Black holes in higher dimensional spacetimes

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The plan

- Motivation
- 4D black holes (solutions, uniqueness, stability)
- Exact solutions in higher D: Solutions and generating techniques
- Exact solutions in higher D: Non-uniqueness

Today's lecture

- Exact solutions in higher D: Linear instabilities
- Approximate solutions in higher D: The blackfold approach
- Approximate solutions in higher D: The large D limit

Next lecture

Part I

Disclaimer

- These lectures will concern (mostly) **Einstein's equations** in **vacuum**, i.e., pure GR formulated in D spacetime dimensions:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 0$$

- Occasionally, we may consider gravity coupled to other fields or turn on a cosmological constant.

- *Refer to* **Brito**'*s lectures for astrophysical implications of the presence of matter in strong gravitational fields.*

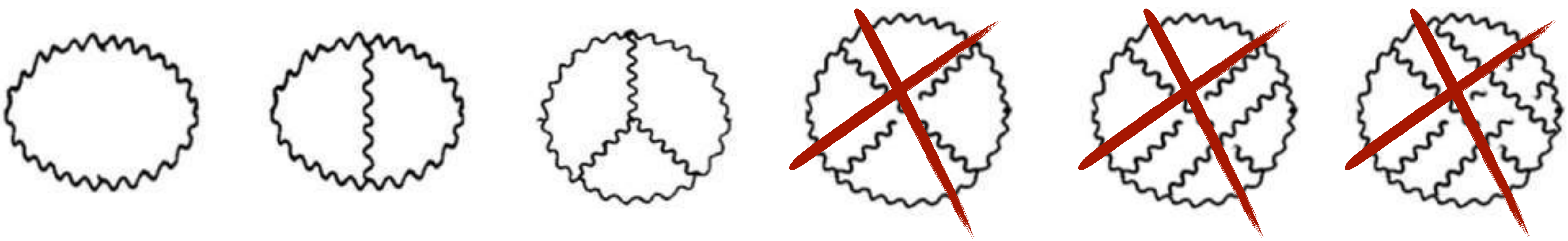
- *Refer to* **Vitagliano**'*s lectures for alternative theories of gravity.*

Motivation: Why D>4?

- D is the only available parameter in the vacuum Einstein equations.

- By considering $D \neq 4$ we can gain understanding about GR.

Example: Yang-Mills theory with SU(N) gauge group simplifies when $N \to \infty$.



Example: In certain cases, more symmetries available dramatically simplifies study of **rotating** *BHs.*

Motivation: Why D>4?

- **Extra dimensions are required** by several modern promising proposals:

  - AdS/CFT correspondence

  - String theory / M-theory

  - Braneworld models

  - TeV scale gravity

4D black holes

4D black holes: Basics

- A **black hole** spacetime is a geometry that possesses a region from which light (null geodesics) cannot be emanated to infinity. The (hyper)surface that encloses this domain is called the **event horizon**.

- Typically there will be **curvature singularities** hidden behind the horizon. This is acceptable as long as the spacetime 'visible' to a distant observer is regular.

- *Note:* There are no asymptotically flat BHs in 3D.
  (Presence of an apparent horizon requires a negative cosmological constant.) [Ida (2000)]

4D black holes: Schwarzschild (1916)

- The simplest BH solution (spherically symmetric):

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d{\Omega_2}^2$$

- describes a **non-rotating** black hole
- has a curvature singularity at $r = 0$
- has an event horizon at $r = 2GM$

- Birkhoff's theorem (1923) guarantees this is the **unique** spherically symmetric solution of the vacuum Einstein equations.

  Note: This excludes **time-dependent** *spherically symmetric solutions.*

4D black holes: Kerr (1963)

- An axisymmetric & stationary (possessing a timelike Killing vector) BH solution:

$$ds^2 = -\left(1 - \frac{2GM\,r}{\rho^2}\right)dt^2 - \frac{4GM\,a\,r\sin^2\theta}{\rho^2}dt\,d\phi + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]d\phi^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

$$\Delta \equiv r^2 - 2GM\,r + a^2 \qquad\qquad \rho^2 \equiv r^2 + a^2\cos^2\theta$$

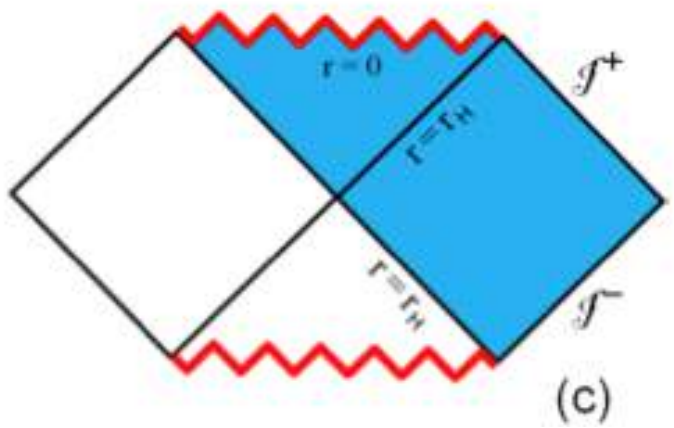
- **rotating** generalization of Schwarzschild
- is parametrized by mass M and angular momentum $J=Ma$
- has a ring-like singularity at $\rho^2 = 0$
- has horizons wherever $\Delta = 0$ → $r_\pm = GM \pm \sqrt{G^2M^2 - a^2}$

