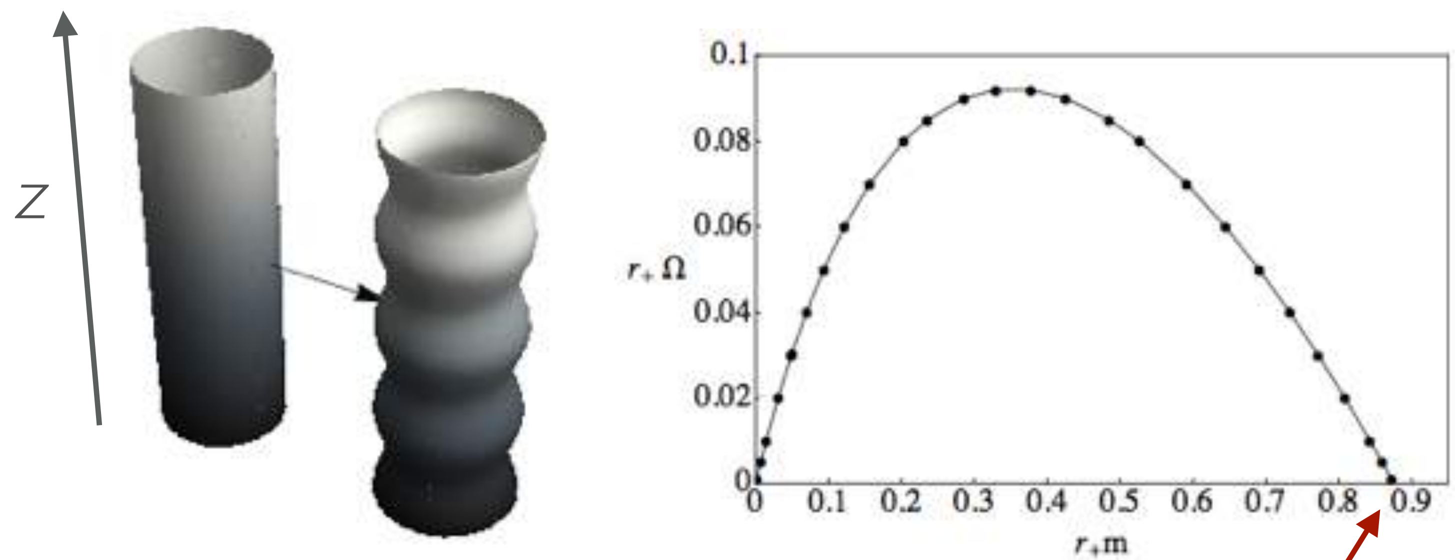
- Similarly, higher dimensional Schwarzschild-(anti-)de Sitter is also mode-stable.

Exact solutions in higher D: Gregory-Laflamme instability

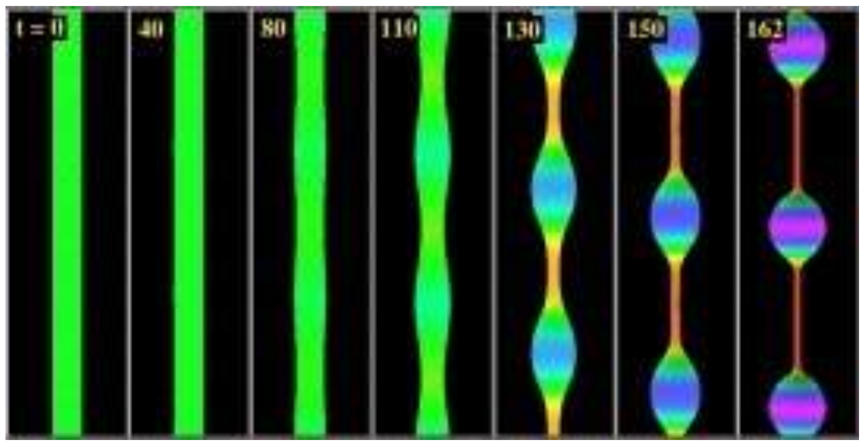
[Gregory, Laflamme (1993)]

- By studying linear (modal) perturbations for the black string, assuming a separated ansatz, $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$ $\qquad h_{\mu\nu} = e^{imz} e^{\Omega t} H_{\mu\nu}(r)$

  Gregory and Laflamme showed that an **instability** appeared for **long wavelengths**.



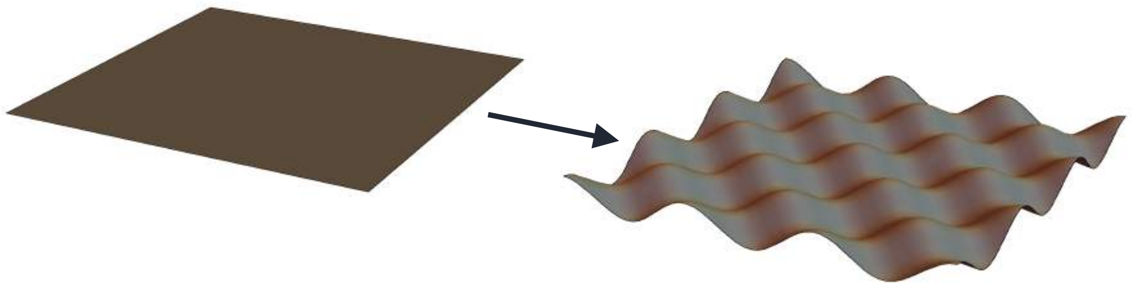
from [Gregory (2011)]



[Lehner, Pretorius (2010)]

time independent = stationary

- An analogous result applies to black branes.

