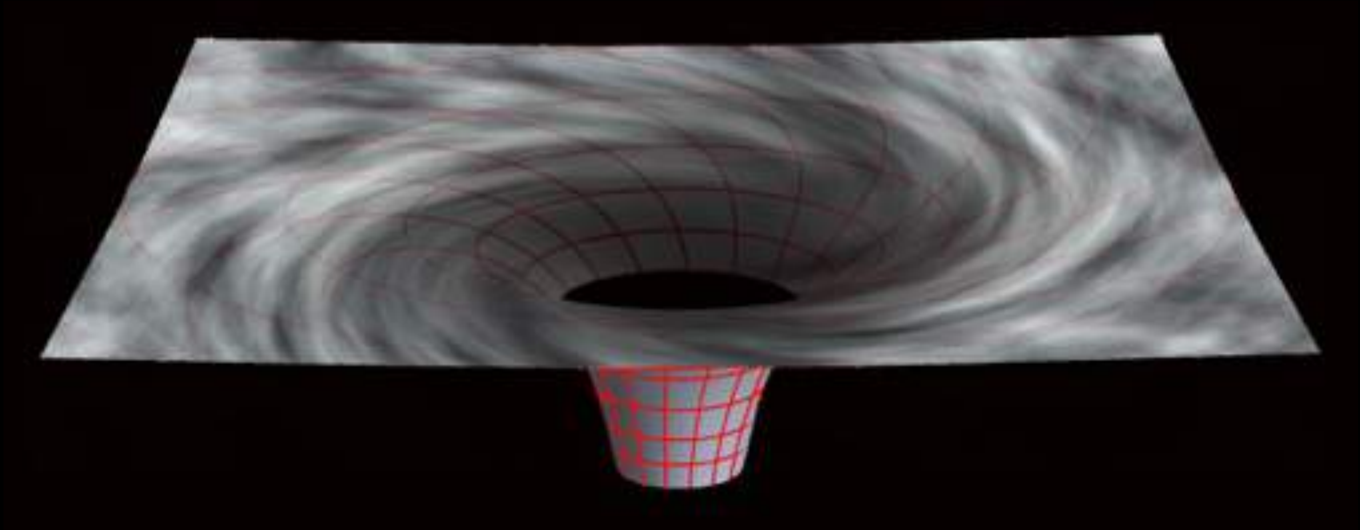
Rotating shells, non-spherical gravitational collapse and cosmic censorship

Jorge V. Rocha (Centra-IST, U.Lisboa)



- **JVR** Int. J. Mod. Phys. D24, 1542002 (2015) **[1501.06724 [gr-qc]]**
- **T. Delsate**, **JVR** and **R. Santarelli** Phys. Rev. D89, 121501(R) (2014) **[1405.1433 [gr-qc]]**
- **JVR, R. Santarelli**, and **T. Delsate** Phys. Rev. D89, 104006 (2014) **[1402.4161 [gr-qc]]**

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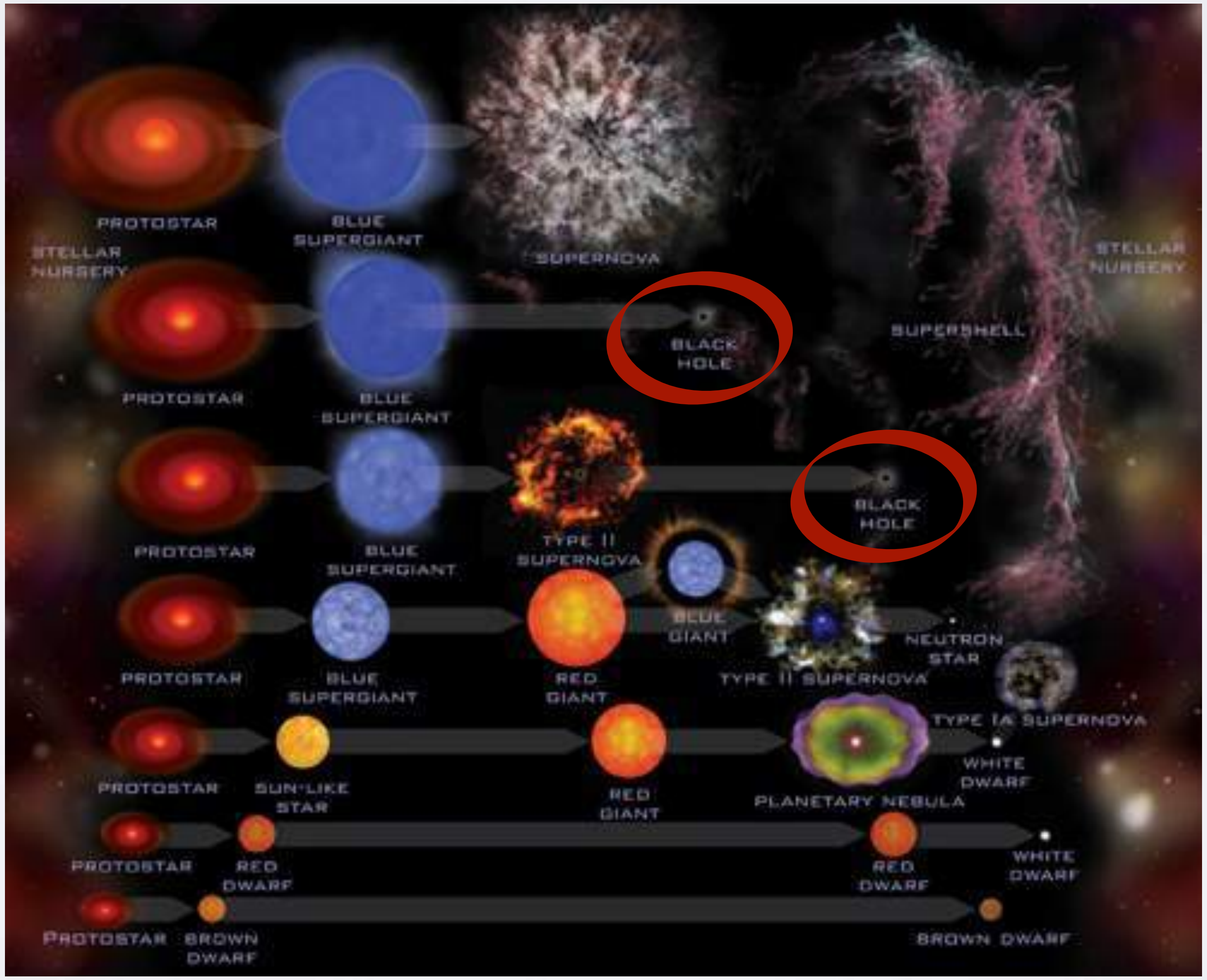

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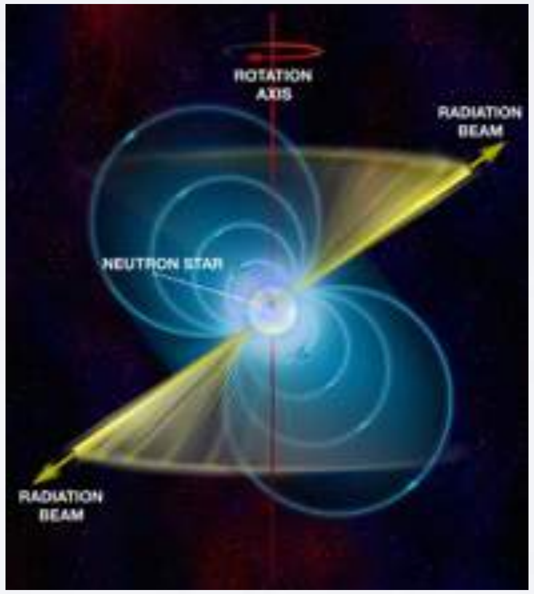
- BHs are theoretically predicted as the endpoint of gravitational collapse of sufficiently massive stars.

- The vast majority of celestial objects are rotating. Black holes are no exception.

ESO / J. Pérez



ESO

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2. known 'violations' of the cosmic censorship conjecture (CCC) occur in non-rotating — thus non-generic — settings;
3. rotation introduces instabilities (e.g. superradiance).

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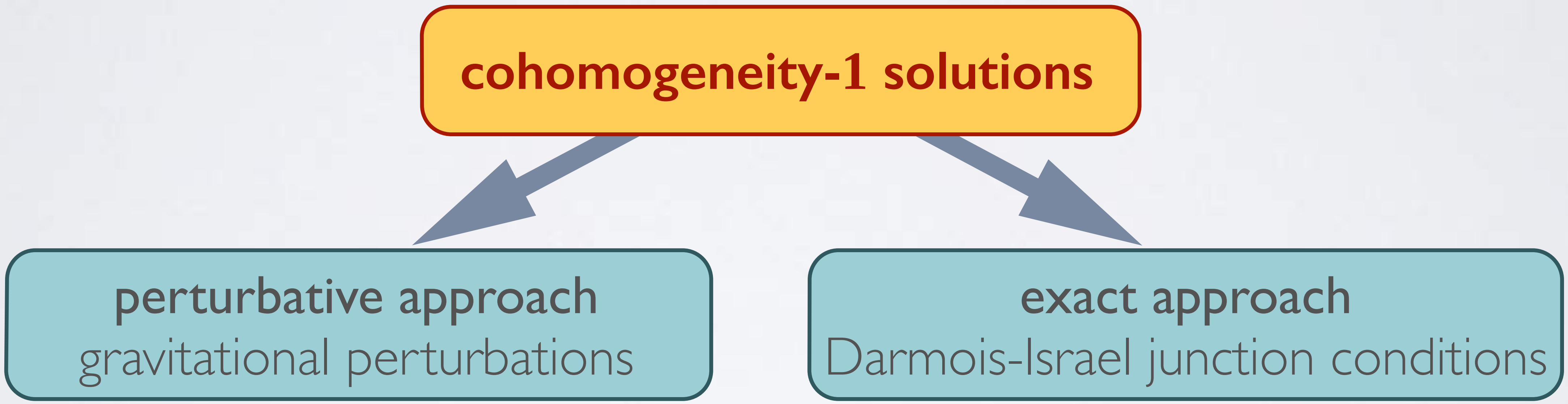