Naked singularity if $a > M$ (set $G = 1$)

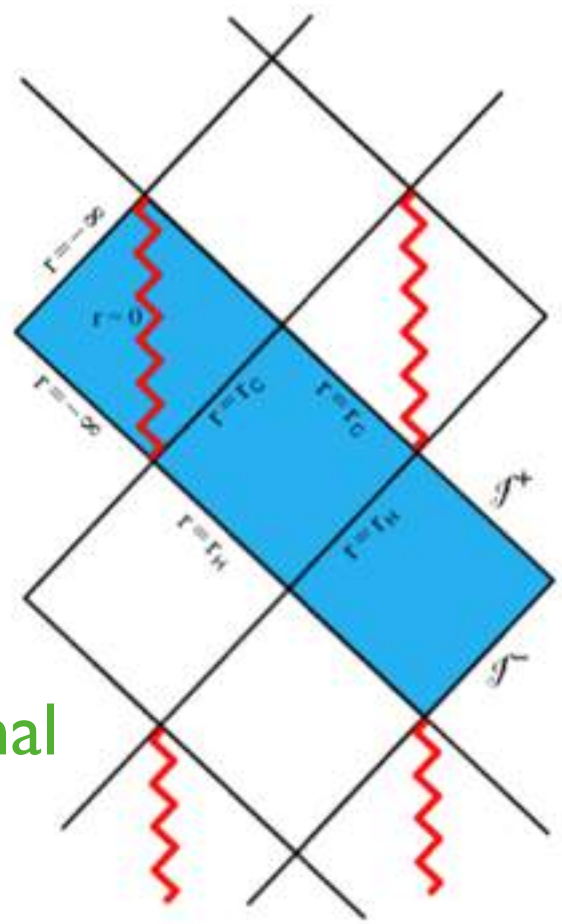
4D black holes: Carter-Penrose diagrams

- Causal structure is conveniently encoded in Carter-Penrose diagrams.

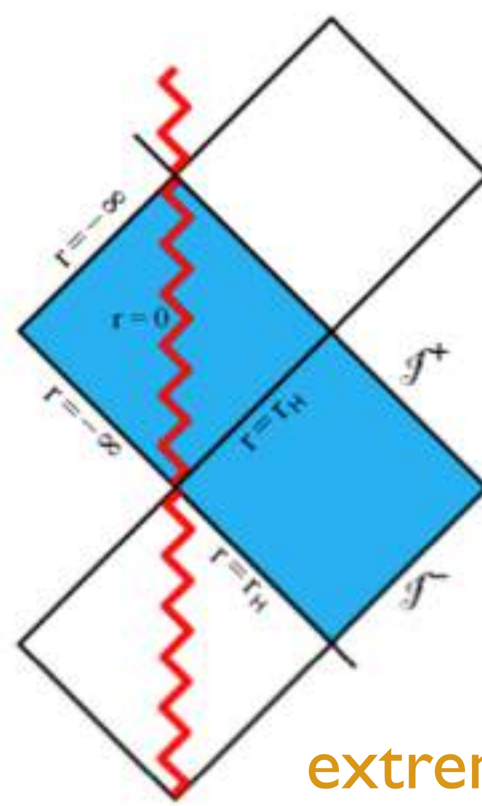
Schwarzschild:



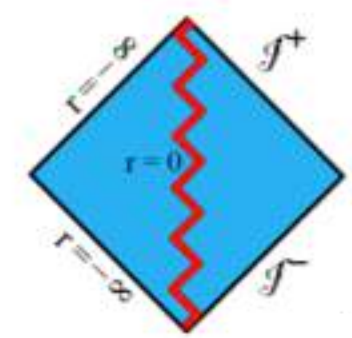
Kerr:



under-extremal



extremal



over-extremal

from [R. Myers (2011)]

4D black holes: Kerr-Newman (1965)

- The Kerr-Newman metric can be obtained from the Kerr solution by a simple replacement:

$$2GMr \to 2GMr - GQ^2$$

- **charged** generalization of Kerr
- is parametrized by mass M, angular momentum $J=Ma$ and charge Q
- same causal structure as Kerr

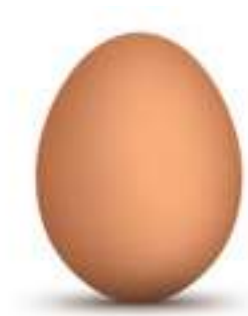
4D black holes: Uniqueness

- A **static**, asymptotically flat vacuum spacetime, non-singular on and outside an event horizon, must be Schwarzschild. [Israel (1967)] [Bunting, Masood-Ul-Alam (1987)]