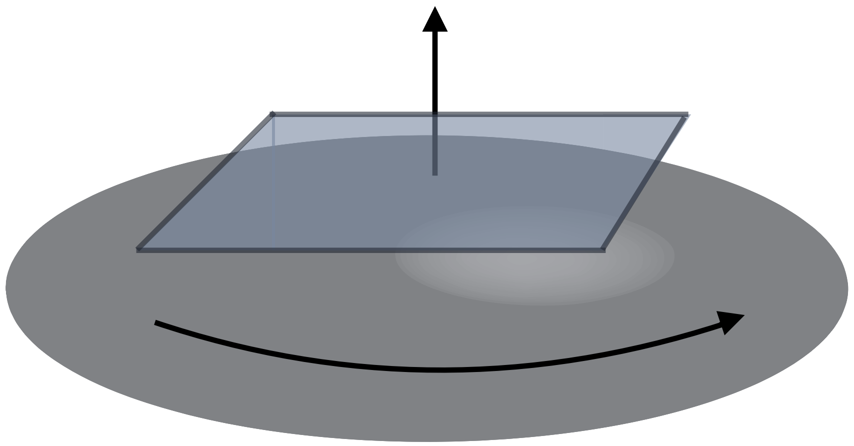


Exact solutions in higher D: Ultraspinning instability

[Emparan, Myers (2003)]

- An **ultraspinning** black hole **flattens** out near the poles.

- Locally it resembles a black brane, which is unstable…

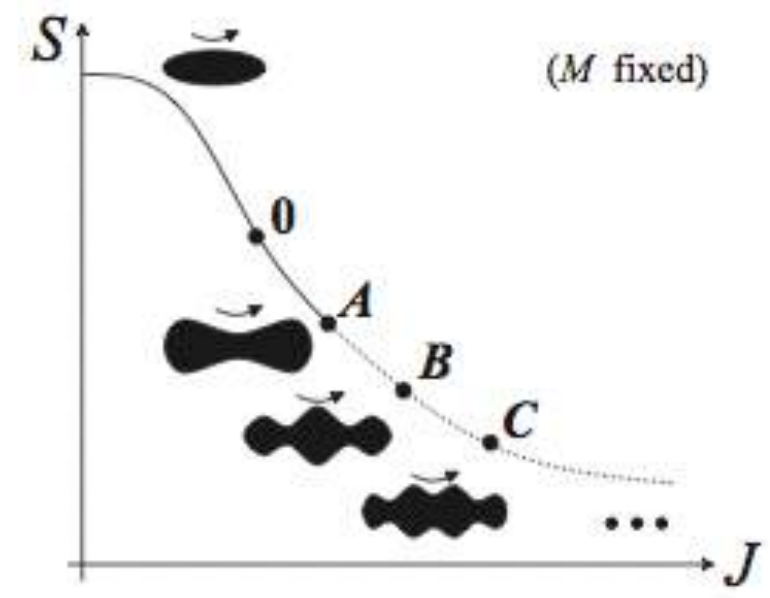


- To firmly demonstrate such an instability, resort to numerical techniques (spectral methods) for studying gravitational perturbations of higher dimensional **rotating** BHs.

Exact solutions in higher D: Ultraspinning instability

- Singly spinning MP solutions branch off — at certain spin parameters — to new **stationary (bumpy) BHs**, but only for D ≥ 6.

  [Dias, Figueras, Monteiro, Santos, Emparan (2009)]
  [Dias, Figueras, Monteiro, Santos (2010)]