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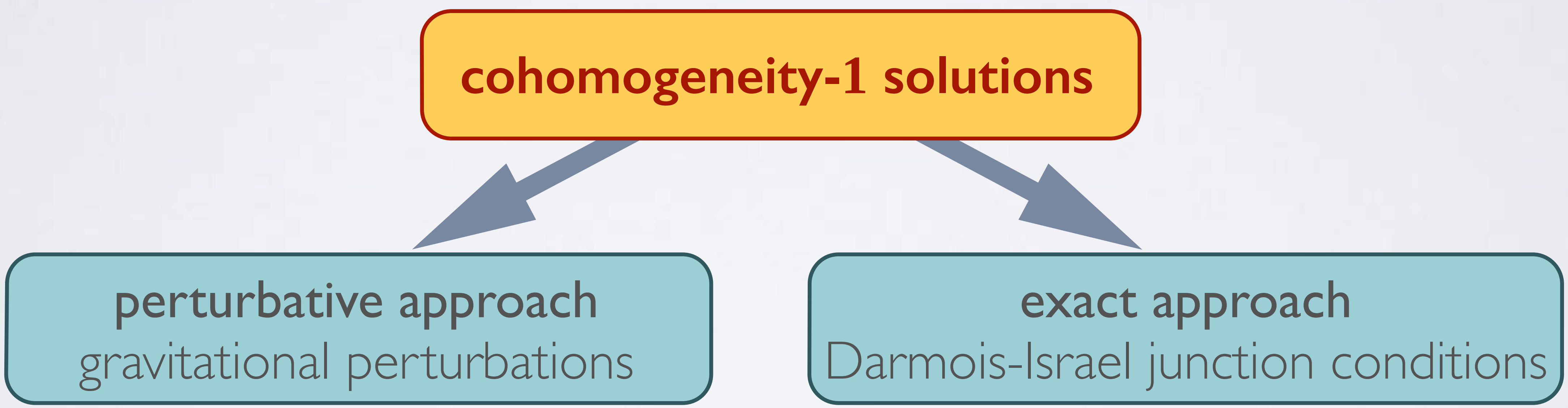

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- The price to pay for the convenience provided by cohomogeneity-1 spacetimes is the restriction to higher (odd) dimensions, $D = 2N+3$ with $N = 1, 2, 3, \ldots$

Background: Cohomogeneity-1 black holes

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- Constant t and r sections are squashed $(2N+1)$-spheres.

- S^{2N+1} can be written as a S^1 bundle over CP^N.

Background: Cohomogeneity-1 black holes

- The metric for these cohomogeneity-1 BHs is

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -f(r)^2 dt^2 + g(r)^2 dr^2 + r^2 \widehat{g}_{ab} dx^a dx^b + h(r)^2 \left[d\psi + A_a dx^a - \Omega(r) dt\right]^2$$

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where

$$g(r)^2 = \left(1 + \frac{r^2}{\ell^2} - \frac{2M\Xi}{r^{2N}} + \frac{2Ma^2}{r^{2N+2}}\right)^{-1}, \qquad f(r) = \frac{r}{g(r)h(r)},$$

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$\widehat{g}_{ab}$ denotes the Fubini-Study metric on CP^N and $A_a dx^a$ is its Kahler potential.

For N=1: $\widehat{g}_{ab}dx^a dx^b = \frac{1}{4}\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$, $\quad A = \frac{1}{2}\cos\theta\, d\phi$.

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- n.b. These solutions accommodate a non-vanishing cosmological constant:

$$R_{\mu\nu} = -(D-1)\ell^{-2} g_{\mu\nu}$$

Background: Thin shells in cohomogeneity-1 spacetimes

- The cohomogeneity-1 property makes an exact (thin-shell) calculation possible, 'gluing' an interior to an exterior geometry.

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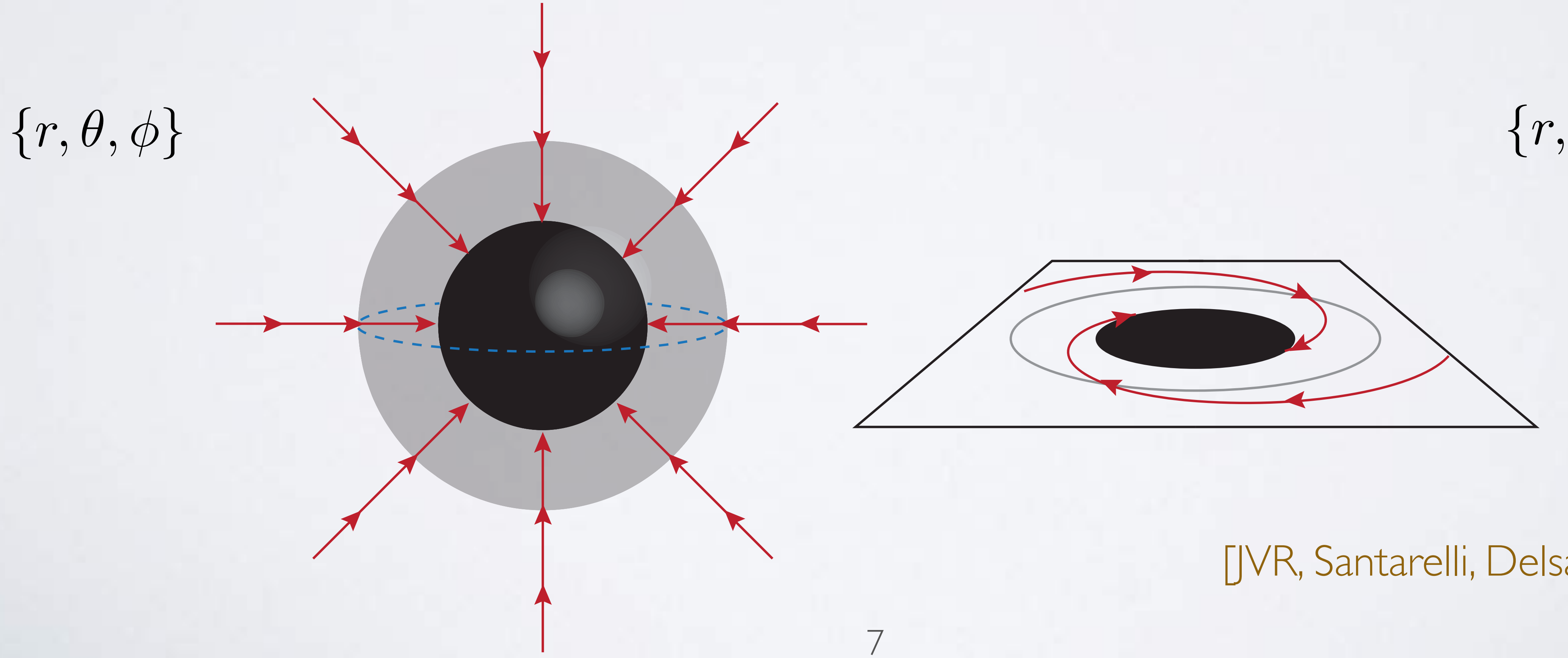
- n.b. The dynamics on the $CP^1 \cong S^2$ and on the S^1 separate.
All traces of the rotation show up in the $\{r, \psi\}$ plane.



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Rotating thin shells: Junction conditions

- Use junction conditions along a timelike hypersurface, $t = \mathcal{T}(\tau), r = \mathcal{R}(\tau)$:

$$\mathfrak{g}_{ij}^{(+)} = \mathfrak{g}_{ij}^{(-)} \equiv \mathfrak{g}_{ij} \,,$$

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energy density (ρ), pressure (P), intrinsic momentum / heat flow (φ), pressure anisotropy (ΔP)

Rotating thin shells: Equation of state and shell equation of motion

- The stress-energy tensor components are dictated by the metric components:

$$\rho = -\frac{(\beta_+ - \beta_-)(\mathcal{R}^2 h)'}{8\pi\mathcal{R}^3}$$

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$$\dot{\mathcal{R}}^2 + V_{\text{eff}}(\mathcal{R}) = 0$$

Rotating thin shells: Effective potential

- For generic values of N, and a linear equation of state:

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$$V_{\rm eff} \approx 1 + \frac{\mathcal{R}^2}{\ell^2} - \left(\frac{\Delta M}{m_0}\right)^2 \left(\frac{\mathcal{R}^2}{m_0}\right)^{3w} - \frac{1}{4}\left(\frac{m_0}{\mathcal{R}^2}\right)^{2+3w}$$

- For N=1 and small values of $\mathcal{R}$:

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Rotating thin shells: Stationary shell around a BH in AdS

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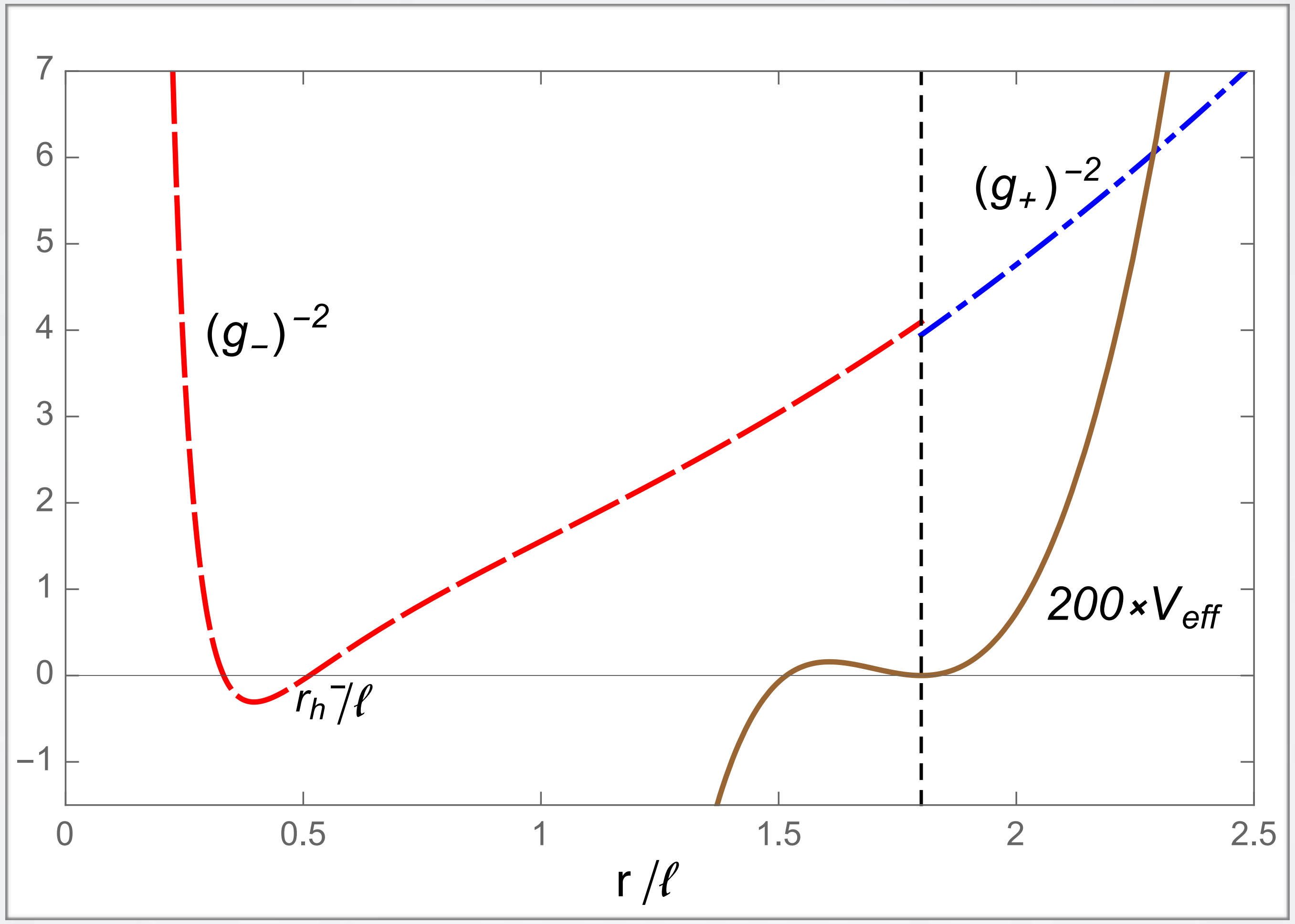
$m_0/\ell^2 = 0.324,\ w = 0.285,\ Ma^2/\ell^4 = 0.02,\ \mathcal{R}_*/\ell = 1.8$