- Similarly, uniqueness of Kerr and Kerr-Newman as **stationary**, asymptotically flat vacuum and electrovacuum spacetimes, respectively, has been proven. [Carter (1971)] [Robinson (1974)]

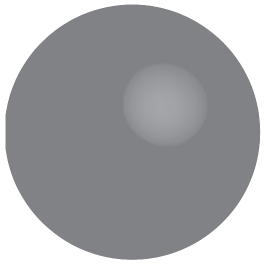
- These results go under the name of **"no hair theorems"** because they imply that the most general vacuum stationary BH is parametrized by only **2 parameters**.
  They are almost bald: not many possibilities for hair-styling.

  Note: These theorems assume non-degenerate horizon and analyticity of spacetime.




4D black holes: Topology and Rigidity

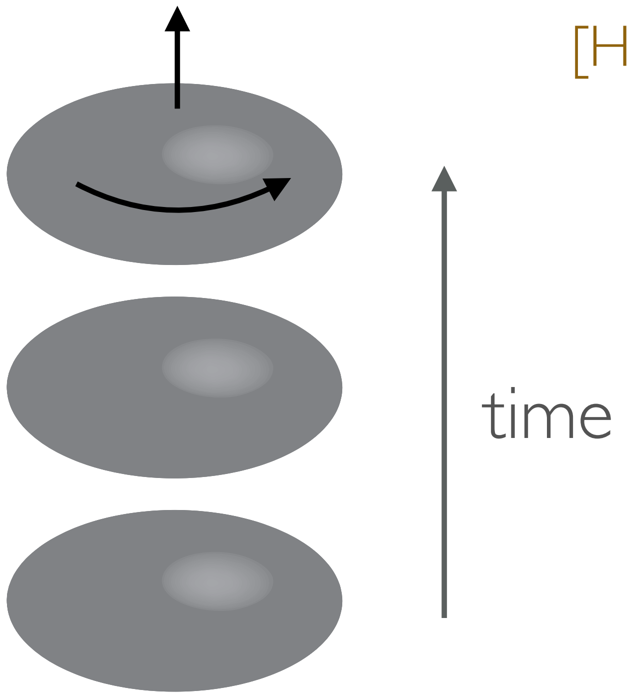
- Hawking's topology theorem asserts that cross sections of the event horizon of a stationary BH (*obeying Dominant Energy Condition*) are **spherical**. [Hawking (1972)]



Note: The generalization to D>4 only requires the event horizon to be a manifold of positive Tamabe type. (This allows topology $S^2 x S^1$*)* [Galloway, Schoen (2006)]

- The rigidity theorem for stationary asymptotically flat solutions of the Einstein-Maxwell equations guarantees that [Hawking (1972)]

stationarity $\Longrightarrow$ **axisymmetry**



Note: This result is also valid in D>4.

[Hollands, Ishibashi, Wald (2006)]

4D black holes: Stability

- Several notions of stability:

mode stability < **linear stability** < **non-linear stability**

see [Berti, Cardoso, Starinets (2009)]

- Mode stability of Schwarzschild proved long ago. [Regge-Wheeler (1957)] [Zerilli (1970)] [Moncrief (1974)]
- Mode stability of Kerr also proved. [Whiting (1989)]
- Strong evidence supporting mode stability of Kerr-Newman. [Zilhão et al. (2014)] [Dias, Godazgar, Santos (2015)]

- Schwarzschild is linearly stable. [Dafermos-Holzegel-Rodnianski (2013)] [Dotti (2014)]
- ***Linear stability of Kerr is still an open issue!***

- Compare: *Non-linear stability of Minkowski was proved in a 400+ pages monograph.* [Christodoulou-Klainerman (1994)]

Exact solutions in higher D:

Solutions and generating techniques

Exact solutions in higher D: Novelties

	D = 4	D > 4
# dof	2	$\frac{D(D-3)}{2}$
# rotation planes	1	$\lfloor \frac{D-1}{2} \rfloor$
Newtonian potential	$-\frac{GM}{r}$	$-\frac{GM}{r^{D-3}}$
centrifugal potential	$\frac{J^2}{M^2 r^2}$	

Exact solutions in higher D: Constructing simple solutions

- The **generalization of Schwarzschild** to higher dimensions is straightforward:

[Tangherlini (1963)]

$$ds^2 = -\left(1 - \frac{\mu}{r^{D-3}}\right) dt^2 + \left(1 - \frac{\mu}{r^{D-3}}\right)^{-1} dr^2 + r^2 d\Omega_{D-2}{}^2$$

- **Adding flat directions** we still get a solution of the vacuum Einstein equations.