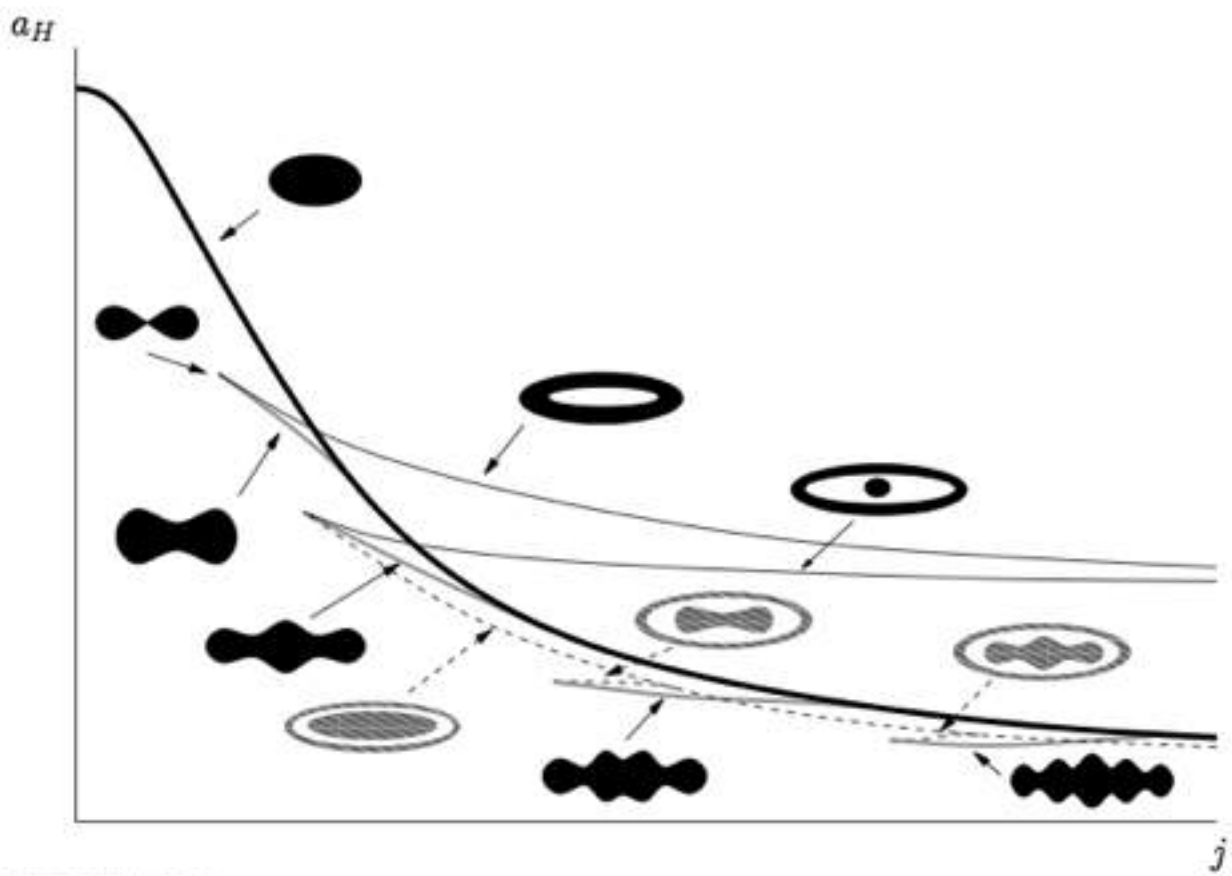


- Considering **equal angular momenta** BHs, it has been shown these are afflicted by ultraspinning instability for D ≥ 7. [Dias, Figueras, Monteiro, Reall, Santos (2010)]

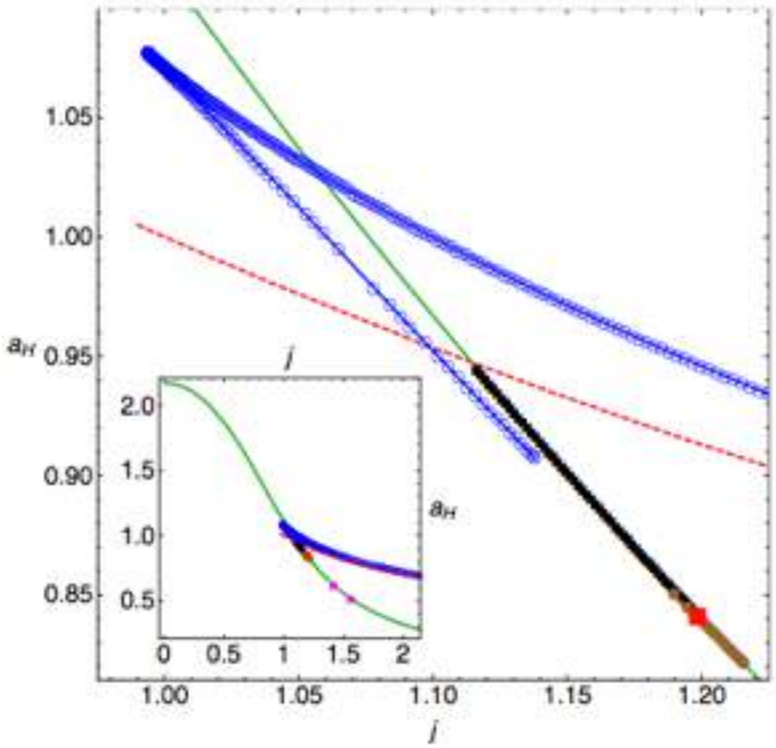
Note 1: EAM Myers-Perry in 5D proved to be stable against linear perturbations.
[Murata, Soda (2008)]

Note 2: The cohomogeneity-1 property allows to study greybody factors with relative ease.
[Jorge, Oliveira, Rocha (2015)]