[Delsate, JVR, Santarelli (2014)]

Rotating thin shells: Full collapse in asymptotically flat spacetime

- Take asymptotically flat limit, $\ell \to \infty$.
- Collapse starting from rest at infinity imposes:
  - $w = 0$ — i.e., matter on the shell has EoS of dust
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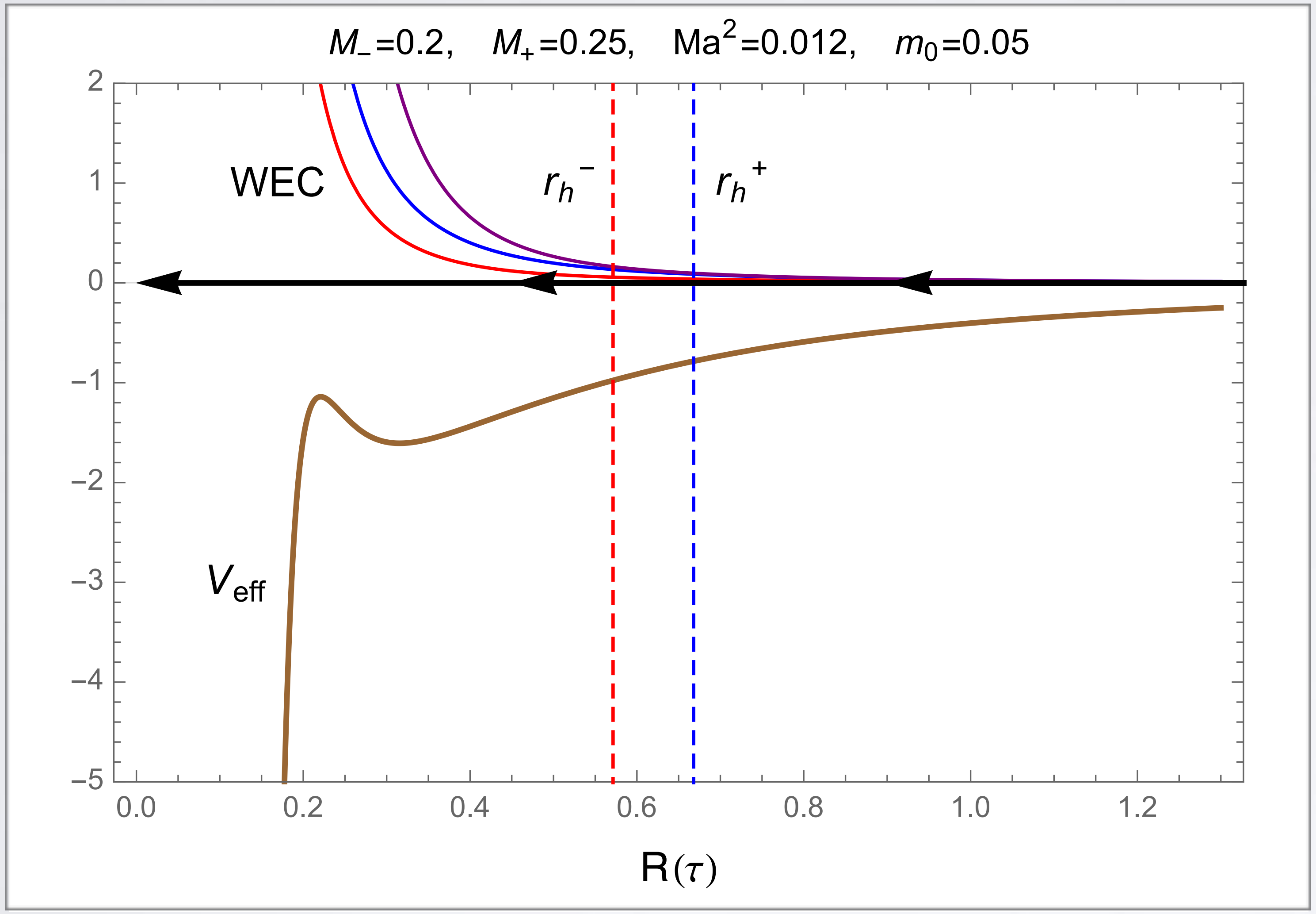

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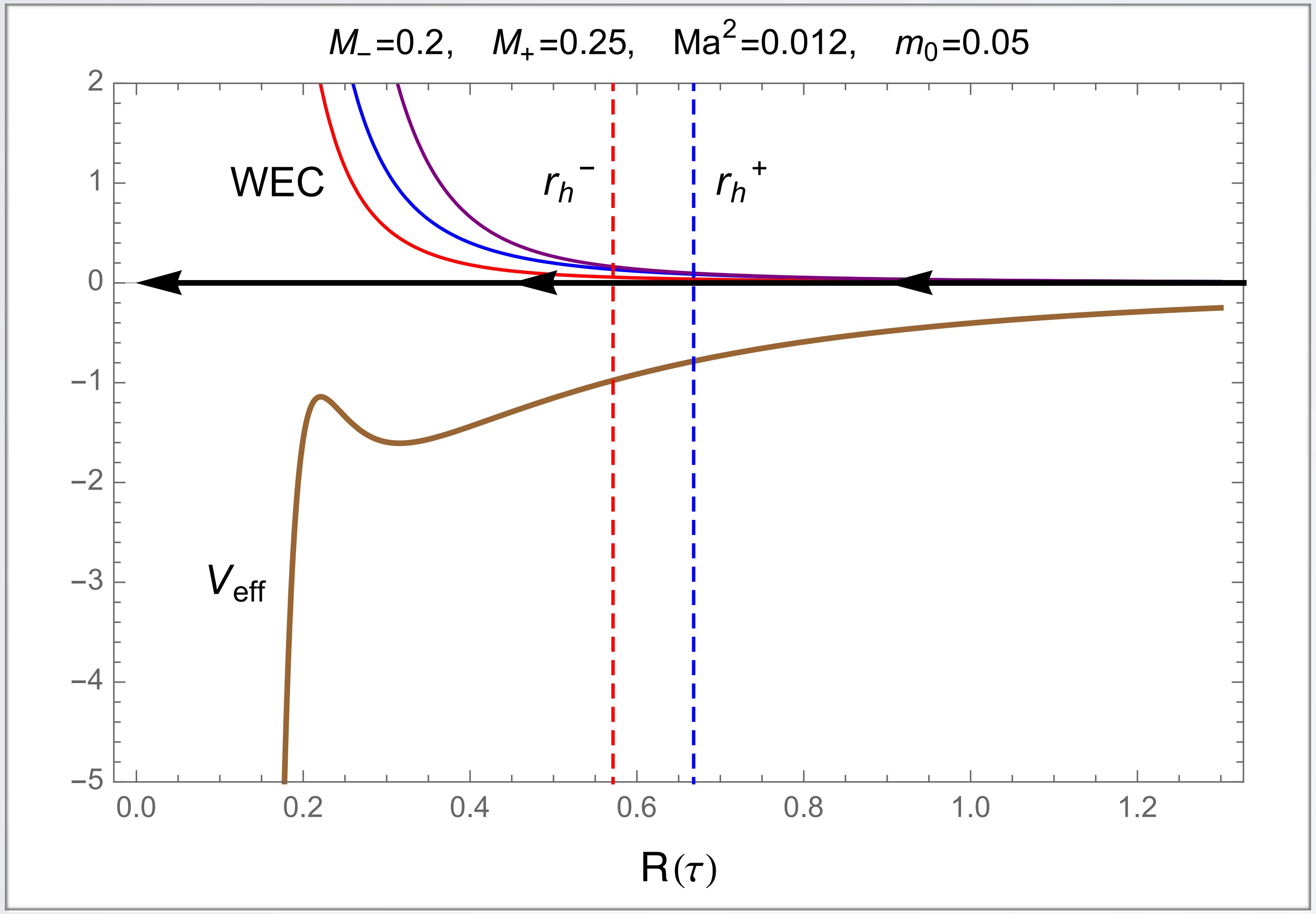

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Weak energy conditions (WEC) are satisfied

Rotating thin shells: Cosmic censorship

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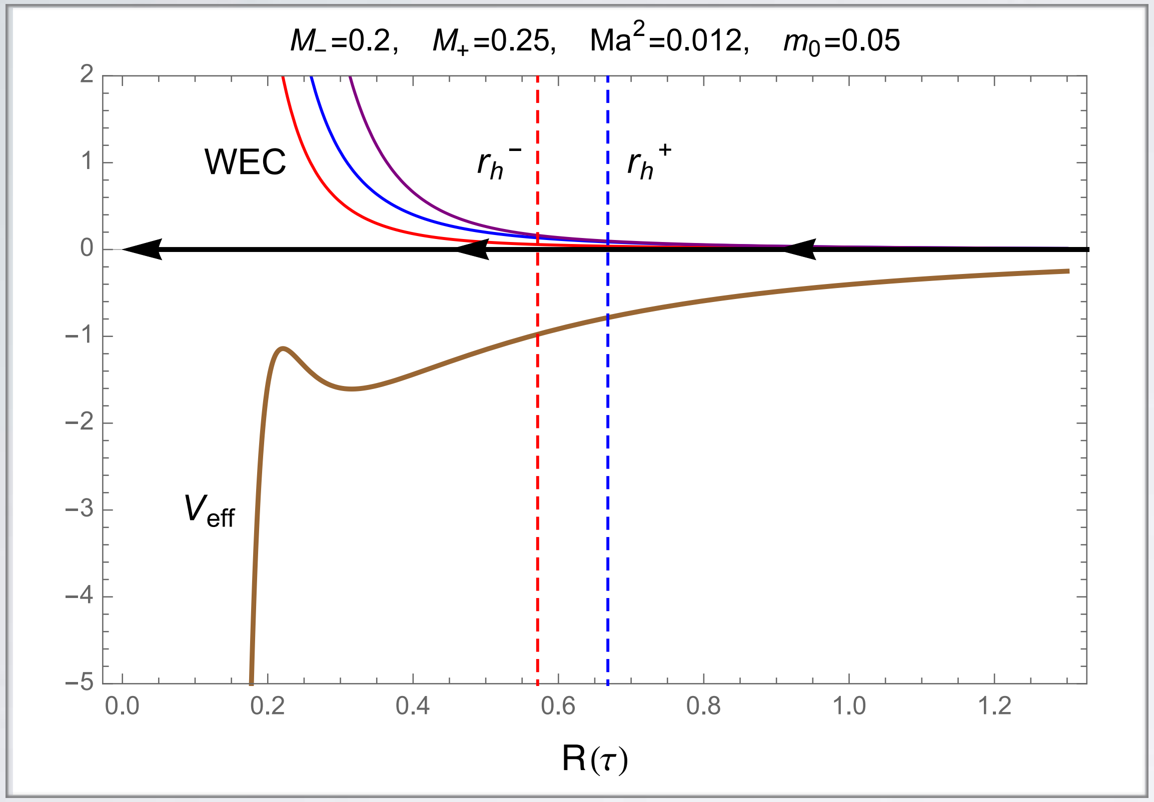
CCC is preserved