→ black strings & black branes

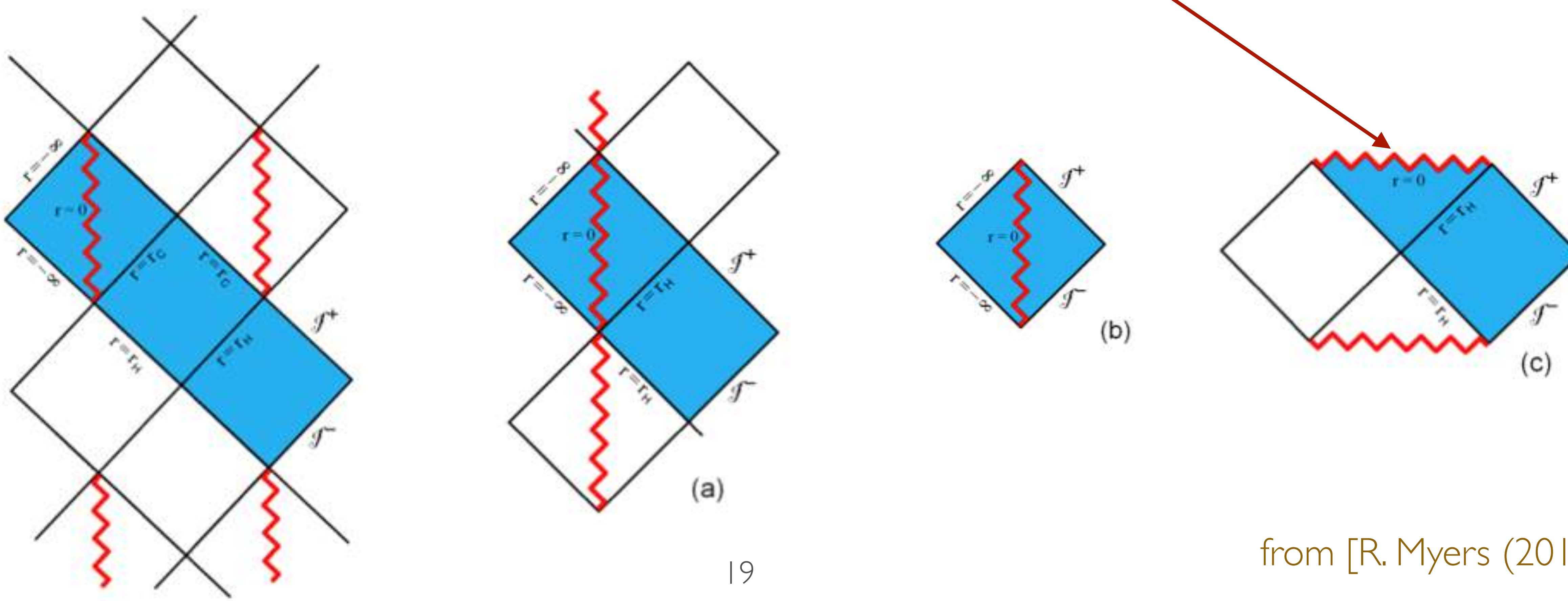
Exact solutions in higher D: Not so simple solutions

- The **generalization of Kerr** to higher dimensions is *not at all* straightforward. The resulting solution is known as the **Myers-Perry** black hole. [Myers-Perry (1986)]

- The metric (too complicated to show here) describes a BH in any dimension D>4, **rotating in all possible rotation planes**.

➡ parametrized by mass M, and $\lfloor \frac{D-1}{2} \rfloor$ angular momenta $J_i \sim M a_i$