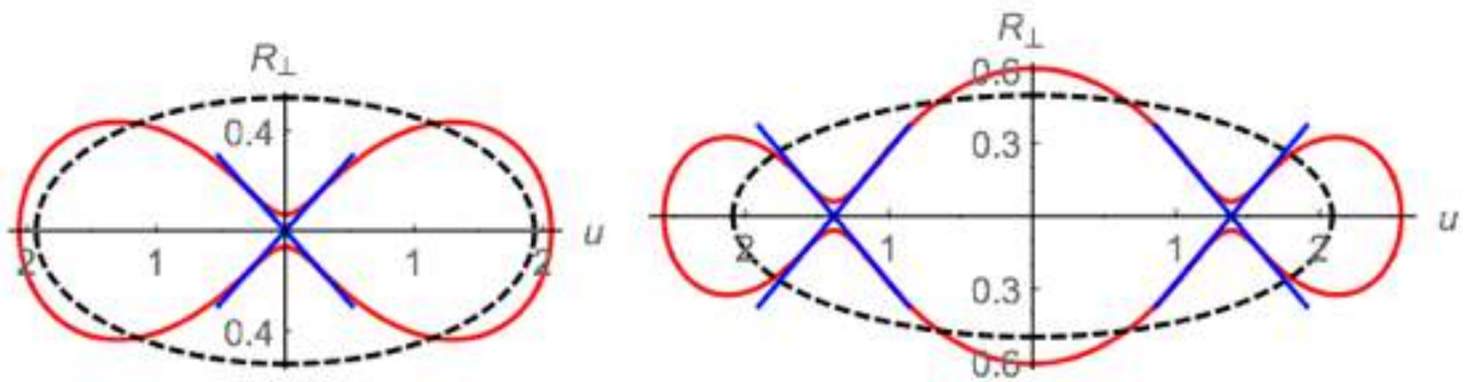
Exact solutions in higher D: Phase structure



from [Emparan, Harmark, Niarchos, Obers, Rodríguez (2007)]



[Dias, Santos, Way (2014)]



[Emparan, Figueras, Martínez (2014)]

Exact solutions in higher D: Bar-mode instability

- **Quickly** spinning BHs should also be unstable against a **non-axisymmetric** instability. [Emparan, Myers (2003)]

- This was numerically confirmed by non-linear time evolutions. [Shibata, Yoshino (2010)]
- Also by linear QNM analysis, for D=6, 7. *But not D=5.* [Dias, Hartnett, Santos (2014)]
- This bar-mode instability kicks in at **smaller rotation** than the ultraspinning instability.

Exact solutions in higher D: Instabilities of the 5D black ring

- Fat rings are unstable towards radial perturbations.

[Elvang, Emparan, Virmani (2006)]
[Figueras, Murata, Reall (2011)]

- Very thin rings must be Gregory-Laflamme unstable.
- Recently, moderately thin rings were shown to suffer from linear instabilities.

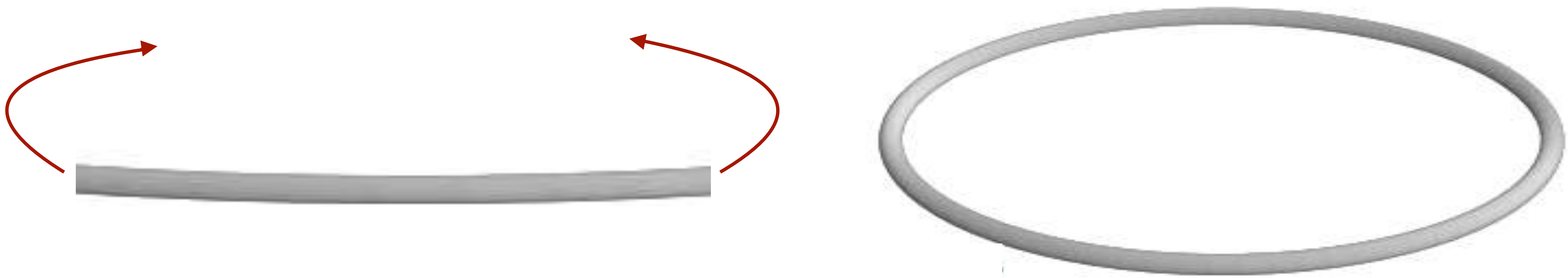
[Santos, Way (2015)]

Approximate solutions in higher D:
The blackfold approach

[Emparan, Harmark, Niarchos, Obers (2009), (2010)]

Approximate solutions in higher D: Blackfold heuristics

- Black rings can be thought of as black strings bend into a circle.



- Similarly, one can take a black brane and bend it into different shapes…

blackfold = **black** hole + mani**fold**

Approximate solutions in higher D: Separation of scales

- Approach works if

typical curvature of the background R >> radius of BH horizon r_0

$$ds^2_{(far)} = ds^2_{background} + O(r/r_0)$$

$$ds^2_{(near)} = ds^2_{Schw} + O(r/R)$$

- Solutions can be **matched** in the overlapping region $r_0 << r << R$.

This cumbersome procedure can be circumvented by using **symmetry** and **conservation** principles: $\nabla_\mu T^{\mu\nu} = 0$

Approximate solutions in higher D: Effective stress-energy tensor

- What could $T^{\mu\nu}$ possibly be?

- We are effectively **replacing the black hole** (or black string, or black brane) **by a point source** (or a line source, etc.).

  $T^{\mu\nu}$ is the stress-energy tensor of a domain wall, enclosing empty space, that originates an exterior field equal to that of the black hole (or black string, etc.).

- This can be computed using the Brown-York quasilocal stress-energy tensor.