Rotating thin shells: Diverse scenarios



Full collapse

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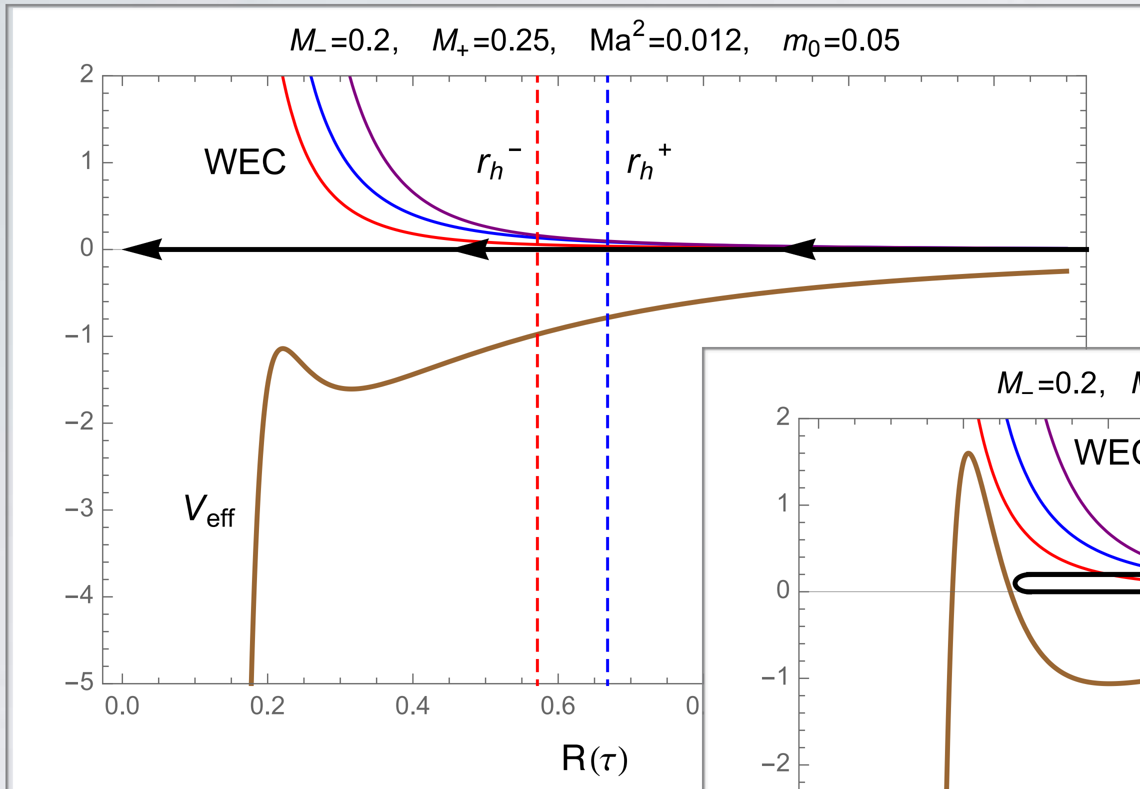

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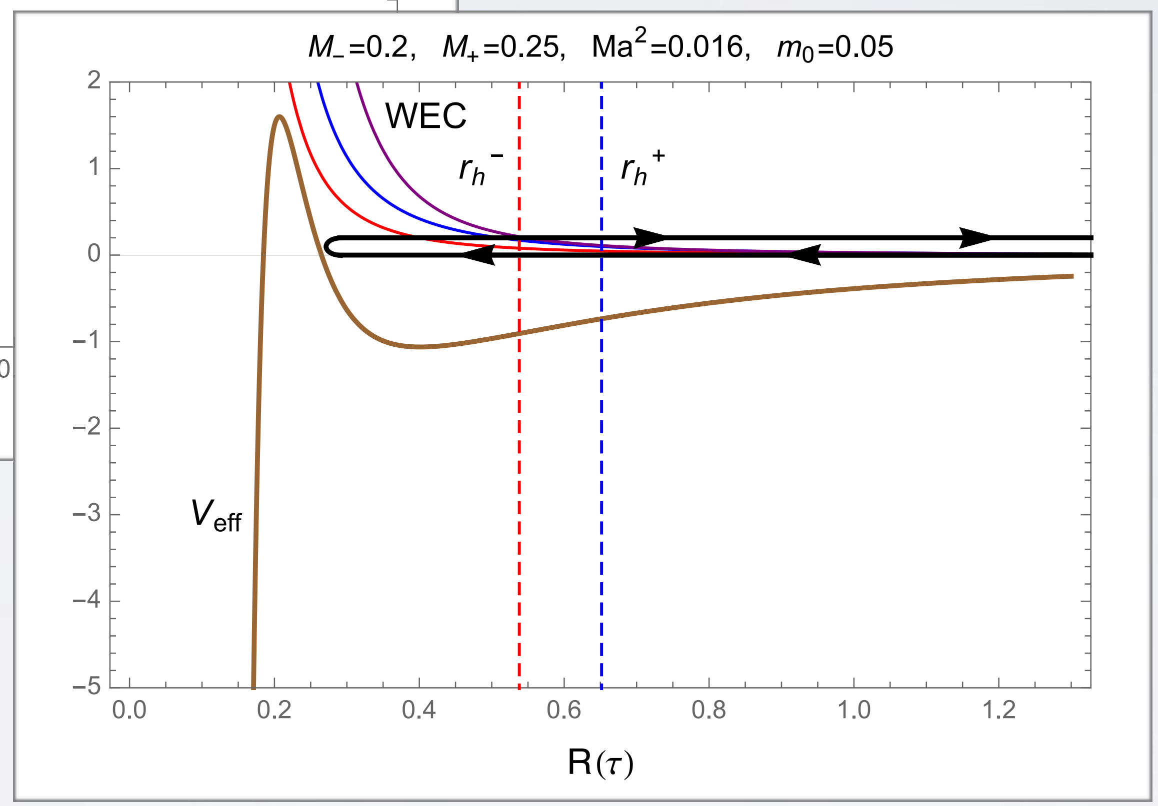

Bounce

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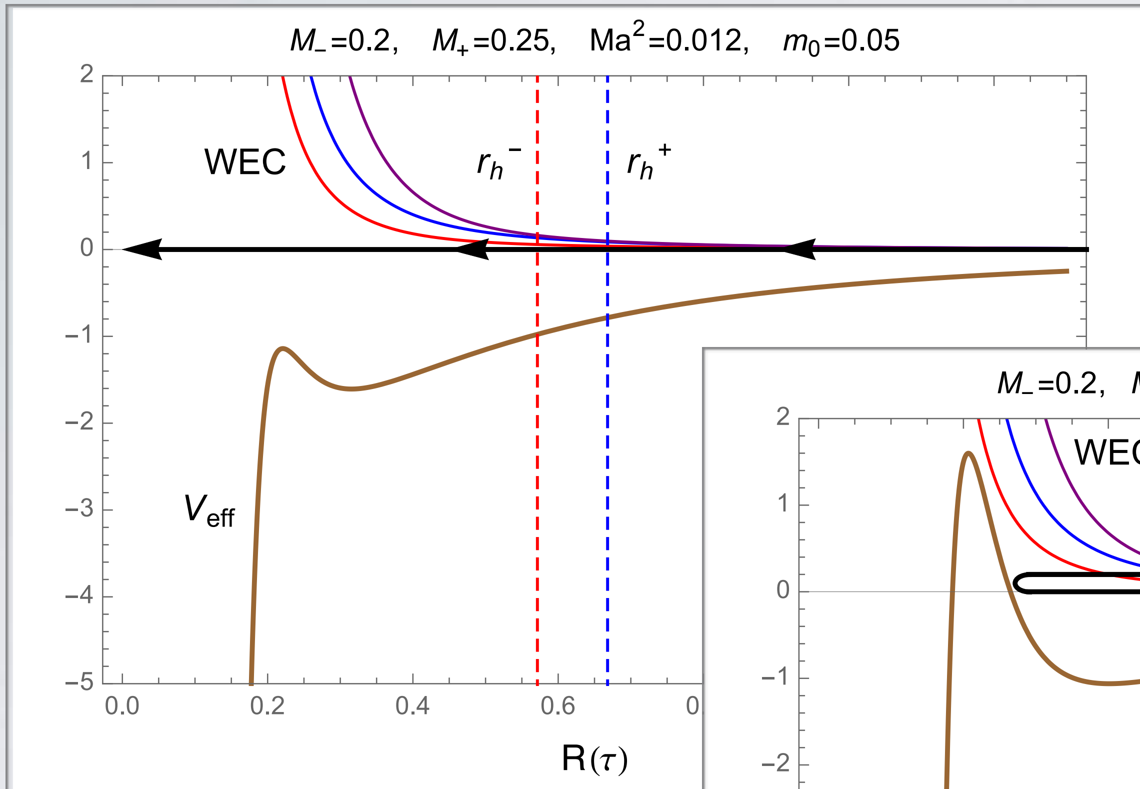