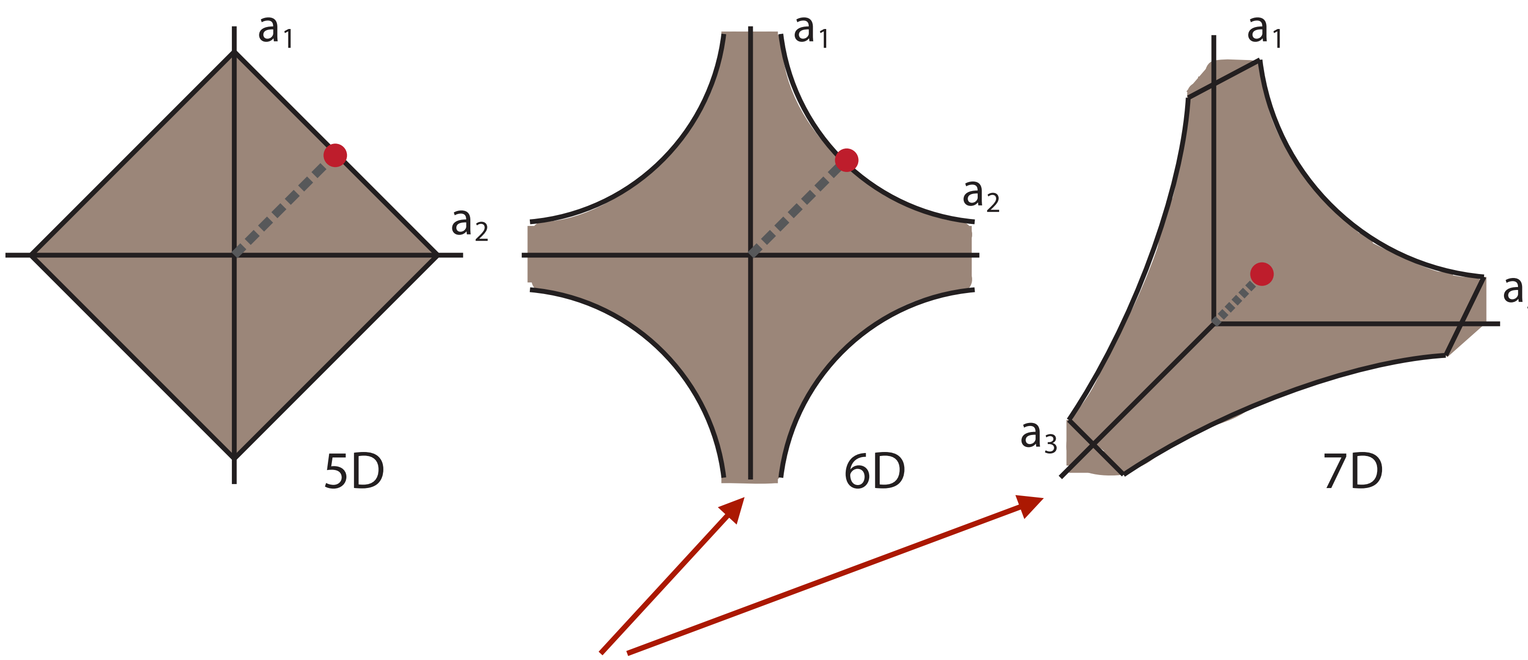
➡ same causal structure as Kerr, except when at least one $a_i=0$ (for even D, two for odd D)



from [R. Myers (2011)]

Exact solutions in higher D: Myers-Perry

- Parameter space is larger $\Longrightarrow$ conditions for $\exists$ horizon are more complicated



Note 1: For D>5, BH can be **ultraspinning** *in some directions.*

Note 2: When all spins are identical there is **enhanced symmetry** *and metric depends on a* **single** *'radial' coordinate (cohomogeneity-1 spacetime).*

- These solutions were obtained by taking a Kerr-Schild ansatz: $g_{\mu\nu} = \eta_{\mu\nu} + H(x^\lambda)k_\mu k_\nu$

  This reduces the Einstein equations to a set of **linear** equations!

Exact solutions in higher D: Solution generating techniques

"It often happens when one is trying to solve an equation that an **algorithm** *will exist for constructing* **new solutions from a given solution**.*"*

[Wald in "General Relativity" (1984)]

- Simplest sol. gen. tech. : add flat directions to a known solution

- Another simple sol. gen. tech. of great utility in Kaluza-Klein theories:

uplift + boost + reduce

Exact solutions in higher D: Solution generating techniques

- Most sol. gen. techs. rely on **using available symmetries** (which may be hidden).

  - **Ehlers transformation:** assumes one Killing vector [Ehlers (1957)]
  - **Geroch transformation:** 2 commuting Killing vectors [Geroch (1971)]

Note: Generated solutions may not be physically relevant.

  - **Kinnersley-Chitre:** subgroup preserving asymptotic flatness [Kinnersley, Chitre (1978)]

can be used to generate Kerr

  - **Inverse scattering method:** D-2 commuting Killing vectors [Belinskii, Zakharov (1978)]
  - **Bäcklund transformation:** D-2 commuting Killing vectors [Harrison (1978)] [Neugebauer (1979)]

algebraic procedure!

Exact solutions in higher D: Stationary and axisymmetric ansatz

- Consider **stationary**, **axisymmetric** solutions of Einstein eqs. in **vacuum**.

- Assume D − 2 commuting Killing vector fields, $\partial/\partial x^i$.
  Then metric can be written in canonical form:

$$ds^2 = \sum_{i,j=0}^{D-3} G_{ij}(\rho, z)dx^i dx^j + e^{2\nu(\rho,z)} \left[d\rho^2 + dz^2\right] , \qquad \det G = -\rho^2$$

- Metric only depends on coordinates *(ρ, z)* and has block diagonal form:

Killing sector →

$$g_{\mu\nu} = \left(\begin{array}{c|cc} G_{ij} & \multicolumn{2}{c}{0} \\ \hline 0 & e^{2\nu} & 0 \\ & 0 & e^{2\nu} \end{array}\right)$$

← conformal factor

Exact solutions in higher D: Static and axisymmetric case

- The vacuum Einstein eqs. reduce to a **decoupled** set of equations for the Killing sector G_{ij} and for the conformal factor $e^{2\nu}$.

- Moreover, $e^{2\nu}$ is automatically determined once a solution for G_{ij} is found.