[Brown, York (1993)]

Approximate solutions in higher D: Effective stress-energy tensor

- Take a the Schwarzschild solution in d dimensions , define $n \equiv d - 3$,

  and make a flat **black brane** in D dimensions by adding p flat directions:

$$ds^2 = -\left(1 - \frac{r_0^n}{r^n}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{r_0^n}{r^n}\right)} + r^2 d\Omega_{n+1}^2 + \sum_{i=1}^{p} (dz^i)^2$$

$$D \equiv d + p = n + p + 3$$

- This black brane can be **boosted** in the z^i directions, and we still get a solution of the Einstein eqs.

- The effective stress-energy tensor for this brane takes the form of a **perfect fluid**:

$$T^{ab} = (\rho + P) u^a u^b + P \eta^{ab} \qquad \rho = \frac{\Omega_{n+1}}{16\pi G}(n+1) r_0^n \qquad P = -\frac{\Omega_{n+1}}{16\pi G} r_0^n$$

- Consideration of slightly bent branes (with gradients in r_0 , u^a and η^{ab}) gives **dissipative corrections** to the stress-energy tensor.

Approximate solutions in higher D: Known BHs from blackfolds

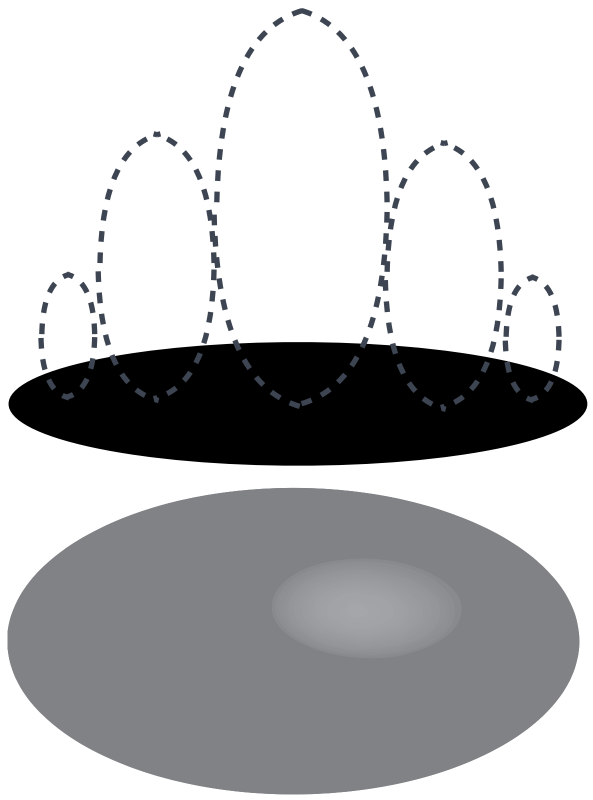
- **Note:** The separation of scales $R >> r_0$ implies that neutral blackfolds are always in an ultraspinning regime.

- **Myers-Perry as a black 2-fold (disk):**

  - At each point of the disk we put an S^{n+1} sphere.

  - For the boundary of the worldvolume to be free (no surface tension) the pressure must go to zero there.

  - Resulting blackfold has horizon topology S^{n+3}.



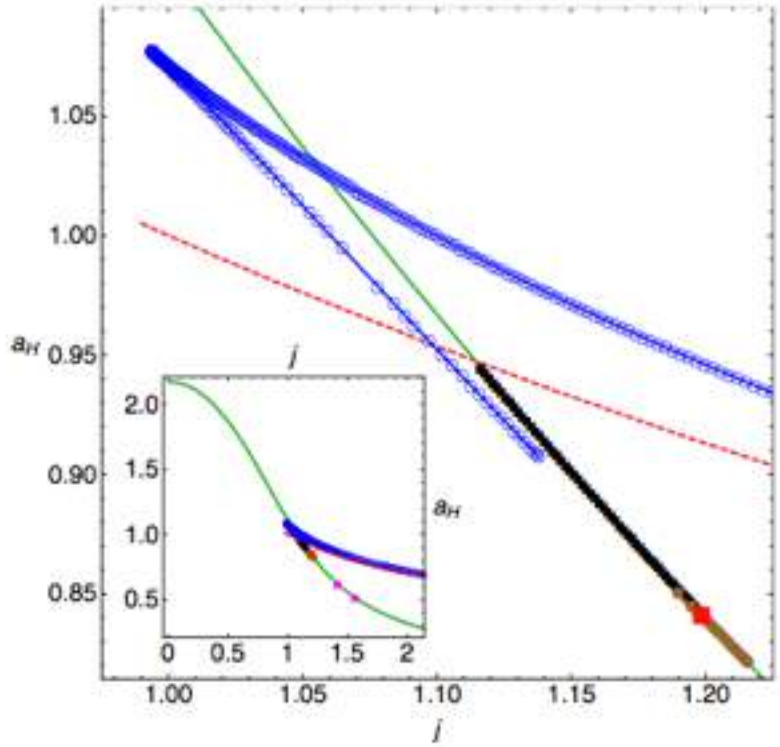
  - Expressions for mass, angular momentum and entropy precisely agree with those of the $D=n+5$ Myers-Perry BH in the ultra pinning limit.