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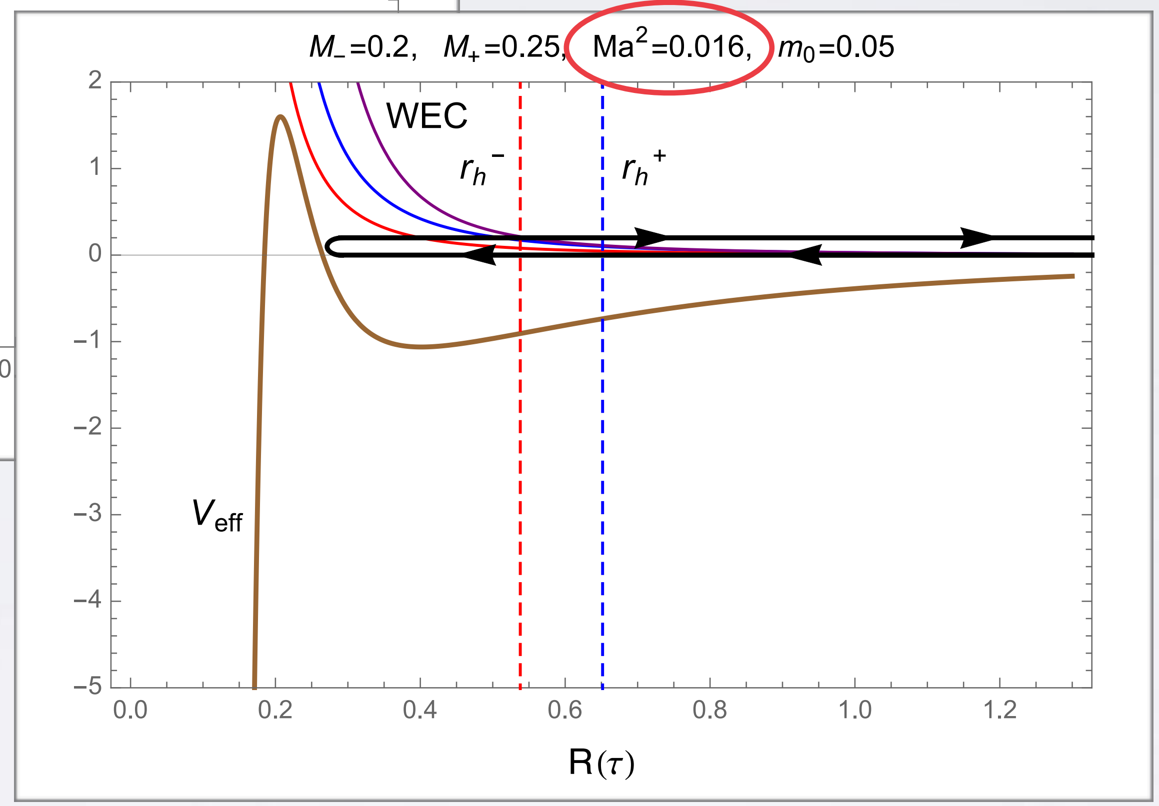

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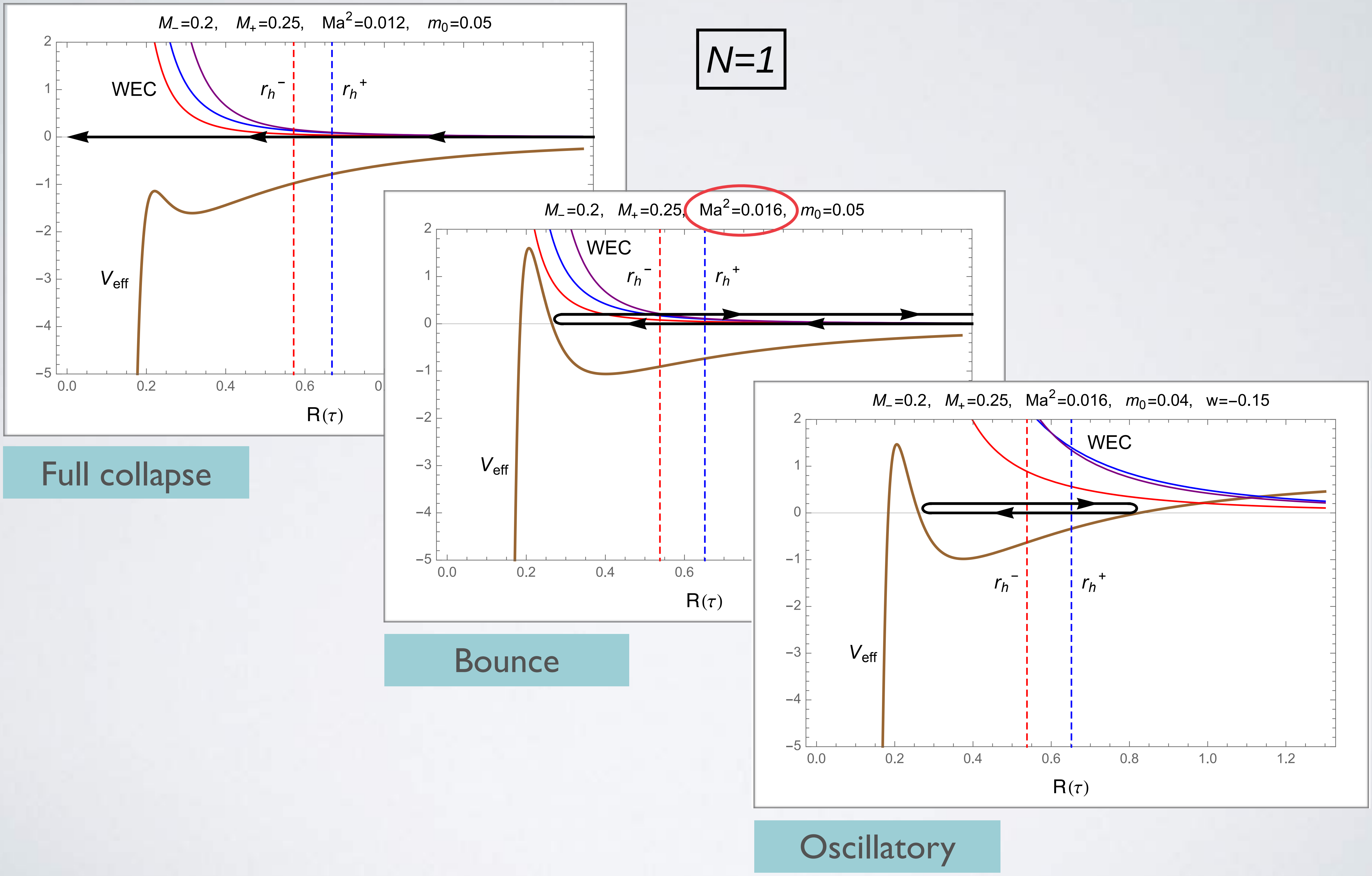

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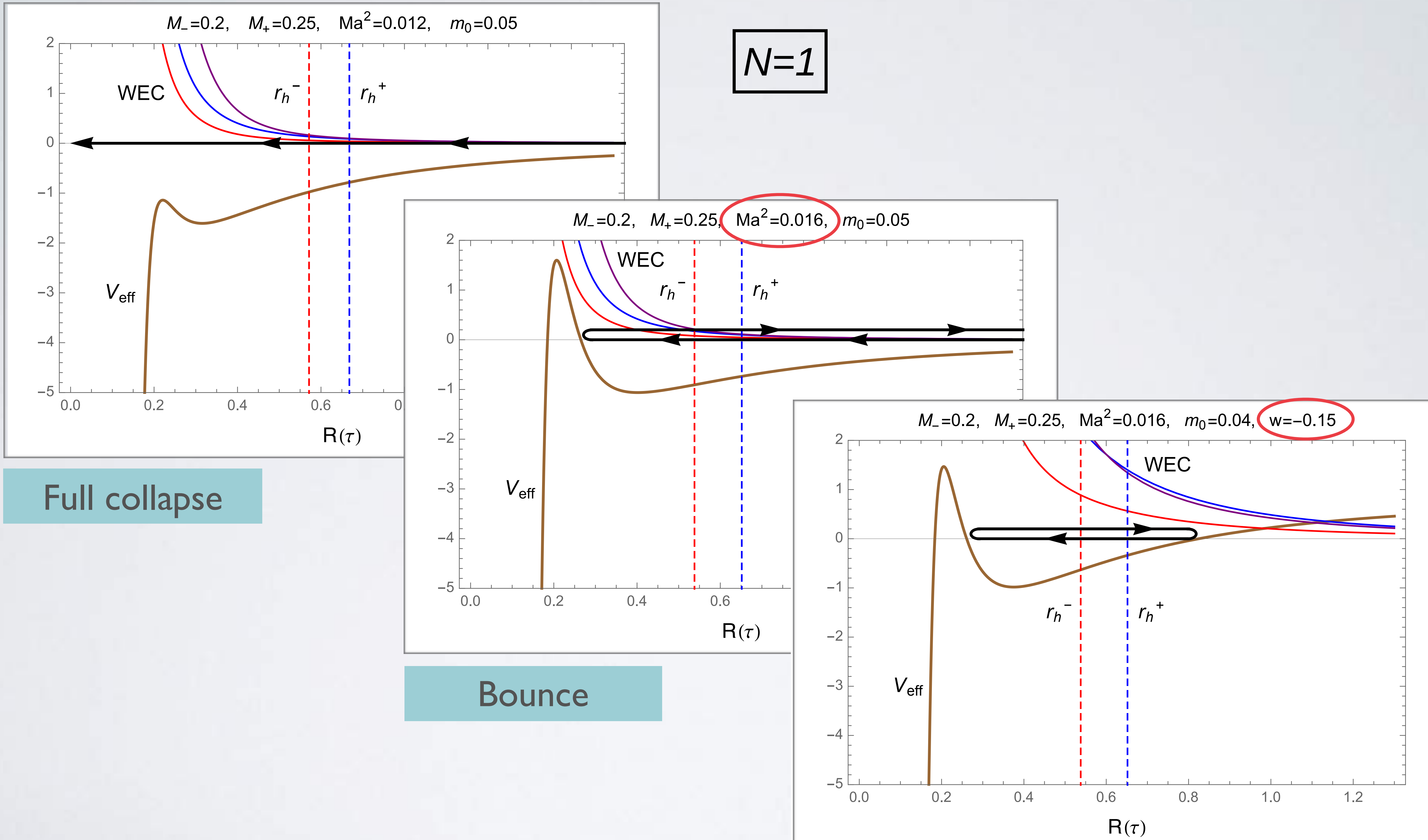

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Rotating thin shells: Scanning parameter space