- Obtaining **static** (diagonal) solutions is simple. Writing

$$G = \mathrm{diag}\{-e^{2U_0}, e^{2U_1}, e^{2U_2}, \ldots\}$$

the problem reduces to **finding D−2 solutions,** $U_i\,(\rho, z)$, **of the Laplace equation** in an auxiliary (cylindrically symmetric) 3d flat space:

$$\nabla^2_{3d} U_i = 0$$

[Weyl (1917)]
[Emparan, Reall (2002)]

Exact solutions in higher D: Rod structure

- The potentials U_i are entirely specified by the location of **zero-thickness rods** along the axis of the auxiliary space.

- The constraint $\det G = -\rho^2$ translates into $\sum_i U_i = \log \rho$.

  Meaning: **sources must add up to give an infinite rod**.

  Note: This simple picture also holds in the **stationary** *case.*

- Upshot: **Vacuum solutions of the Einstein equations with D−2 orthogonal commuting KVFs are fully determined by rod-like sources, only subject to the above constraint.**

[Emparan, Reall (2002)]

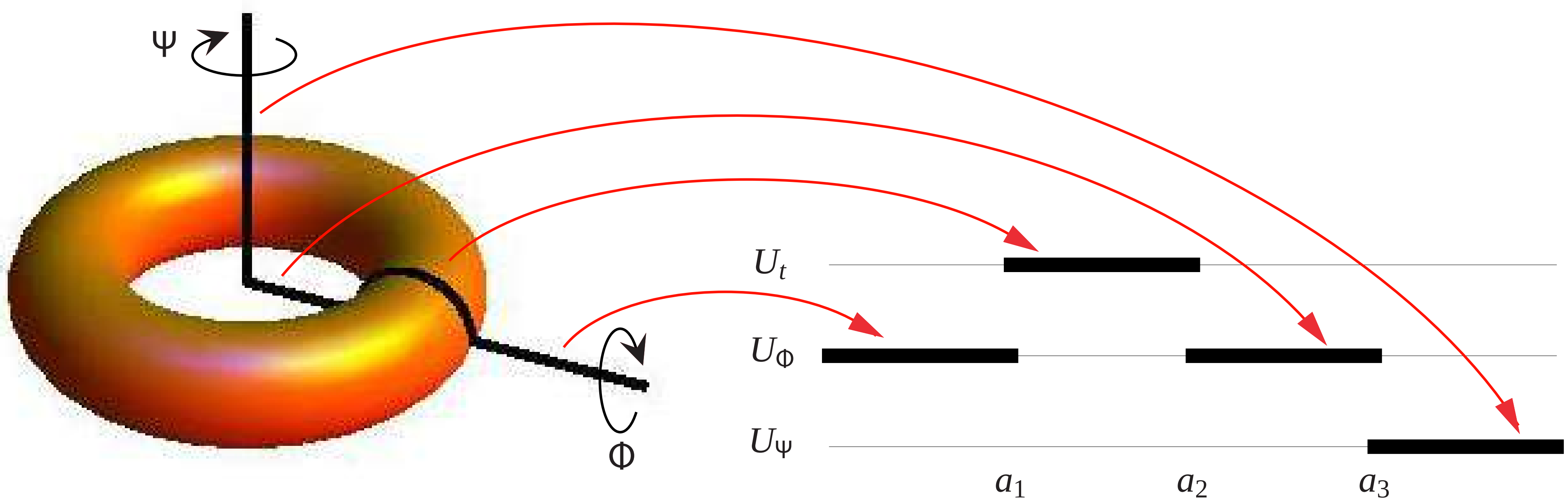
Note: The class of metrics considered can be **asymptotically flat only if D ≤ 5**.
If D > 5 there are necessarily KK directions.

Exact solutions in higher D: Static black ring [Emparan, Reall (2002)]

- Some thumb rules:

finite timelike rods ⟷ event horizons

semi-infinite spacelike rods ⟷ axes of rotation

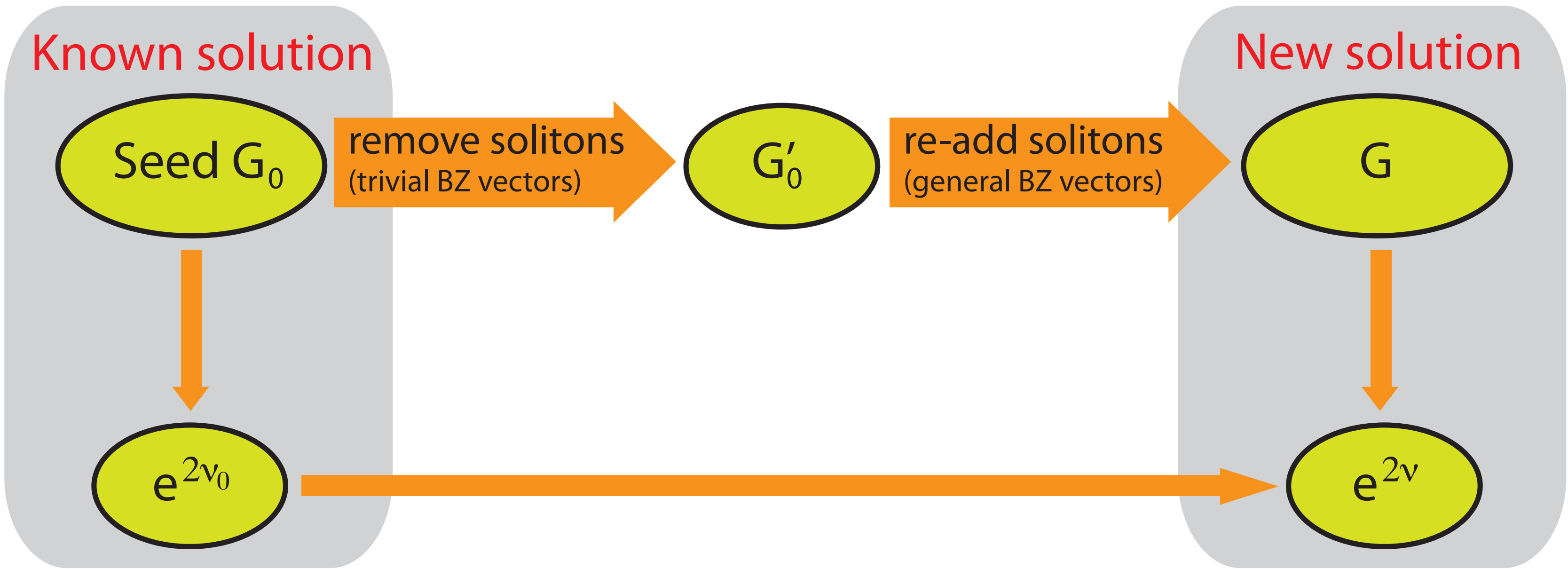


Note: The **static** *black ring is not regular. A* **conical singularity disk** *bounded by the ring provides the necessary force to balance the system.*

Exact solutions in higher D: Inverse scattering method

- The Belinskii-Zakharov approach consists in replacing the original (non-linear) equation for G_{ij} by an **equivalent system of linear equations** (Lax pair).

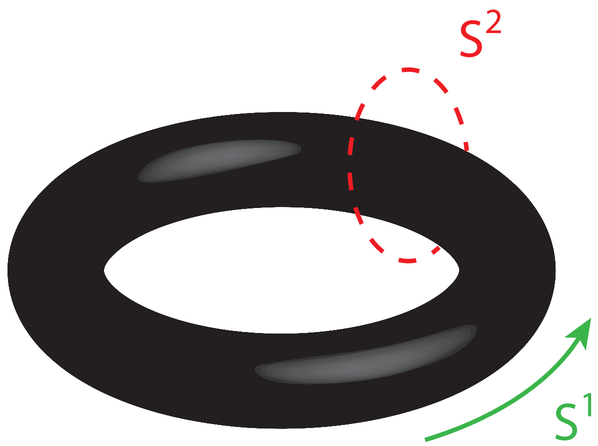
- If the seed is diagonal and the 'dressing' procedure is restricted to the so-called class of **solitonic transformations**, then the whole scheme is **purely algebraic**.



Note 1: The seed solution need not be regular.

Note 2: Might need to impose some constraints to generate a regular solution.

Exact solutions in higher D: Rotating black ring [Emparan, Reall (2002)]

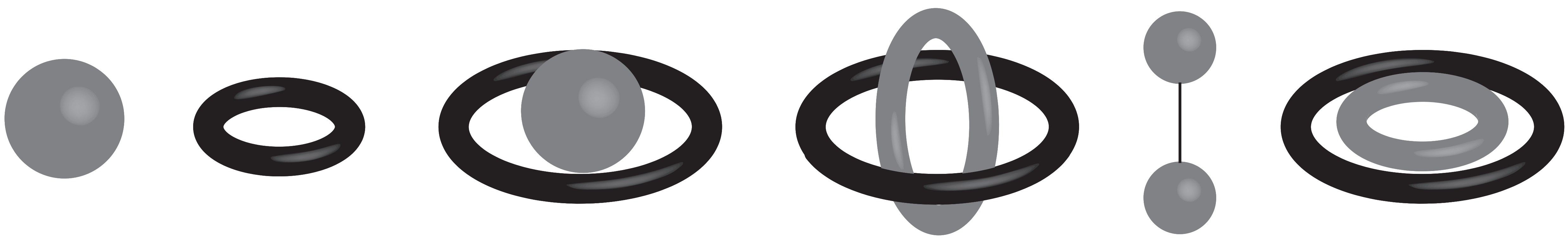


- Originally discovered by a Wick rotation of a C-metric and some educated guesswork (a.k.a. black magic).
- However, the black ring can be systematically generated using the ISM.

  [Tomizawa, Nozawa (2006)]
- In essence*, one takes the static black ring solution and performs a solitonic transformation that adds further parameters to the solution, thus obtaining a **rotating generalization**.
- Can add rotation along S^1, along S^2, add more black holes…

*In practice it's a bit more involved.