- **Black ring as a black 1-fold (circle):**

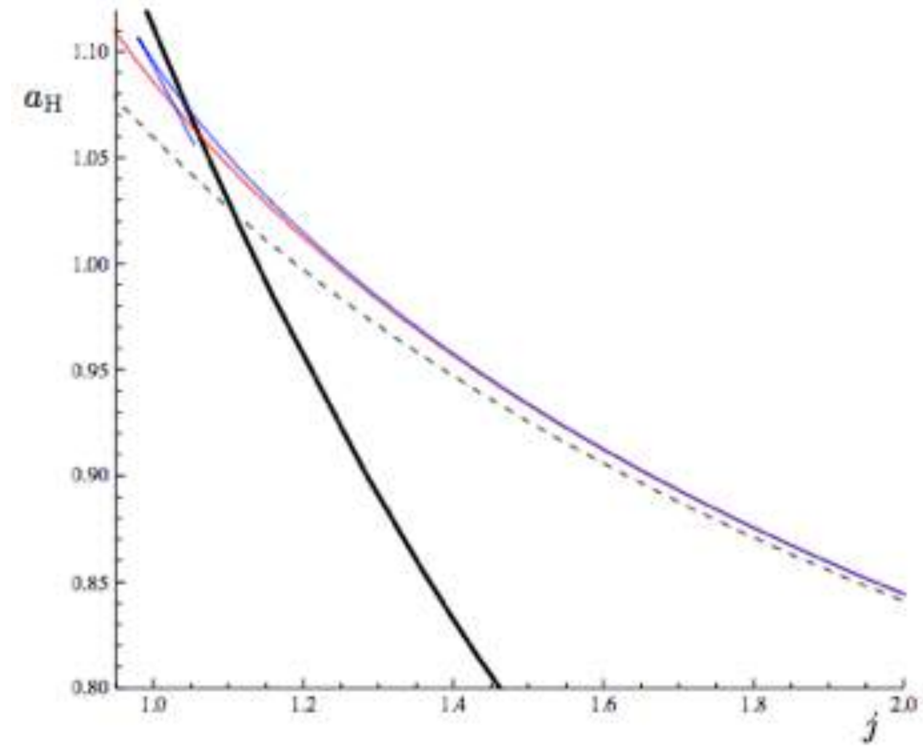
  - Similar to the blackfold disk but now the black string is actually bent…

Approximate solutions in higher D: Comparison with numerics

D=6:

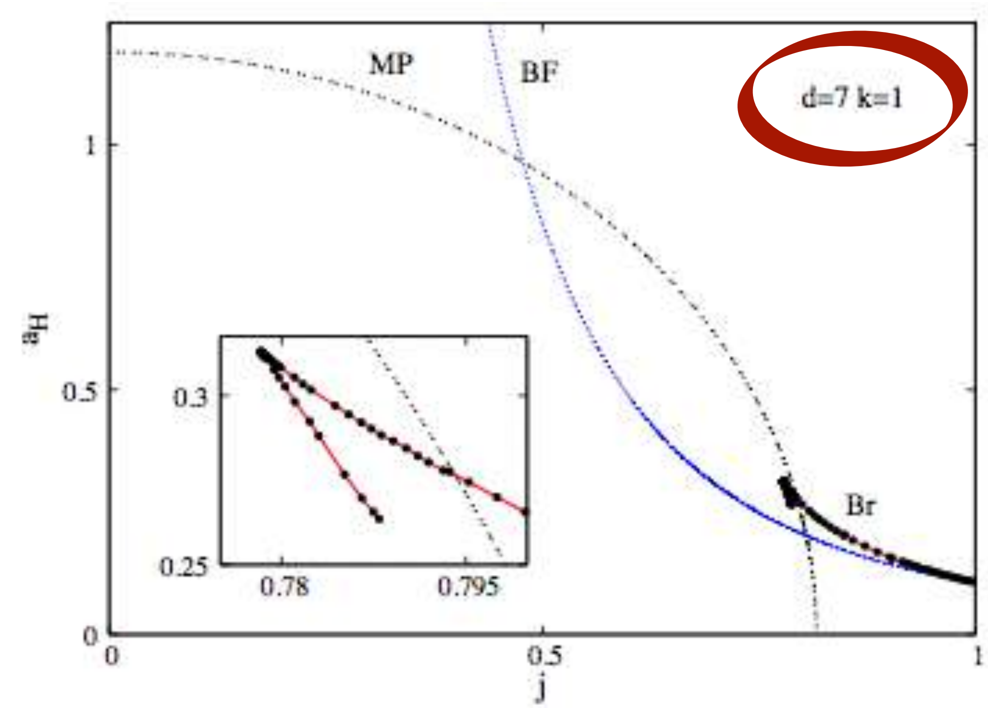


[Dias, Santos, Way (2014)]