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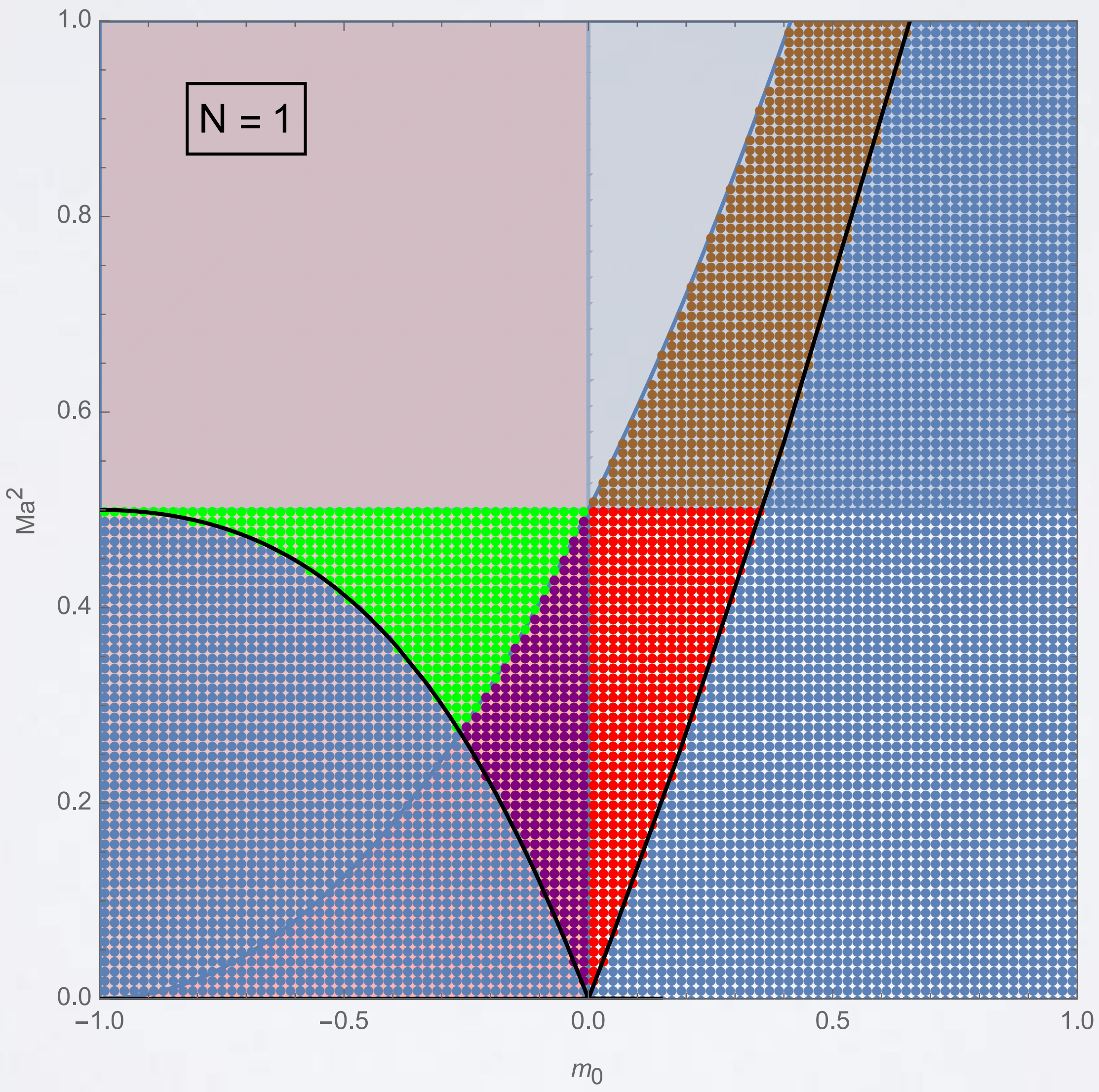

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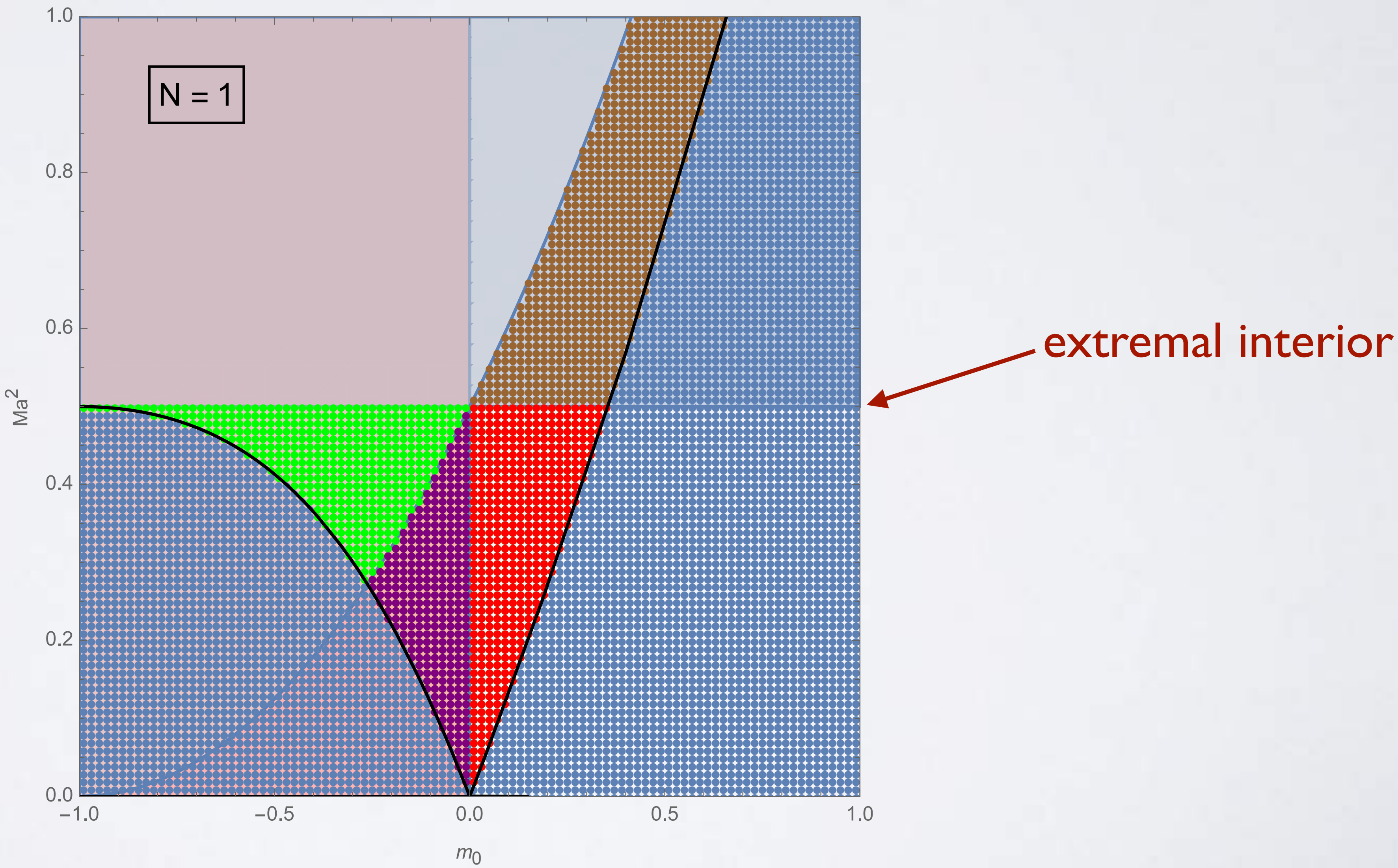

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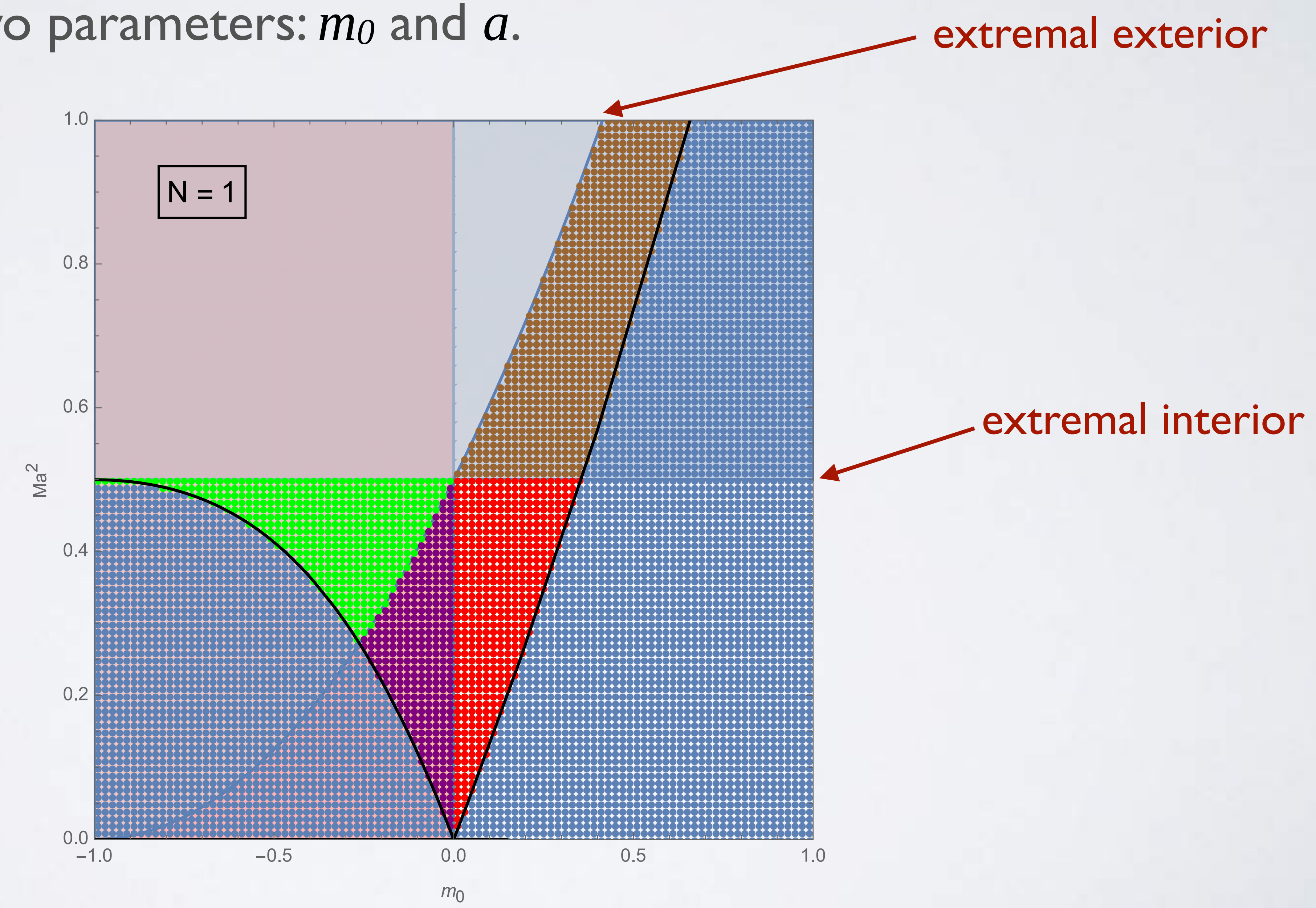