Exact solutions in higher D: The 5D black hole bestiary



[Pomeransky (2006)], [Tomizawa et al. (2006)], [Tomizawa, Nozawa (2006)], [Pomeransky, Sen'kov (2006)], [Elvang, Figueras (2007)], [Elvang & Rodriguez (2008)], [Herdeiro et al. (2008)], [Evslin, Krishnan (2009)]

- In certain Einstein-Maxwell-dilaton theories the ISM can also be applied to construct (magnetic) **dipole black rings** and **electrically charged black rings**.

  [Rocha, Rodriguez, Virmani (2011)] [Rocha, Rodriguez, Varela, Virmani (2013)]

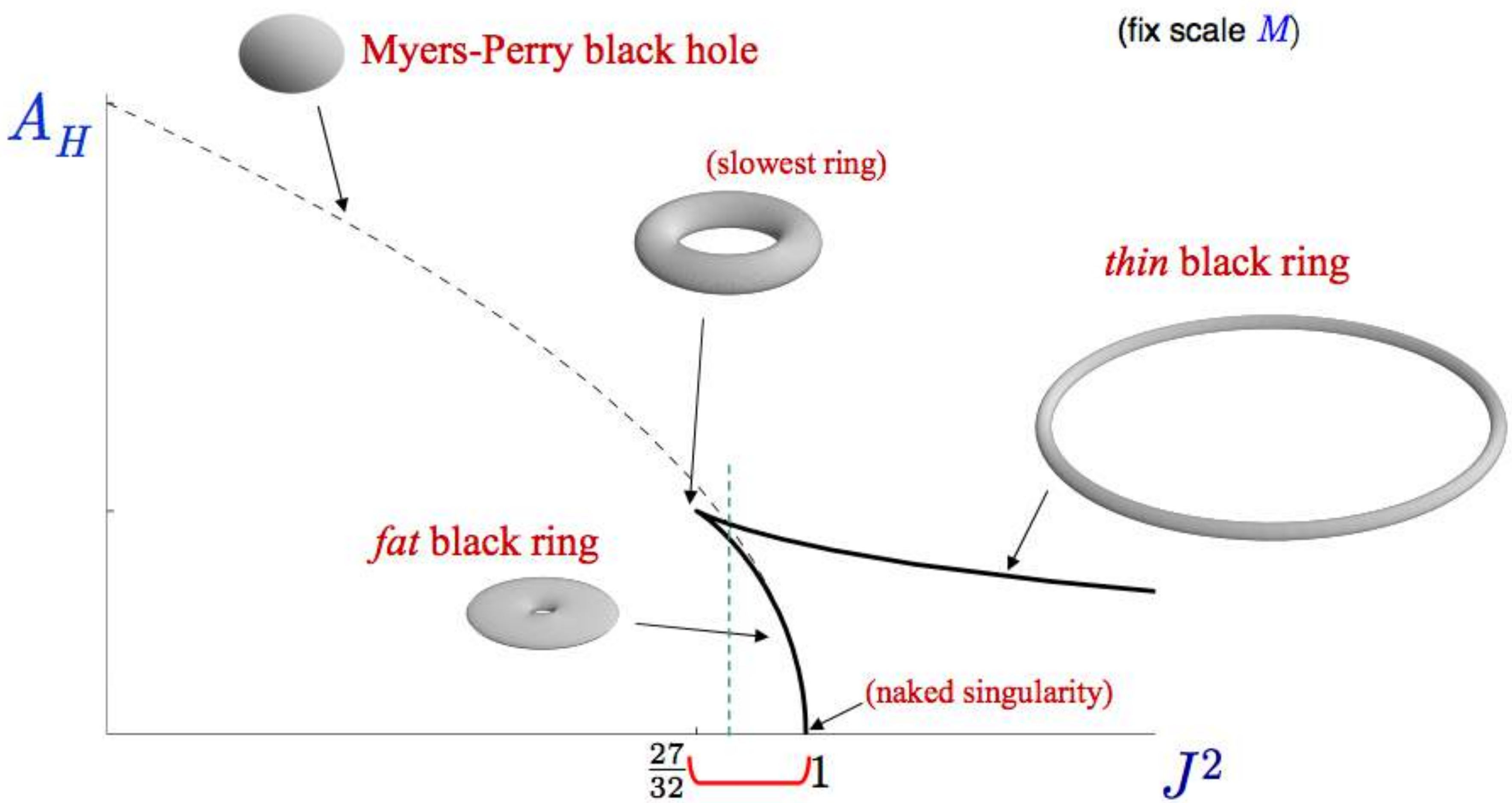
- A plethora of black objects is expected to populate D ≥ 6.

Exact solutions in higher D:

Non-uniqueness

Exact solutions in higher D: Non-uniqueness

- In 5D, for a certain range of parameters 3 BHs Coexist with the same charges:



Phase diagram for singly rotating BH solutions in 5D from [R. Emparan (2009)]