[Armas, Harmark (2014)]

	spherical horizon	*black rings*	*black ringoids*		
	MP/'pinched'	$k=0$	$k=1$	$k=2$	$k=3$
$d=5$	S^3	$\mathbf{S}^2\times\mathbf{S}^1$			
$d=6$	S^4	$S^3\times S^1$			
$d=7$	S^5	$S^4\times S^1$	$\mathbf{S}^2\times\mathbf{S}^3$		
$d=8$	S^6	$S^5\times S^1$	$S^3\times S^3$		
$d=9$	S^7	$S^6\times S^1$	$S^4\times S^3$	$\mathbf{S}^2\times\mathbf{S}^5$	
$d=10$	S^8	$S^7\times S^1$	$S^5\times S^3$	$S^3\times S^5$	
$d=11$	S^9	$S^8\times S^1$	$S^6\times S^3$	$S^4\times S^5$	$\mathbf{S}^2\times\mathbf{S}^7$



[Kleihaus, Kunz, Radu (2014)]

Approximate solutions in higher D: Helical black rings

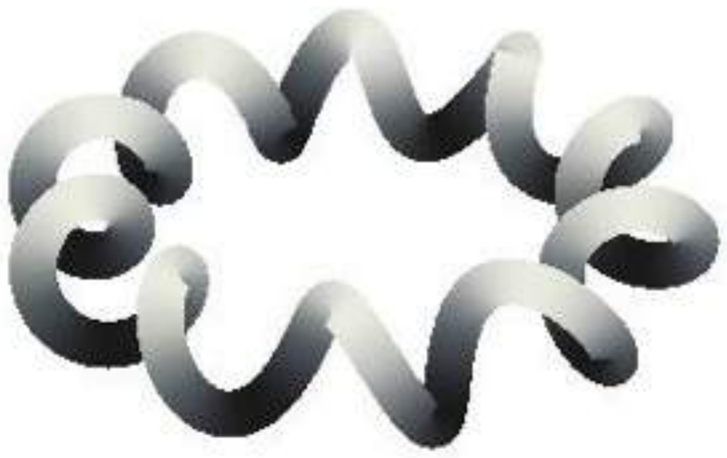
- Consider D=5.
  A stationary (compact) black 1-fold must wrap along a spatial isometry of IR^4.
  Embed it as

$$ds^2 = \sum_{i=1}^{2}(dr_i^2 + r_i^2 d\phi_i^2) \qquad r_i = R_i \qquad \phi_i = n_i\sigma \qquad \sigma \in [0, 2\pi)$$

- For the curve to close in on itself (and to avoid multiple covering) the n_i's must be integers (and coprime).

- If $n_1=n_2=1$ we just recover the circular planar black ring.
  Otherwise, one obtains a **helical** black ring.

- Helical BRs have the **largest entropy** among BFs with given mass and angular momenta.
  Moreover, they **saturate the rigidity theorem**.



Approximate solutions in higher D:
The large D limit

[Emparan, Suzuki, Tanabe (2013), (2014)]

Approximate solutions in higher D: Large D limit

- Take the Tangherlini solution in D dimensions and keep the length scale r_0 fixed as $D \to \infty$

$$ds^2 = -\left(1 - \frac{r_0^{D-3}}{r^{D-3}}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{r_0^{D-3}}{r^{D-3}}\right)} + r^2 d\Omega_{D-2}^2$$

- In this limit the area of the unitary sphere vanishes exponentially

$$\Omega_{D-2} = \frac{2\pi^{(D-1)/2}}{\Gamma\left(\frac{D-1}{2}\right)} \to \frac{D}{\sqrt{2}\pi}\left(\frac{2\pi e}{D}\right)^{D/2} \to 0 \qquad \Longrightarrow \quad \textbf{zero cross section}$$

- Moreover, gravitational potential $\left(\frac{r_0}{r}\right)^{D-3}$ vanishes exponentially, away from r_0 .

$\Longrightarrow$ **spacetime outside r_0 is flat**

$\Longrightarrow$ **no attraction**

Approximate solutions in higher D: Separation of scales (again)

- Besides the lengthscale r_0 we also have a (widely separated) scale

$$\text{surface gravity on the horizon} = \frac{D-3}{2r_0} \sim \frac{D}{r_0}$$

- Once again we can tackle problems in these spacetimes with **matched asymptotic expansions**.

- The study of black hole scattering reveals that waves with low frequencies ($\omega < \frac{D}{r_0}$) are **strongly reflected** by the BH (and high frequencies are almost perfectly absorbed).

$\Longrightarrow$ **BH is just a hole cut out in flat space with reflecting boundary conditions**

Approximate solutions in higher D: Universal quasinormal modes

- Consider a massless scalar field, with (rescaled) radial profile $\phi(r)$.
- The Klein-Gordon eq. can be written in the conventional form