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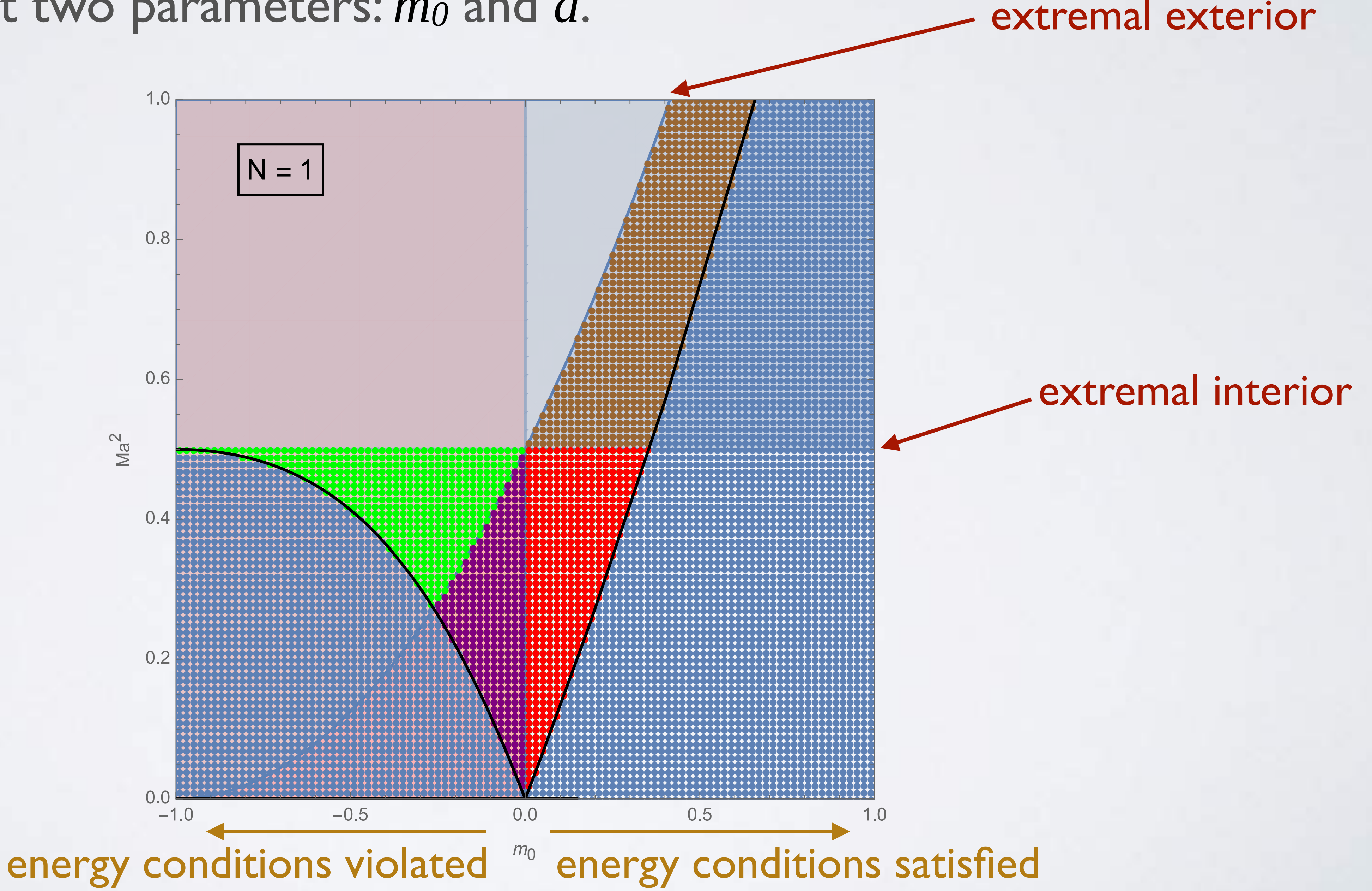

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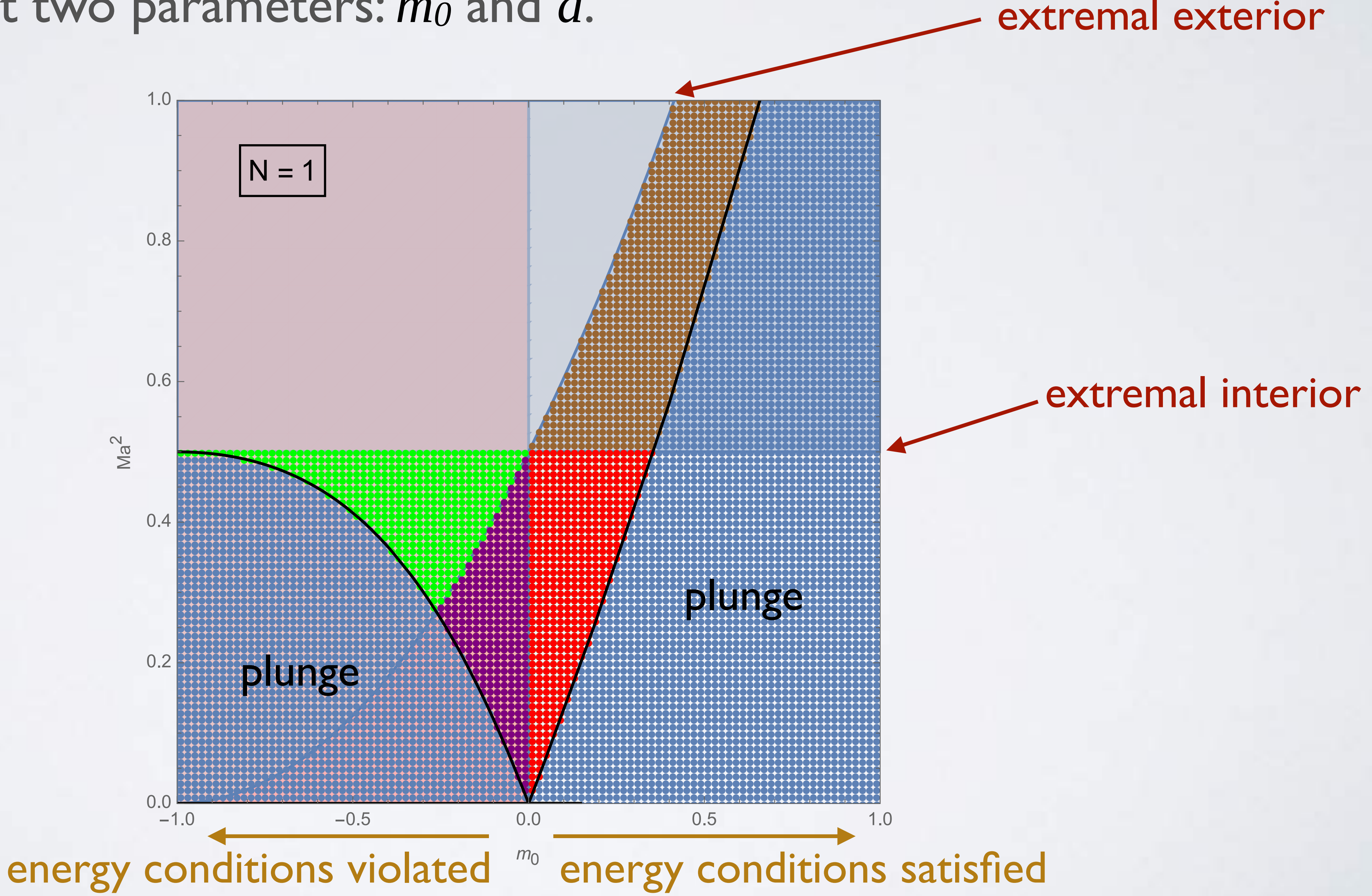

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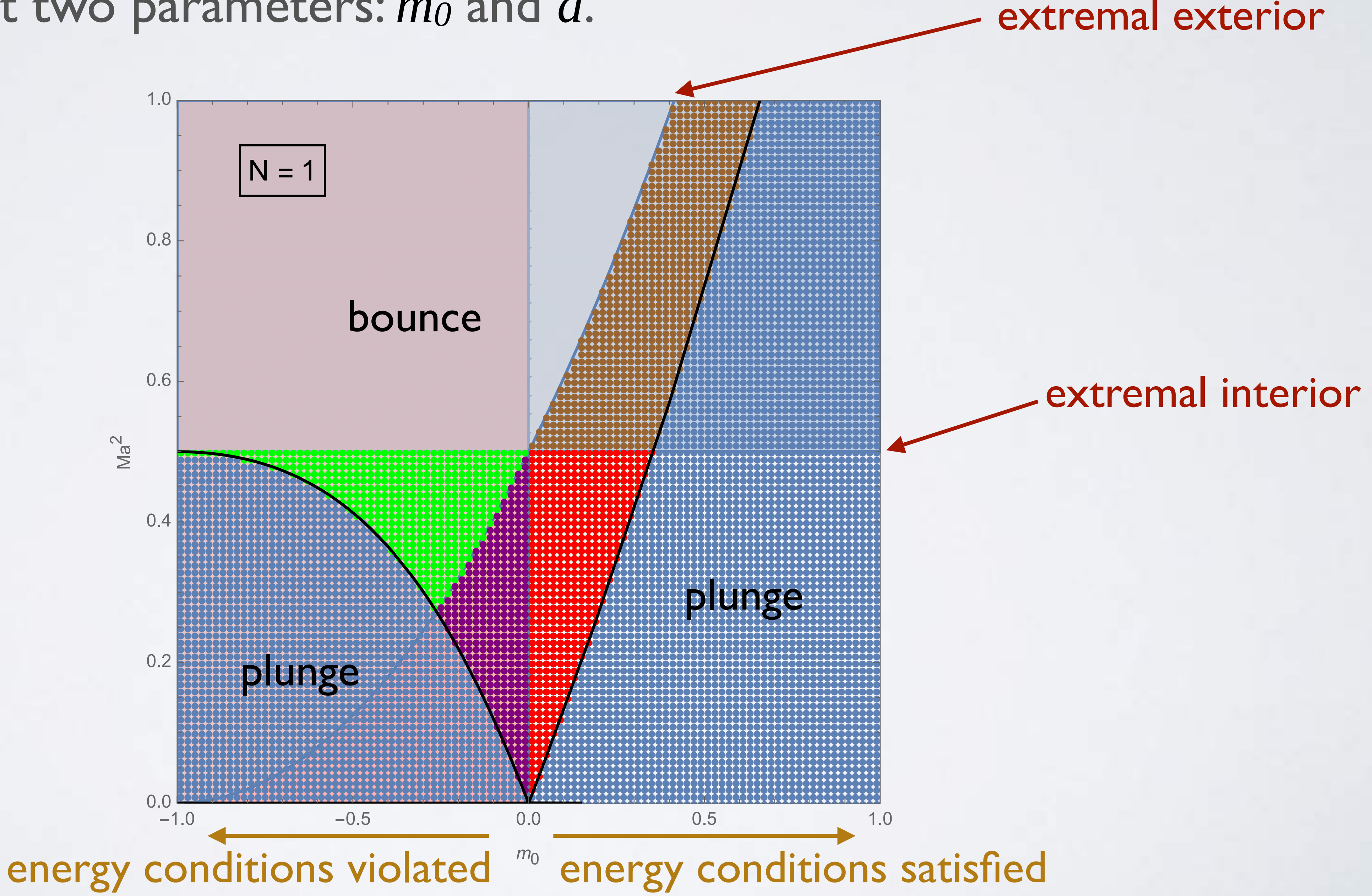

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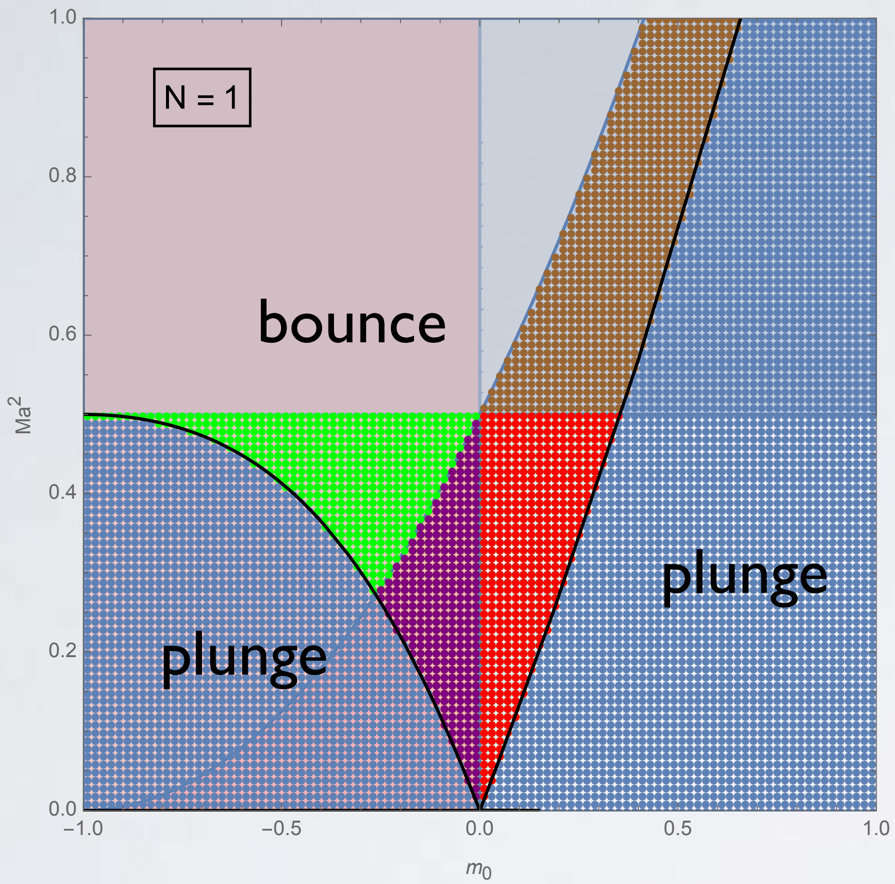

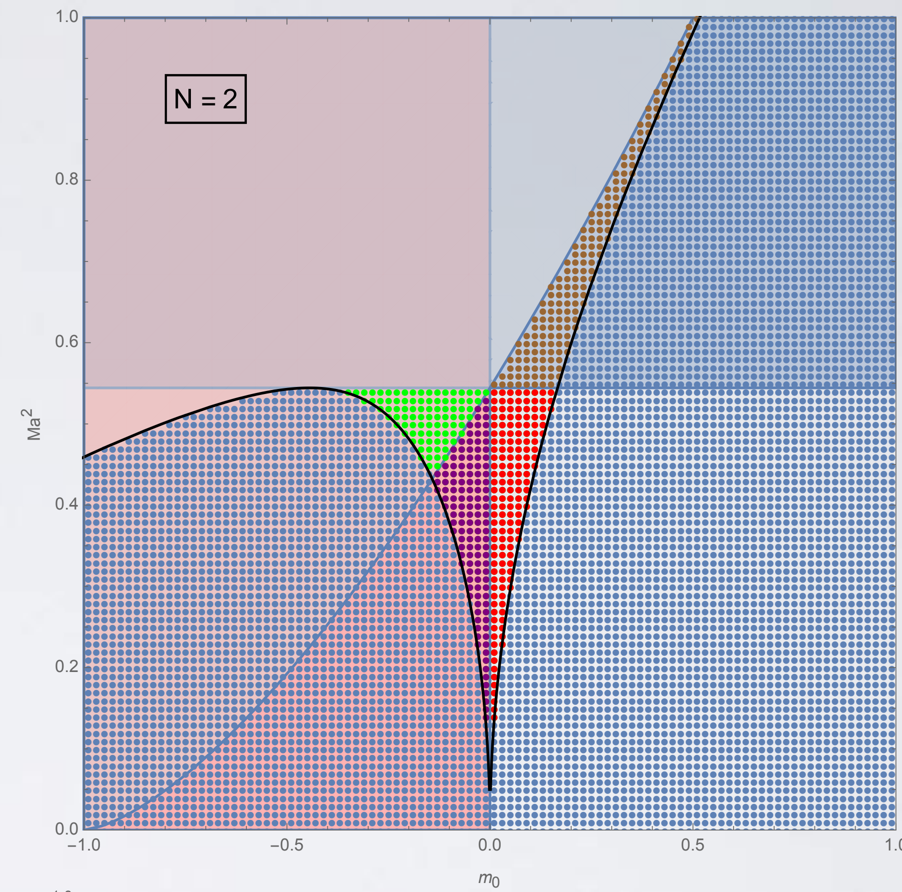

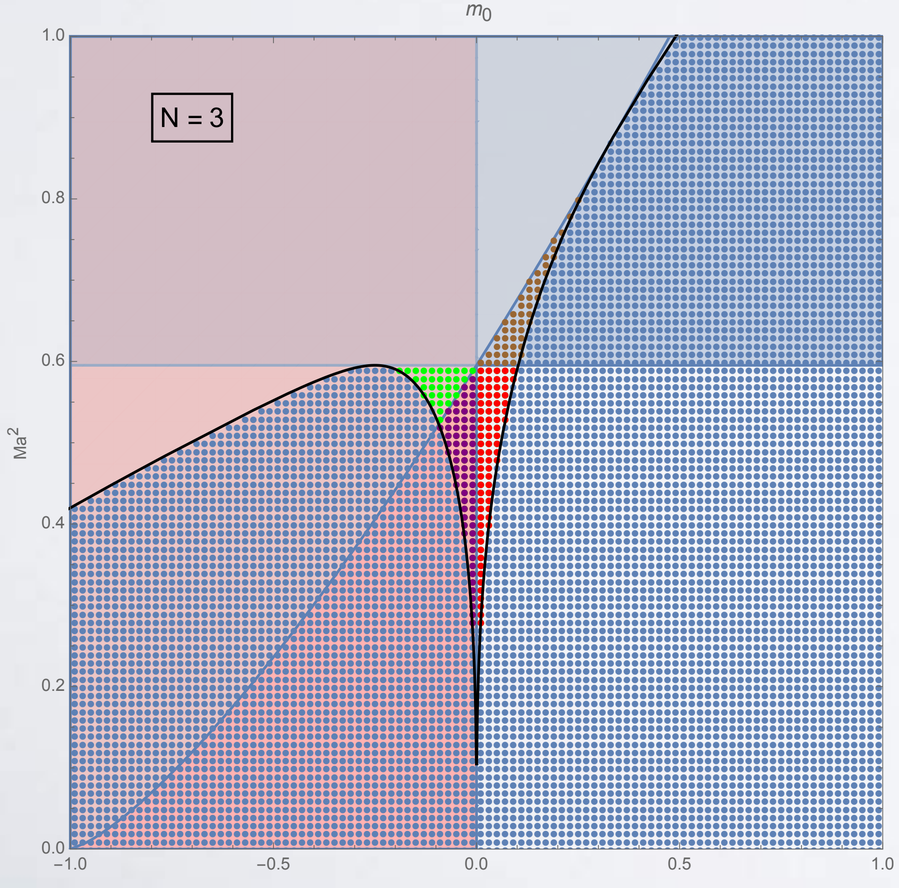

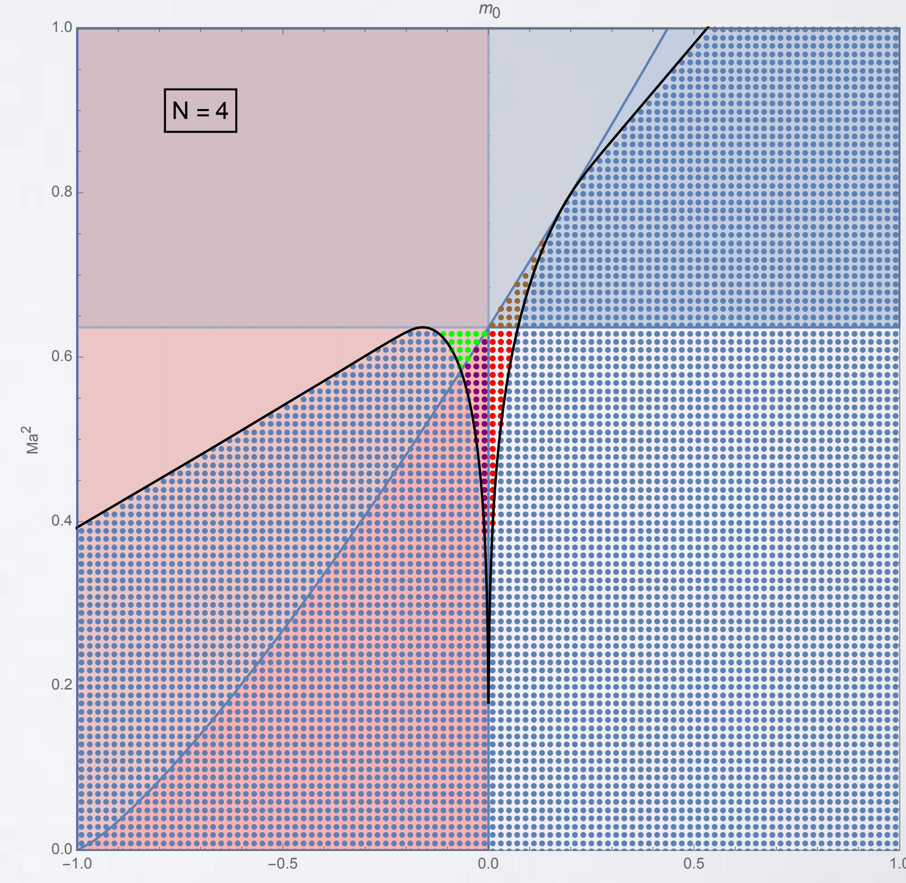

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