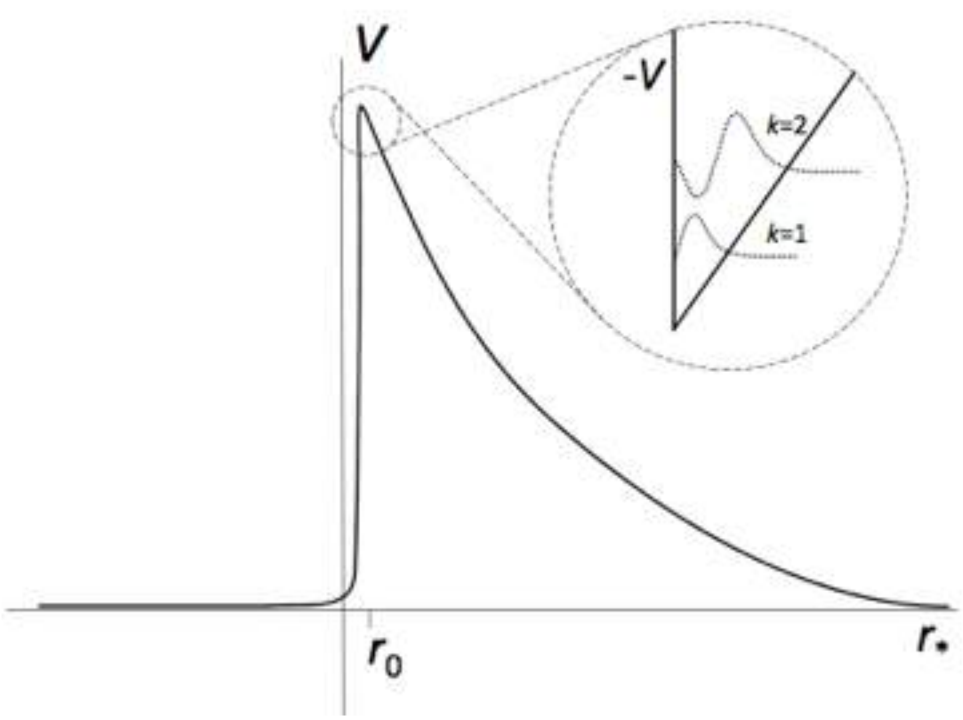
$$\frac{d^2\phi}{dr_*^2} + (\omega^2 - V(r_*))\phi = 0$$

- The potential vanishes at the horizon and at infinity, with a maximum somewhere in between determined by the angular momentum quantum number.

  In the large D limit it simplifies: $$V(r_*) \to \frac{D^2\varpi_\ell^2}{r_*^2}\Theta(r_* - r_0) \qquad \varpi_\ell \equiv \frac{1}{2} + \frac{\ell}{D}$$

- The least damped QNMs can be equivalently computed (via analytic continuation) by the lowest bound states in the inverted potential.

$\Longrightarrow$ **Universal set of QNMs for static, non-extremal, asymptotically flat BHs**

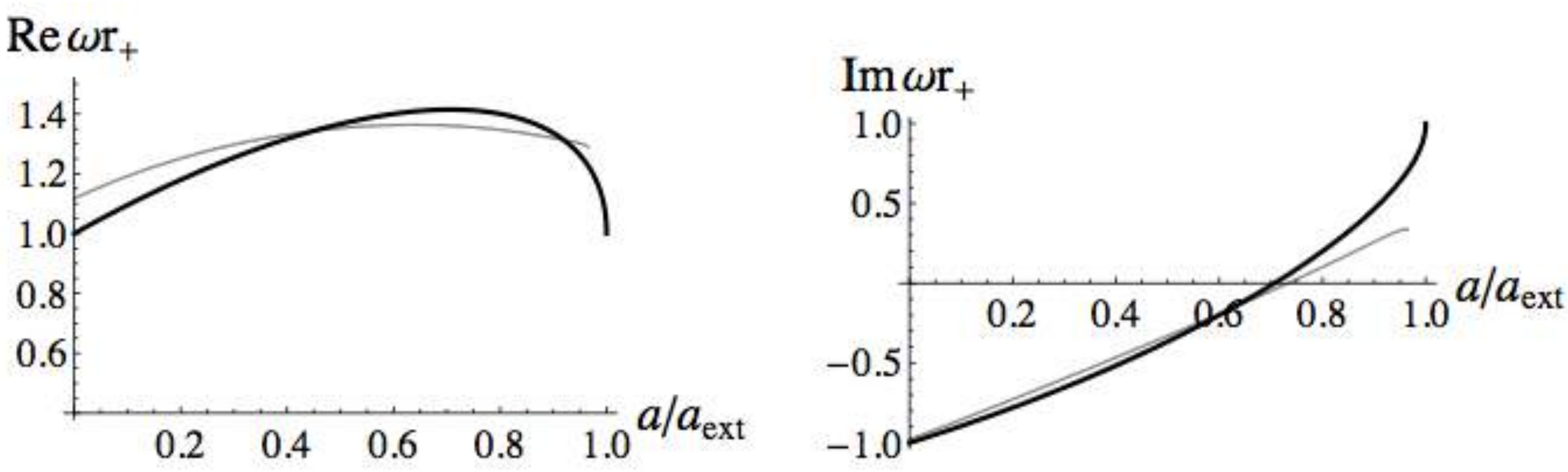


Approximate solutions in higher D: Comparing with numerics

- Good agreement with numerics, obtained for QNMs of Equal Angular Momenta BHs.

[Emparan, Suzuki, Tanabe (2014)]
[Hartnett, Santos (2013)]
[Dias, Hartnett, Santos (2014)]

$$D = 15 \qquad (\ell, m) = (2, 2)$$



from [Emparan, Suzuki, Tanabe (2014)]

Epilogue

- Gravity in higher dimensions is **much richer** than in 4D.

Not noticing it would be like navigating to India and not discovering Brazil.